



DC Machines Solved

Electrical And Electronics Engineering (University of Zambia)

CHAPTER 25 D.C. MACHINES

Exercise 157, Page 446

1. A 4-pole, wave-connected armature of a d.c. machine has 750 conductors and is driven at 720 rev/min. If the useful flux per pole is 15 mWb, determine the generated e.m.f.

$Z = 750$, $c = 2$ (for a wave winding), $p = 2$ pairs, $n = \frac{720}{60}$ rev/s and $\Phi = 15 \times 10^{-3}$ Wb

$$\text{Generated e.m.f. } E = \frac{2p\Phi n Z}{c} = \frac{2(2)(15 \times 10^{-3}) \left(\frac{720}{60} \right) (750)}{2} = 270 \text{ volts}$$

2. A 6-pole generator has a lap-wound armature with 40 slots with 20 conductors per slot. The flux per pole is 25 mWb. Calculate the speed at which the machine must be driven to generate an e.m.f. of 300 V.

$p = 6/2 = 3$, lap means $c = 2p$, $Z = 40 \times 20 = 800$, $\Phi = 25 \times 10^{-3}$ Wb, $E = 300$ V

Generated e.m.f., $E = \frac{2p\Phi n Z}{c}$ from which,

$$\text{speed, } n = \frac{E c}{2p\Phi Z} = \frac{(300) 2p}{2p(25 \times 10^{-3})(800)} = \frac{300}{(25 \times 10^{-3})(800)} = 15 \text{ rev/s or } 900 \text{ rev/min}$$

3. A 4-pole armature of a d.c. machine has 1000 conductors and a flux per pole of 20 mWb. Determine the e.m.f. generated when running at 600 rev/min, when the armature is (a) wave-wound, (b) lap-wound.

$p = 4/2 = 2$, $Z = 1000$, $\Phi = 20 \times 10^{-3}$ Wb, $n = 600/60 = 10$ rev/s

$$\begin{aligned} \text{(a) For wave wound, } c = 2, \text{ hence, generated e.m.f., } E &= \frac{2p\Phi n Z}{c} = \frac{2(2)(20 \times 10^{-3})(10)(1000)}{2} \\ &= 400 \text{ V} \end{aligned}$$

(b) For **lap wound**, $c = 2p$, hence, **generated e.m.f.**, $E = \frac{2p\Phi n Z}{c} = \Phi n Z = (20 \times 10^{-3})(10)(1000)$
 $= 200 \text{ V}$

4. A d.c. generator running at 25 rev/s generates an e.m.f. of 150 V. Determine the percentage increase in the flux per pole required to generate 180 V at 20 rev/s.

Generated e.m.f, $E \propto \Phi \omega$ and since $\omega = 2\pi n$, then $E \propto \Phi n$

Let $E_1 = 150 \text{ V}$, $n_1 = 25 \text{ rev/s}$ and flux per pole at this speed be Φ_1

Let $E_2 = 180 \text{ V}$, $n_2 = 20 \text{ rev/s}$ and flux per pole at this speed be Φ_2

Since $E \propto \Phi n$ then $\frac{E_1}{E_2} = \frac{\Phi_1 n_1}{\Phi_2 n_2}$ i.e. $\frac{150}{180} = \frac{\Phi_1 (25)}{\Phi_2 (20)}$

$$\Phi_2 = \frac{\Phi_1 (25)(180)}{(20)(150)} = 1.5\Phi_1$$

from which,

Hence, **the flux has increased by 50%**

Exercise 158, Page 450

1. Determine the terminal voltage of a generator which develops an e.m.f. of 240 V and has an armature current of 50 A on load. Assume the armature resistance is $40\text{ m}\Omega$.

Terminal voltage, $V = E - I_a R_a = 240 - (50)(40 \times 10^{-3})$
 $= 240 - 2 = 238\text{ volts}$

2. A generator is connected to a $50\text{ }\Omega$ load and a current of 10 A flows. If the armature resistance is $0.5\text{ }\Omega$, determine (a) the terminal voltage, and (b) the generated e.m.f.

(a) **Terminal voltage, $V = I_a R_L = (10)(50) = 500\text{ V}$**

(b) **Generated e.m.f., $E = V + I_a R_a = 500 + (10)(0.5) = 505\text{ V}$**

3. A separately excited generator develops a no-load e.m.f. of 180 V at an armature speed of 15 rev/s and a flux per pole of 0.20 Wb. Calculate the generated e.m.f. when
- (a) the speed increases to 20 rev/s and the flux per pole remaining unchanged,
- (b) the speed remains at 15 rev/s and the pole flux is decreased to 0.125 Wb, and
- (c) the speed increases to 25 rev/s and the pole flux is decreased to 0.18 Wb.

(a) Since $E \propto \Phi n$ then $\frac{E_1}{E_2} = \frac{\Phi_1 n_1}{\Phi_2 n_2}$ i.e. $\frac{180}{E_2} = \frac{(0.2)(15)}{(0.2)(20)}$

from which, **generated e.m.f., $E_2 = \frac{(180)(20)}{(15)} = 240\text{ V}$**

(b) $\frac{E_1}{E_2} = \frac{\Phi_1 n_1}{\Phi_2 n_2}$ i.e. $\frac{180}{E_2} = \frac{(0.2)(15)}{(0.125)(15)}$

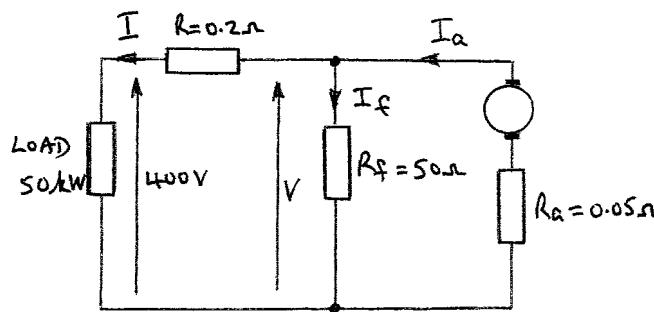
from which, **generated e.m.f., $E_2 = \frac{(180)(0.125)}{(0.20)} = 112.5\text{ V}$**

$$(c) \frac{E_1}{E_2} = \frac{\Phi_1 n_1}{\Phi_2 n_2} \quad \text{i.e.} \quad \frac{180}{E_2} = \frac{(0.20)(15)}{(0.18)(25)}$$

from which, **generated e.m.f.,** $E_2 = \frac{(180)(0.18)(25)}{(0.20)(15)} = 270 \text{ V}$

4. A shunt generator supplies a 50 kW load at 400 V through cables of resistance 0.2 Ω. If the field winding resistance is 50 Ω and the armature resistance is 0.05 Ω, determine (a) the terminal voltage, (b) the e.m.f. generated in the armature.

The circuit is shown below.



(a) Load current, $I = \frac{P}{V} = \frac{50000}{400} = 125 \text{ A}$

Volt drop in cable to load = $I R = (125)(0.2) = 25 \text{ V}$

Hence, **terminal voltage, V** = $400 + 25 = 425 \text{ V}$

(b) Armature current, $I_a = I_f + I$ where field current, $I_f = \frac{V}{R_f} = \frac{425}{50} = 8.5 \text{ A}$

Hence, $I_a = I_f + I = 8.5 + 125 = 133.5 \text{ A}$

and **generated e.m.f., E** = $V + I_a R_a = 425 + (133.5)(0.05)$
 $= 425 + 6.675 = 431.68 \text{ V}$

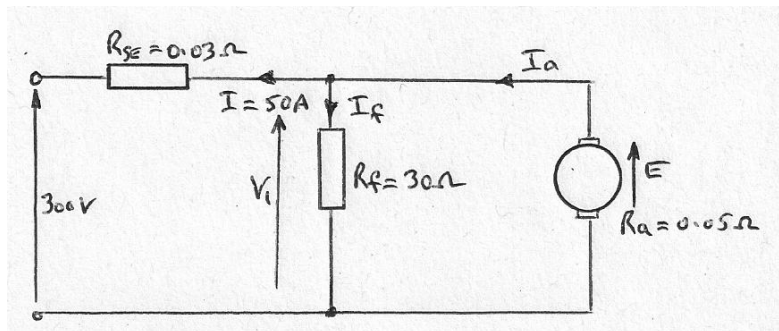
5. A short-shunt compound generator supplies 50 A at 300 V. If the field resistance is 30 Ω, the series resistance 0.03 Ω and the armature resistance 0.05 Ω, determine the e.m.f. generated.

The circuit is shown below.

$$\text{Volt drop in series winding} = I R_{se} = (50)(0.03) = 1.5 \text{ V}$$

P.d. across the field winding = p.d. across armature

$$= V_1 = 300 + 1.5 = 301.5 \text{ V}$$



$$\text{Field current, } I_f = \frac{V_1}{R_f} = \frac{301.5}{30} = 10.05 \text{ A}$$

$$\text{Armature current, } I_a = I + I_f = 50 + 10.05 = 60.05 \text{ A}$$

$$\begin{aligned} \text{Generated e.m.f., } E &= V_1 + I_a R_a \\ &= 301.5 + (60.05)(0.05) \\ &= 301.5 + 3.00 = \mathbf{304.5 \text{ volts}} \end{aligned}$$

6. A d.c. generator has a generated e.m.f. of 210 V when running at 700 rev/min and the flux per pole is 120 mWb. Determine the generated e.m.f. (a) at 1050 rev/min, assuming the flux remains constant, (b) if the flux is reduced by one-sixth at constant speed, and (c) at a speed of 1155 rev/min and a flux of 132 mWb.

$$\begin{aligned} \text{(a) Since } E \propto \Phi n \text{ then } \frac{E_1}{E_2} &= \frac{\Phi_1 n_1}{\Phi_2 n_2} \quad \text{i.e.} \quad \frac{210}{E_2} = \frac{(120 \times 10^{-3}) \left(\frac{700}{60} \right)}{(120 \times 10^{-3}) \left(\frac{1050}{60} \right)} \\ \text{from which,} \quad \text{generated e.m.f.,} \quad E_2 &= \frac{(210)(1050)}{(700)} = \mathbf{315 \text{ V}} \end{aligned}$$

$$\begin{aligned} \text{(b) } \frac{E_1}{E_2} &= \frac{\Phi_1 n_1}{\Phi_2 n_2} \quad \text{i.e.} \quad \frac{210}{E_2} = \frac{(120 \times 10^{-3}) \left(\frac{700}{60} \right)}{\left(\frac{5}{6} \times 120 \times 10^{-3} \right) \left(\frac{700}{60} \right)} \end{aligned}$$

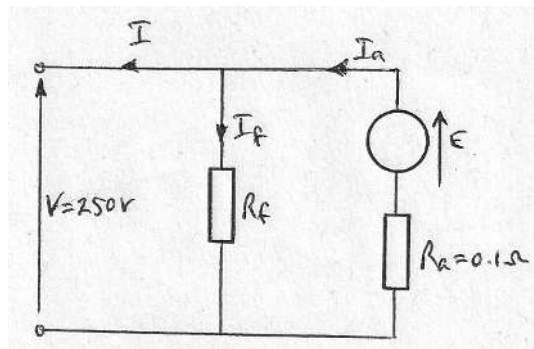
from which, **generated e.m.f.**,
$$E_2 = \frac{(210) \left(\frac{5}{6} \right)}{1} = 175 \text{ V}$$

(c)
$$\frac{E_1}{E_2} = \frac{\Phi_1 n_1}{\Phi_2 n_2} \quad \text{i.e.} \quad \frac{210}{E_2} = \frac{(120 \times 10^{-3}) \left(\frac{700}{60} \right)}{(132 \times 10^{-3}) \left(\frac{1155}{60} \right)}$$

from which, **generated e.m.f.**,
$$E_2 = \frac{(210)(132)(1155)}{(120)(700)} = 381.2 \text{ V}$$

7. A 250 V d.c. shunt-wound generator has an armature resistance of 0.1Ω . Determine the generated e.m.f. when the generator is supplying 50 kW, neglecting the field current of the generator.

The circuit is shown below.



Load current,
$$I = \frac{P}{V} = \frac{50000}{250} = 200 \text{ A}$$

If the field current is neglected, armature current, $I_a = 200 \text{ A}$ and terminal voltage, $V = 250 \text{ V}$

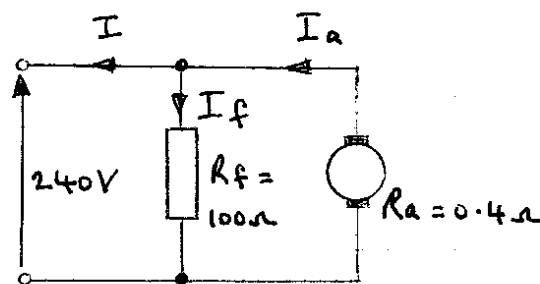
Hence, **generated e.m.f.**,
$$E = V + I_a R_a = 250 + (200)(0.1)$$

$$= 250 + 20 = 270 \text{ V}$$

Exercise 159, Page 452

1. A 15 kW shunt generator having an armature circuit resistance of $0.4\ \Omega$ and a field resistance of $100\ \Omega$, generates a terminal voltage of 240 V at full load. Determine the efficiency of the generator at full load, assuming the iron, friction and windage losses amount to 1 kW.

The circuit diagram is shown below.



Output power = 15000 W = $V I$

$$\text{from which, current, } I = \frac{15000}{V} = \frac{15000}{240} = 62.5 \text{ A}$$

$$\text{Field current, } I_f = \frac{V}{R_f} = \frac{240}{100} = 2.4 \text{ A}$$

$$\text{Armature current, } I_a = I_f + I = 2.4 + 62.5 = 64.9 \text{ A}$$

$$\begin{aligned} \text{Efficiency, } \eta &= \left(\frac{VI}{VI + I_a^2 R_a + I_f V + C} \right) \times 100\% = \left(\frac{15000}{15000 + (64.9)^2 (0.4) + (2.4)(240) + 1000} \right) \\ &= \left(\frac{15000}{18260.804} \right) \times 100\% = 82.14\% \end{aligned}$$

Exercise 160, Page 452

1. A d.c. motor operates from a 350 V supply. If the armature resistance is $0.4\ \Omega$ determine the back e.m.f. when the armature current is 60 A

For a motor, $V = E + I_a R_a$ hence, back e.m.f., $E = V - I_a R_a$

$$= 350 - (60)(0.4)$$

$$= 350 - 24 = \mathbf{326\ volts}$$

2. The armature of a d.c. machine has a resistance of $0.5\ \Omega$ and is connected to a 200 V supply. Calculate the e.m.f. generated when it is running (a) as a motor taking 50 A and (b) as a generator giving 70 A.

(a) **As a motor, generated e.m.f.** (or back e.m.f.), $E = V - I_a R_a = 200 - (50)(0.5)$

$$= 200 - 25 = \mathbf{175\ V}$$

(b) **As a generator, generated e.m.f.**, $E = V + I_a R_a = 200 + (70)(0.5) = 200 + 35 = \mathbf{235\ V}$

3. Determine the generated e.m.f. of a d.c. machine if the armature resistance is $0.1\ \Omega$ and it (a) is running as a motor connected to a 230 V supply, the armature current being 60 A, and (b) is running as a generator with a terminal voltage of 230 V, the armature current being 80 A.

(a) **As a motor, generated e.m.f.**, $E = V - I_a R_a = 230 - (60)(0.1)$

$$= 230 - 6 = \mathbf{224\ V}$$

(b) **As a generator, generated e.m.f.**, $E = V + I_a R_a = 230 + (80)(0.1) = 230 + 8 = \mathbf{238\ V}$

Exercise 161, Page 454

1. The shaft torque required to drive a d.c. generator is 18.7 Nm when it is running at 1250 rev/min. If its efficiency is 87% under these conditions and the armature current is 17.3 A, determine the voltage at the terminals of the generator

$$\text{Efficiency, } \eta = \frac{\text{output power}}{\text{input power}} = \frac{VI}{T(2\pi n)} \times 100\%$$

$$\text{i.e. } 87 = \frac{(V)(17.3)}{(18.7) \left(2\pi \times \frac{1250}{60} \right)} \times 100\%$$

$$\text{from which, terminal voltage, } V = \frac{(87)(18.7) \left(2\pi \times \frac{125}{60} \right)}{17.3 \times 100} = 123.1 \text{ V}$$

2. A 220 V, d.c. generator supplies a load of 37.5 A and runs at 1550 rev/min. Determine the shaft torque of the diesel motor driving the generator, if the generator efficiency is 78%.

$$\text{Efficiency, } \eta = \frac{VI}{T(2\pi n)} \times 100\% \quad \text{i.e. } 78 = \frac{(220)(37.5)}{T \left(2\pi \times \frac{1550}{60} \right)} \times 100\%$$

$$\text{from which, shaft torque, } T = \frac{(220)(37.5)}{(78) \left(2\pi \times \frac{1550}{60} \right)} \times 100 = 65.2 \text{ N m}$$

3. A 4-pole d.c. motor has a wave-wound armature with 800 conductors. The useful flux per pole is 20 mWb. Calculate the torque exerted when a current of 40 A flows in each armature conductor.

$$p = 2, c = 2 \text{ for a wave winding, } \Phi = 20 \times 10^{-3} \text{ Wb, } Z = 800 \text{ and } I_a = 40 \text{ A}$$

$$\text{From equation (7), torque, } T = \frac{p\Phi Z I_a}{\pi c}$$

$$= \frac{(2)(20 \times 10^{-3})(800)(40)}{\pi(2)} = 203.7 \text{ N m}$$

4. Calculate the torque developed by a 240 V d.c. motor whose armature current is 50 A, armature resistance is 0.6Ω and is running at 10 rev/s.

$$V = 240 \text{ V, } I_a = 50 \text{ A, } R_a = 0.6 \Omega \text{ and } n = 10 \text{ rev/s.}$$

$$\text{Back e.m.f., } E = V - I_a R_a = 240 - (50)(0.6) = 210 \text{ V}$$

$$\text{Torque, } T = \frac{E I_a}{2\pi n} = \frac{(240)(50)}{2\pi \times 10} = 167.1 \text{ N m}$$

5. An 8-pole lap-wound d.c. motor has a 200 V supply. The armature has 800 conductors and a resistance of 0.8Ω . If the useful flux per pole is 40 mWb and the armature current is 30 A, calculate (a) the speed, and (b) the torque developed.

$$V = 200 \text{ V, } Z = 800, R_a = 0.8 \Omega, \Phi = 40 \times 10^{-3} \text{ Wb, } I_a = 30 \text{ A and } c = 2p \text{ for a lap winding.}$$

$$(a) \text{ Back e.m.f., } E = V - I_a R_a = 200 - (30)(0.8) = 176 \text{ V}$$

$$\text{E.m.f., } E = \frac{2p\Phi n Z}{c} \quad \text{i.e.} \quad 176 = \frac{2p(40 \times 10^{-3})n(800)}{2p}$$

$$\text{from which, speed, } n = \frac{176}{(40 \times 10^{-3})(800)} = 5.5 \text{ rev/s or } 5.5 \times 60 = 330 \text{ rev/min}$$

$$(b) \text{ Torque, } T = \frac{E I_a}{2\pi n} = \frac{(176)(30)}{2\pi \times 5.5} = 152.8 \text{ N m}$$

6. A 150 V d.c. generator supplies a current of 25 A when running at 1200 rev/min. If the torque on the shaft driving the generator is 35.8 Nm, determine (a) the efficiency of the generator, and (b) the power loss in the generator.

$$(a) \text{ Efficiency of generator} = \frac{\text{output power}}{\text{input power}} \times 100\% = \frac{VI}{T(2\pi n)} \times 100\%$$

$$= \frac{(150)(25)}{(35.8) \left(2\pi \times \frac{1200}{60} \right)} \times 100\% = 83.4\%$$

$$(b) \quad \text{Input power} = VI + \text{losses}$$

$$\text{Hence, } T(2\pi n) = VI + \text{losses}$$

$$\text{i.e. } \text{losses} = T(2\pi n) - VI$$

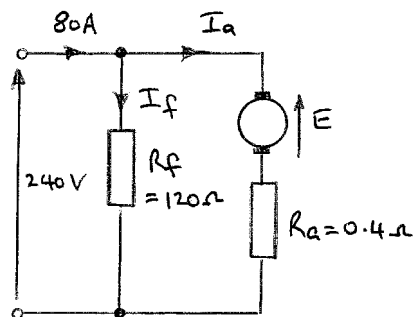
$$= (35.8) \left(2\pi \times \frac{1200}{60} \right) - (150)(25)$$

$$\text{i.e. } \text{power loss} = 4498.8 - 3750 = 748.8 \text{ W}$$

Exercise 162, Page 460

1. A 240 V shunt motor takes a current of 80 A. If the field winding resistance is $120\ \Omega$ and the armature resistance is $0.4\ \Omega$, determine (a) the current in the armature, and (b) the back e.m.f.

The circuit is shown below.



(a) Field current,
$$I_f = \frac{V}{R_f} = \frac{240}{120} = 2\text{ A}$$

Supply current, $I = I_a + I_f$ hence, **armature current**, $I_a = 80 - 2 = 78\text{ A}$

(b) **Back e.m.f.**,
$$E = V - I_a R_a = 240 - (78)(0.4)$$

$$= 240 - 31.2 = \mathbf{208.8\text{ V}}$$

2. A d.c. motor has a speed of 900 rev/min when connected to a 460 V supply. Find the approximate value of the speed of the motor when connected to a 200 V supply, assuming the flux decreases by 30% and neglecting the armature volt drop.

E.m.f, $E \propto \Phi n$ Hence, $460 \propto (\Phi)(900)$

Also, $200 \propto (0.7\Phi)(n_2)$

Thus,
$$\frac{460}{200} = \frac{900\Phi}{0.7\Phi n_2}$$

from which, **speed of motor**, $n_2 = \frac{900 \times 200}{460 \times 0.7} = 559 \text{ rev/min}$

3. A series motor having a series field resistance of 0.25Ω and an armature resistance of 0.15Ω is connected to a 220 V supply and at a particular load runs at 20 rev/s when drawing 20 A from the supply. Calculate the e.m.f. generated at this load. Determine also the speed of the motor when the load is changed such that the current increases to 25 A . Assume the flux increases by 25% .

Generated e.m.f. at initial load, $E_1 = V - I_a (R_a + R_f)$

$$= 220 - (20)(0.15 + 0.25) = 212 \text{ V}$$

When current is increased to 25 A , the generated e.m.f. is given by:

$$E_2 = V - I_a (R_a + R_f)$$

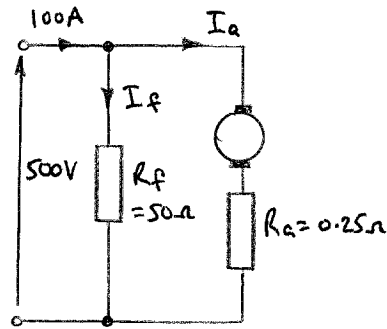
$$= 220 - (25)(0.15 + 0.25) = 210 \text{ V}$$

Now $\frac{E_1}{E_2} = \frac{\Phi_1 n_1}{\Phi_2 n_2}$ i.e. $\frac{212}{210} = \frac{\Phi_1 (20)}{(1.25\Phi_1)(n_2)}$ since flux increases by 25%

from which, **motor speed**, $n_2 = \frac{(20)(210)}{(212)(1.25)} = 15.85 \text{ rev/s or } 950.9 \text{ rev/min}$

4. A 500 V shunt motor takes a total current of 100 A and runs at 1200 rev/min . If the shunt field resistance is 50Ω , the armature resistance is 0.25Ω and the iron, friction and windage losses amount to 2 kW , determine the overall efficiency of the motor.

The circuit is shown below.



Field current, $I_f = \frac{V}{R_f} = \frac{500}{50} = 10 \text{ A}$

Armature current, $I_a = I - I_f = 100 - 10 = 90 \text{ A}$

Iron, friction and windage losses, $C = 2000 \text{ W}$

Efficiency,
$$\eta = \left(\frac{VI - I_a^2 R_a - I_f V - C}{VI} \right) \times 100\%$$

$$= \left(\frac{(500)(100) - (90^2)(0.25) - (10)(500) - 2000}{(500)(100)} \right) \times 100\%$$

$$= \left(\frac{50000 - 2025 - 5000 - 2000}{50000} \right) \times 100\% = \frac{40975}{50000} \times 100\% = 81.95\%$$

5. A 250 V, series-wound motor is running at 500 rev/min and its shaft torque is 130 Nm. If its efficiency at this load is 88%, find the current taken from the supply.

The efficiency of a motor = $\frac{\text{output power}}{\text{input power}} \times 100\%$

The output power of a motor is the power available to do work at its shaft and is given by $T \omega$ or $T(2\pi n)$ watts, where T is the torque in Nm and n is the speed of rotation in rev/s. The input power is the electrical power in watts supplied to the motor, i.e. VI watts.

Thus for a motor, efficiency, $\eta = \frac{T(2\pi n)}{VI} \times 100\%$

$$\text{i.e.} \quad 88 = \left[\frac{(130)(2\pi) \left(\frac{500}{60} \right)}{(250)(I)} \right] \times 100$$

$$\text{Thus, the current supplied, } I = \frac{(130)(2\pi) \left(\frac{500}{60} \right)}{(250)(88)} \times 100 = \mathbf{30.94 \text{ A}}$$

6. In a test on a d.c. motor, the following data was obtained. Supply voltage: 500 V

Current taken from the supply: 42.4 A Speed: 850 rev/min Shaft torque: 187 Nm

Determine the efficiency of the motor correct to the nearest 0.5%

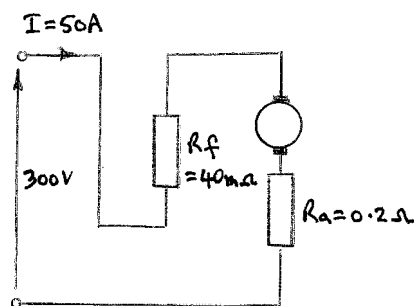
$$\text{From the previous problem, for a motor, efficiency, } \eta = \frac{T(2\pi n)}{VI} \times 100\%$$

$$\text{i.e.} \quad = \left[\frac{(187)(2\pi) \left(\frac{850}{60} \right)}{(500)(42.4)} \right] \times 100$$

$$= \mathbf{78.5\%}$$

7. A 300 V series motor draws a current of 50 A. The field resistance is 40 mΩ and the armature resistance is 0.2 Ω. Determine the maximum efficiency of the motor.

The circuit is shown below.



$$\text{Efficiency, } \eta = \left(\frac{VI - I_a^2 R_a - I_f V - C}{VI} \right) \times 100\%$$

However, for a series motor, $I_f = 0$ and $I_a^2 R_a$ needs to be $I^2(R_a + R_f)$

Hence, $\eta = \left(\frac{VI - I^2(R_a + R_f) - C}{VI} \right) \times 100\%$ and for maximum efficiency, $C = I^2(R_a + R_f)$

$$\begin{aligned} \text{Thus, efficiency, } \eta &= \left(\frac{VI - 2I^2(R_a + R_f)}{VI} \right) \times 100\% = \left(\frac{(300)(50) - 2(50)^2(0.04 + 0.2)}{(300)(50)} \right) \times 100\% \\ &= \left(\frac{15000 - 1200}{15000} \right) \times 100\% = \frac{13800}{15000} \times 100\% \end{aligned}$$

i.e. **maximum efficiency = 92%**

8. A series motor drives a load at 1500 rev/min and takes a current of 20 A when the supply voltage is 250 V. if the total resistance of the motor is 1.5Ω and the iron, friction and windage losses amount to 400 W, determine the efficiency of the motor.

$$\begin{aligned} \text{Efficiency, } \eta &= \left(\frac{VI - I^2R - C}{VI} \right) \times 100\% = \left(\frac{(250)(20) - 20^2(1.5) - 400}{(250)(20)} \right) \times 100\% \\ &= \left(\frac{5000 - 600 - 400}{5000} \right) \times 100\% = \frac{4000}{5000} \times 100\% \\ &= \mathbf{80\%} \end{aligned}$$

9. A series-wound motor is connected to a d.c. supply and develops full-load torque when the current is 30 A and speed is 1000 rev/min. If the flux per pole is proportional to the current flowing, find the current and speed at half full-load torque, when connected to the same supply.

Torque, $T = \Phi I_a$

$$\text{Hence, } \frac{T_1}{T_2} = \frac{\Phi_1 I_1}{\Phi_2 I_2} \quad \text{and since, } \Phi \propto I \quad \text{then} \quad \frac{T_1}{0.5 T_1} = \frac{30^2}{I_2^2}$$

from which, $I_2^2 = (0.5)(30)^2$ and **current at half full-load torque**, $I_2 = \sqrt{(0.5)(30^2)} = 21.2 \text{ A}$

$$\frac{n_1}{n_2} = \frac{\frac{1}{I_1}}{\frac{1}{I_2}} = \frac{I_2}{I_1}$$

Speed, $n \propto \frac{1}{I}$ hence,

$$\frac{1000}{n_2} = \frac{21.2}{30}$$

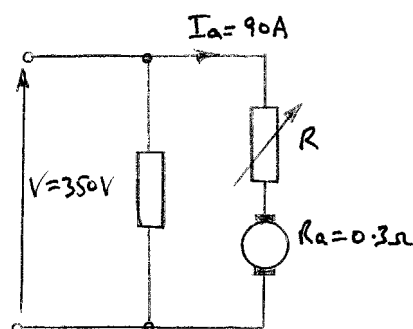
i.e.

$$\text{from which, speed at half full-load torque, } n_2 = \frac{(1000)(30)}{21.2} = 1415 \text{ rev/min}$$

Exercise 163, Page 463

1. A 350 V shunt motor runs at its normal speed of 12 rev/s when the armature current is 90 A. The resistance of the armature is 0.3Ω . (a) Find the speed when the current is 45 A and a resistance of 0.4Ω is connected in series with the armature, the shunt field remaining constant. (b) Find the speed when the current is 45 A and the shunt field is reduced to 75% of its normal value by increasing the resistance of the field.

The circuit is shown below.



(a) Back e.m.f. at 90 A, $E_1 = V - I_a R_a = 350 - (90)(0.3) = 323 \text{ V}$

When $I_a = 45 \text{ A}$, $E_2 = 350 - (45)(0.3 + 0.4) = 318.5 \text{ V}$

$$\text{Now } \frac{E_1}{E_2} = \frac{\Phi_1 n_1}{\Phi_2 n_2} \quad \text{i.e.} \quad \frac{323}{318.5} = \frac{\Phi_1 (12)}{\Phi_1 (n_2)} \quad \text{since } \Phi_2 = \Phi_1$$

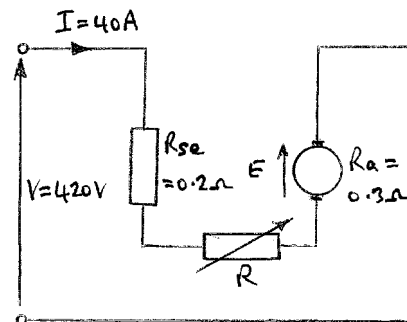
$$\text{from which, speed, } n_2 = \frac{(12)(318.5)}{(323)} = \mathbf{11.83 \text{ rev/s}}$$

$$(b) \text{ When } I_a = 45 \text{ A, } E_2 = 350 - (45)(0.3) = 336.5 \text{ V}$$

$$\text{Now } \frac{E_1}{E_2} = \frac{\Phi_1 n_1}{\Phi_2 n_2} \quad \text{i.e.} \quad \frac{323}{336.5} = \frac{\Phi_1 (12)}{(0.75 \Phi_1)(n_2)} \quad \text{since } \Phi_2 = 0.75 \Phi_1$$

$$\text{from which, speed, } n_2 = \frac{(12)(336.5)}{(323)(0.75)} = \mathbf{16.67 \text{ rev/s}}$$

2. A series motor runs at 900 rev/min when the voltage is 420 V and the current is 40 A. The armature resistance is 0.3Ω and the series field resistance is 0.2Ω . Calculate the resistance to be connected in series to reduce the speed to 720 rev/min with the same current.



$$\begin{aligned} \text{At 900 rev/min, e.m.f., } E_1 &= V - I(R_a + R_{se}) \\ &= 420 - (40)(0.3 + 0.2) = 400 \text{ V} \end{aligned}$$

$$\text{At 720 rev/min, since } I \text{ is unchanged, } \Phi \text{ is unchanged, thus, } \frac{E_1}{E_2} = \frac{n_1}{n_2}$$

$$\text{i.e. } \frac{400}{E_2} = \frac{900}{720} \quad \text{from which, } E_2 = \frac{(400)(720)}{(900)} = 320 \text{ V}$$

Also, $E_2 = V - I(R_a + R_{se} + R)$ where R is the extra series resistance

i.e. $320 = 420 - 40(0.3 + 0.2 + R)$

i.e. $40(0.5 + R) = 420 - 320$

from which, $0.5 + R = \frac{420 - 320}{40} = 2.5$

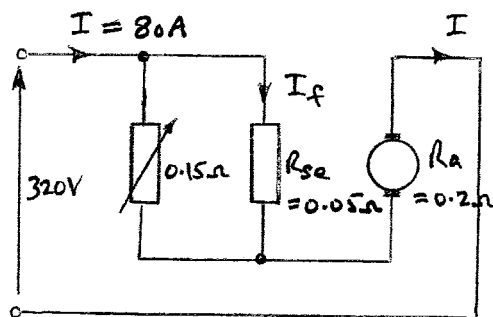
Hence, **the extra resistance, $R = 2.5 - 0.5 = 2 \Omega$**

3. A 320 V series motor takes 80 A and runs at 1080 rev/min at full load. The armature resistance is 0.2Ω and the series winding resistance is 0.05Ω . Assuming the flux is proportional to the field current, calculate the speed when developing full-load torque, but with a 0.15Ω diverter in parallel with the field winding.

The circuit is shown below.

At 320 V, $E_1 = V - I(R_a + R_{se})$ without the diverter connected

$$= 320 - 80(0.2 + 0.05) = 300 \text{ V}$$



With a 0.15Ω diverter in parallel with R_{se} ,

the equivalent resistance, $R = \frac{(0.15)(0.05)}{0.15 + 0.05} = 0.0375 \Omega$

By current division, $I_f = \left(\frac{0.15}{0.15 + 0.05} \right) I = 0.75 I$

Torque, $T \propto I_a \Phi$ and for full-load torque, $I_{a1} \Phi_1 = I_{a2} \Phi_2$

Since $\Phi \propto I_f$, $\Phi_1 \propto I_{a_1}$ and $\Phi_2 \propto 0.75 I_{a_2}$

then $(80)(80) = (I_{a_2})(0.75 I_{a_2})$

from which, $I_{a_2}^2 = \frac{80^2}{0.75}$ and $I_{a_2} = \frac{80}{\sqrt{0.75}} = 92.38 \text{ A}$

Hence, $E_2 = V - I_{a_2} (R_a + R)$
 $= 320 - 92.38(0.2 + 0.0375)$
 $= 320 - 21.94 = 298.06 \text{ V}$

Now $\frac{E_1}{E_2} = \frac{\Phi_1 n_1}{\Phi_2 n_2} = \frac{I_{a_1} n_1}{0.75 I_{a_2} n_2}$

Hence, $\frac{300}{298.06} = \frac{(80) \left(\frac{1080}{60} \right)}{0.75 (92.38) (n_2)}$

and new speed, $n_2 = \frac{(80) \left(\frac{1080}{60} \right) (298.06)}{(300) (0.75) (92.38)} = 20.65 \text{ rev/s or } 1239 \text{ rev/min}$
