
7 Magnetic circuits

At the end of this chapter you should be able to:

- describe the magnetic field around a permanent magnet
- state the laws of magnetic attraction and repulsion for two magnets in close proximity
- define magnetic flux, Φ , and magnetic flux density, B , and state their units
- perform simple calculations involving $B = \frac{\Phi}{A}$
- define magnetomotive force, F_m , and magnetic field strength, H , and state their units
- perform simple calculations involving $F_m = NI$ and $H = \frac{NI}{l}$
- define permeability, distinguishing between μ_0 , μ_r and μ
- understand the B–H curves for different magnetic materials
- appreciate typical values of μ_r
- perform calculations involving $B = \mu_0\mu_r H$
- define reluctance, S , and state its units
- perform calculations involving $S = \frac{\text{mmf}}{\Phi} = \frac{l}{\mu_0\mu_r A}$
- perform calculations on composite series magnetic circuits
- compare electrical and magnetic quantities
- appreciate how a hysteresis loop is obtained and that hysteresis loss is proportional to its area

7.1 Magnetic fields

A **permanent magnet** is a piece of ferromagnetic material (such as iron, nickel or cobalt) which has properties of attracting other pieces of these materials. A permanent magnet will position itself in a north and south direction when freely suspended. The north-seeking end of the magnet is called the **north pole**, N , and the south-seeking end the **south pole**, S .

The area around a magnet is called the **magnetic field** and it is in this area that the effects of the **magnetic force** produced by the magnet can be detected. A magnetic field cannot be seen, felt, smelt or heard and therefore is difficult to represent. Michael Faraday suggested that the magnetic field could be represented pictorially, by imagining the field to consist of **lines of magnetic flux**, which enables investigation of the distribution and density of the field to be carried out.

The distribution of a magnetic field can be investigated by using some iron filings. A bar magnet is placed on a flat surface covered by, say,

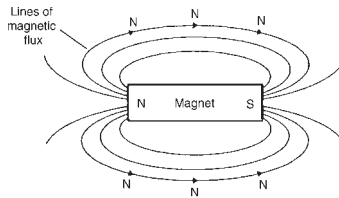


Figure 7.1

cardboard, upon which is sprinkled some iron filings. If the cardboard is gently tapped the filings will assume a pattern similar to that shown in Figure 7.1. If a number of magnets of different strength are used, it is found that the stronger the field the closer are the lines of magnetic flux and vice versa. Thus a magnetic field has the property of exerting a force, demonstrated in this case by causing the iron filings to move into the pattern shown. The strength of the magnetic field decreases as we move away from the magnet. It should be realized, of course, that the magnetic field is three dimensional in its effect, and not acting in one plane as appears to be the case in this experiment.

If a compass is placed in the magnetic field in various positions, the direction of the lines of flux may be determined by noting the direction of the compass pointer. The direction of a magnetic field at any point is taken as that in which the north-seeking pole of a compass needle points when suspended in the field. The direction of a line of flux is from the north pole to the south pole on the outside of the magnet and is then assumed to continue through the magnet back to the point at which it emerged at the north pole. Thus such lines of flux always form complete closed loops or paths, they never intersect and always have a definite direction. The laws of magnetic attraction and repulsion can be demonstrated by using two bar magnets. In Figure 7.2(a), **with unlike poles adjacent, attraction takes place**. Lines of flux are imagined to contract and the magnets try to pull together. The magnetic field is strongest in between the two magnets, shown by the lines of flux being close together. In Figure 7.2(b), **with similar poles adjacent (i.e. two north poles), repulsion occurs**, i.e. the two north poles try to push each other apart, since magnetic flux lines running side by side in the same direction repel.

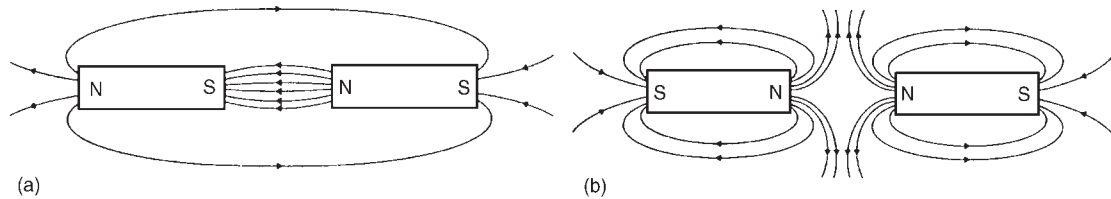


Figure 7.2

7.2 Magnetic flux and flux density

Magnetic flux is the amount of magnetic field (or the number of lines of force) produced by a magnetic source. The symbol for magnetic flux is Φ (Greek letter 'phi'). The unit of magnetic flux is the **weber, Wb**

Magnetic flux density is the amount of flux passing through a defined area that is perpendicular to the direction of the flux:

$$\text{Magnetic flux density} = \frac{\text{magnetic flux}}{\text{area}}$$

The symbol for magnetic flux density is B . The unit of magnetic flux density is the **tesla, T**, where $1 \text{ T} = 1 \text{ Wb/m}^2$ Hence

$$B = \frac{\Phi}{A} \text{ tesla}, \text{ where } A(\text{m}^2) \text{ is the area}$$

Problem 1. A magnetic pole face has a rectangular section having dimensions 200 mm by 100 mm. If the total flux emerging from the pole is 150 μWb , calculate the flux density.

$$\text{Flux } \Phi = 150 \mu\text{Wb} = 150 \times 10^{-6} \text{ Wb}$$

$$\begin{aligned} \text{Cross sectional area } A &= 200 \times 100 = 20000 \text{ mm}^2 \\ &= 20000 \times 10^{-6} \text{ m}^2 \end{aligned}$$

$$\begin{aligned} \text{Flux density } B &= \frac{\Phi}{A} = \frac{150 \times 10^{-6}}{20000 \times 10^{-6}} \\ &= \mathbf{0.0075 \text{ T or } 7.5 \text{ mT}} \end{aligned}$$

Problem 2. The maximum working flux density of a lifting electromagnet is 1.8 T and the effective area of a pole face is circular in cross-section. If the total magnetic flux produced is 353 mWb, determine the radius of the pole face.

$$\text{Flux density } B = 1.8 \text{ T; flux } \Phi = 353 \text{ mWb} = 353 \times 10^{-3} \text{ Wb}$$

$$\begin{aligned} \text{Since } B &= \frac{\Phi}{A}, \text{ cross-sectional area } A = \frac{\Phi}{B} = \frac{353 \times 10^{-3}}{1.8} \text{ m}^2 \\ &= 0.1961 \text{ m}^2 \end{aligned}$$

The pole face is circular, hence area = πr^2 , where r is the radius.

$$\text{Hence } \pi r^2 = 0.1961$$

$$\text{from which } r^2 = \frac{0.1961}{\pi} \text{ and radius } r = \sqrt{\left(\frac{0.1961}{\pi}\right)} = 0.250 \text{ m}$$

i.e. the radius of the pole face is 250 mm

7.3 Magnetomotive force and magnetic field strength

Magnetomotive force (mmf) is the cause of the existence of a magnetic flux in a magnetic circuit,

$$\text{mmf, } F_m = NI \text{ amperes}$$

where N is the number of conductors (or turns) and I is the current in amperes. The unit of mmf is sometimes expressed as ‘ampere-turns’. However since ‘turns’ have no dimensions, the SI unit of mmf is the

ampere. **Magnetic field strength** (or **magnetizing force**),

$$H = NI / l \text{ ampere per metre,}$$

where l is the mean length of the flux path in metres.

Thus **mmf** = $NI = Hl$ amperes.

Problem 3. A magnetizing force of 8000 A/m is applied to a circular magnetic circuit of mean diameter 30 cm by passing a current through a coil wound on the circuit. If the coil is uniformly wound around the circuit and has 750 turns, find the current in the coil.

$$H = 8000 \text{ A/m; } l = \pi d = \pi \times 30 \times 10^{-2} \text{ m; } N = 750 \text{ turns}$$

$$\text{Since } H = \frac{NI}{l} \text{ then, } I = \frac{Hl}{N} = \frac{8000 \times \pi \times 30 \times 10^{-2}}{750}$$

Thus, **current** $I = 10.05 \text{ A}$

7.4 Permeability and $B-H$ curves

For air, or any non-magnetic medium, the ratio of magnetic flux density to magnetizing force is a constant, i.e. $B/H = \text{a constant}$. This constant is μ_0 , the **permeability of free space** (or the magnetic space constant) and is equal to $4\pi \times 10^{-7} \text{ H/m}$, i.e., **for air, or any non-magnetic medium**, the ratio $B/H = \mu_0$ (Although all non-magnetic materials, including air, exhibit slight magnetic properties, these can effectively be neglected.)

For all media other than free space, $B/H = \mu_0 \mu_r$

where μ_r is the relative permeability, and is defined as

$$\mu_r = \frac{\text{flux density in material}}{\text{flux density in a vacuum}}$$

μ_r varies with the type of magnetic material and, since it is a ratio of flux densities, it has no unit. From its definition, μ_r for a vacuum is 1. $\mu_0 \mu_r = \mu$, called the **absolute permeability**

By plotting measured values of flux density B against magnetic field strength H , a **magnetization curve** (or **$B-H$ curve**) is produced. For non-magnetic materials this is a straight line. Typical curves for four magnetic materials are shown in Figure 7.3.

The **relative permeability** of a ferromagnetic material is proportional to the slope of the $B-H$ curve and thus varies with the magnetic field strength. The approximate range of values of relative permeability μ_r for some common magnetic materials are:

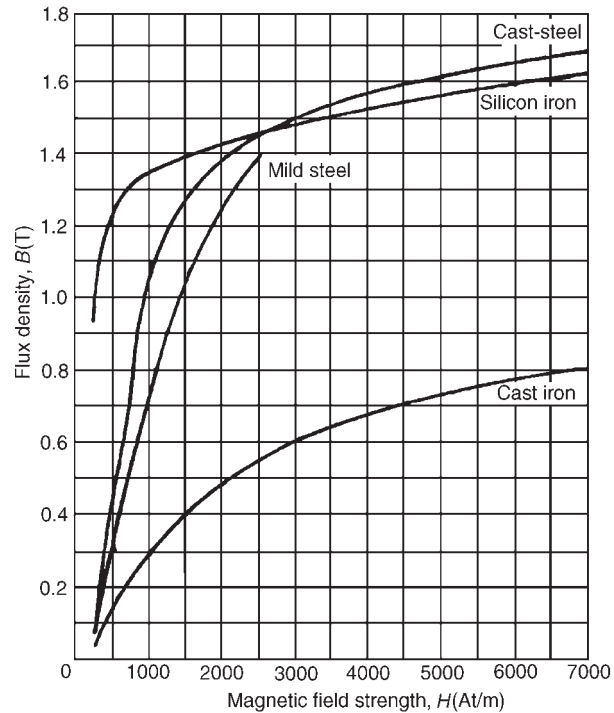


Figure 7.3 *B–H curves for four materials*

Cast iron	$\mu_r = 100-250$	Mild steel	$\mu_r = 200-800$
Silicon iron	$\mu_r = 1000-5000$	Cast steel	$\mu_r = 300-900$
Mumetal	$\mu_r = 200-5000$	Stalloy	$\mu_r = 500-6000$

Problem 4. A flux density of 1.2 T is produced in a piece of cast steel by a magnetizing force of 1250 A/m. Find the relative permeability of the steel under these conditions.

For a magnetic material:

$$B = \mu_0 \mu_r H$$

$$\text{i.e. } \mu_r = \frac{B}{\mu_0 H} = \frac{1.2}{(4\pi \times 10^{-7})(1250)} = \mathbf{764}$$

Problem 5. Determine the magnetic field strength and the mmf required to produce a flux density of 0.25 T in an air gap of length 12 mm.

For air: $B = \mu_0 H$ (since $\mu_r = 1$)

$$\text{Magnetic field strength } H = \frac{B}{\mu_0} = \frac{0.25}{4\pi \times 10^{-7}} = \mathbf{198\,940\, A/m}$$

$$\text{mmf} = Hl = 198\,940 \times 12 \times 10^{-3} = \mathbf{2387\, A}$$

Problem 6. A coil of 300 turns is wound uniformly on a ring of non-magnetic material. The ring has a mean circumference of 40 cm and a uniform cross sectional area of 4 cm². If the current in the coil is 5 A, calculate (a) the magnetic field strength, (b) the flux density and (c) the total magnetic flux in the ring.

- (a) Magnetic field strength $H = \frac{NI}{l} = \frac{300 \times 5}{40 \times 10^{-2}} = \mathbf{3750\, A/m}$
- (b) For a non-magnetic material $\mu_r = 1$, thus flux density $B = \mu_0 H$
i.e. $\mathbf{B} = 4\pi \times 10^{-7} \times 3750 = \mathbf{4.712\, mT}$
- (c) Flux $\Phi = BA = (4.712 \times 10^{-3})(4 \times 10^{-4}) = \mathbf{1.885\, \mu Wb}$

Problem 7. An iron ring of mean diameter 10 cm is uniformly wound with 2000 turns of wire. When a current of 0.25 A is passed through the coil a flux density of 0.4 T is set up in the iron. Find (a) the magnetizing force and (b) the relative permeability of the iron under these conditions.

$l = \pi d = \pi \times 10\text{ cm} = \pi \times 10 \times 10^{-2}\text{ m}$; $N = 2000$ turns; $I = 0.25\text{ A}$;
 $B = 0.4\text{ T}$

- (a) $\mathbf{H} = \frac{NI}{l} = \frac{2000 \times 0.25}{\pi \times 10 \times 10^{-2}} = \frac{5000}{\pi} = \mathbf{1592\, A/m}$
- (b) $B = \mu_0 \mu_r H$, hence $\mu_r = \frac{B}{\mu_0 H} = \frac{0.4}{(4\pi \times 10^{-7})(1592)} = \mathbf{200}$

Problem 8. A uniform ring of cast iron has a cross-sectional area of 10 cm² and a mean circumference of 20 cm. Determine the mmf necessary to produce a flux of 0.3 mWb in the ring. The magnetization curve for cast iron is shown on page 78.

$A = 10\text{ cm}^2 = 10 \times 10^{-4}\text{ m}^2$; $l = 20\text{ cm} = 0.2\text{ m}$; $\Phi = 0.3 \times 10^{-3}\text{ Wb}$

$$\text{Flux density } B = \frac{\Phi}{A} = \frac{0.3 \times 10^{-3}}{10 \times 10^{-4}} = 0.3\text{ T}$$

From the magnetization curve for cast iron on page 78, when $B = 0.3\text{ T}$,
 $H = 1000\text{ A/m}$, hence $\text{mmf} = Hl = 1000 \times 0.2 = \mathbf{200\, A}$

A tabular method could have been used in this problem. Such a solution is shown below.

Part of circuit	Material	Φ (Wb)	A (m ²)	$B = \frac{\Phi}{A}$ (T)	H from graph	l (m)	mmf = HI (A)
Ring	Cast iron	0.3×10^{-3}	10×10^{-4}	0.3	1000	0.2	200

7.5 Reluctance Reluctance S (or R_M) is the ‘magnetic resistance’ of a magnetic circuit to the presence of magnetic flux.

$$\text{Reluctance } S = \frac{F_M}{\Phi} = \frac{NI}{\Phi} = \frac{HI}{BA} = \frac{l}{(B/H)A} = \frac{l}{\mu_0\mu_r A}$$

The unit of reluctance is 1/H (or H⁻¹) or A/Wb

Ferromagnetic materials have a low reluctance and can be used as **magnetic screens** to prevent magnetic fields affecting materials within the screen.

Problem 9. Determine the reluctance of a piece of mumetal of length 150 mm and cross-sectional area 1800 mm² when the relative permeability is 4000. Find also the absolute permeability of the mumetal.

$$\begin{aligned} \text{Reluctance } S &= \frac{l}{\mu_0\mu_r A} = \frac{150 \times 10^{-3}}{(4\pi \times 10^{-7})(4000)(1800 \times 10^{-6})} \\ &= \mathbf{16\ 580/H} \end{aligned}$$

$$\begin{aligned} \text{Absolute permeability, } \mu &= \mu_0\mu_r = (4\pi \times 10^{-7})(4000) \\ &= \mathbf{5.027 \times 10^{-3} \text{ H/m}} \end{aligned}$$

Problem 10. A mild steel ring has a radius of 50 mm and a cross-sectional area of 400 mm². A current of 0.5 A flows in a coil wound uniformly around the ring and the flux produced is 0.1 mWb. If the relative permeability at this value of current is 200 find (a) the reluctance of the mild steel and (b) the number of turns on the coil.

$$l = 2\pi r = 2 \times \pi \times 50 \times 10^{-3} \text{ m}; A = 400 \times 10^{-6} \text{ m}^2; I = 0.5 \text{ A}; \\ \Phi = 0.1 \times 10^{-3} \text{ Wb}; \mu_r = 200$$

$$\begin{aligned} \text{(a) Reluctance } S &= \frac{l}{\mu_0\mu_r A} = \frac{2 \times \pi \times 50 \times 10^{-3}}{(4\pi \times 10^{-7})(200)(400 \times 10^{-6})} \\ &= \mathbf{3.125 \times 10^6/H} \end{aligned}$$

$$(b) \quad S = \frac{\text{mmf}}{\Phi} \text{ i.e. } \text{mmf} = S\Phi$$

so that $NI = S\Phi$ and

$$\text{hence } N = \frac{S\Phi}{I} = \frac{3.125 \times 10^6 \times 0.1 \times 10^{-3}}{0.5} = \mathbf{625 \text{ turns}}$$

Further problems on magnetic circuit quantities may be found in Section 7.9, problems 1 to 14, page 85.

7.6 Composite series magnetic circuits

For a series magnetic circuit having n parts, the **total reluctance** S is given by:

$$S = S_1 + S_2 + \dots + S_n$$

(This is similar to resistors connected in series in an electrical circuit.)

Problem 11. A closed magnetic circuit of cast steel contains a 6 cm long path of cross-sectional area 1 cm^2 and a 2 cm path of cross-sectional area 0.5 cm^2 . A coil of 200 turns is wound around the 6 cm length of the circuit and a current of 0.4 A flows. Determine the flux density in the 2 cm path, if the relative permeability of the cast steel is 750.

For the 6 cm long path:

$$\begin{aligned} \text{Reluctance } S_1 &= \frac{l_1}{\mu_0 \mu_r A_1} = \frac{6 \times 10^{-2}}{(4\pi \times 10^{-7})(750)(1 \times 10^{-4})} \\ &= 6.366 \times 10^5 / \text{H} \end{aligned}$$

For the 2 cm long path:

$$\begin{aligned} \text{Reluctance } S_2 &= \frac{l_2}{\mu_0 \mu_r A_2} = \frac{2 \times 10^{-2}}{(4\pi \times 10^{-7})(750)(0.5 \times 10^{-4})} \\ &= 4.244 \times 10^5 / \text{H} \end{aligned}$$

$$\begin{aligned} \text{Total circuit reluctance } S &= S_1 + S_2 = (6.366 + 4.244) \times 10^5 \\ &= 10.61 \times 10^5 / \text{H} \end{aligned}$$

$$S = \frac{\text{mmf}}{\Phi}, \text{ i.e. } \Phi = \frac{\text{mmf}}{S} = \frac{NI}{S} = \frac{200 \times 0.4}{10.61 \times 10^5} = 7.54 \times 10^{-5} \text{ Wb}$$

$$\text{Flux density in the 2 cm path, } B = \frac{\Phi}{A} = \frac{7.54 \times 10^{-5}}{0.5 \times 10^{-4}} = \mathbf{1.51 \text{ T}}$$

Problem 12. A silicon iron ring of cross-sectional area 5 cm^2 has a radial air gap of 2 mm cut into it. If the mean length of the silicon iron path is 40 cm , calculate the magnetomotive force to produce a flux of 0.7 mWb . The magnetization curve for silicon is shown on page 78.

There are two parts to the circuit—the silicon iron and the air gap. The total mmf will be the sum of the mmf's of each part.

$$\text{For the silicon iron: } B = \frac{\Phi}{A} = \frac{0.7 \times 10^{-3}}{5 \times 10^{-4}} = 1.4 \text{ T}$$

From the B - H curve for silicon iron on page 78, when $B = 1.4 \text{ T}$, $H = 1650 \text{ At/m}$.

Hence the mmf for the iron path = $HI = 1650 \times 0.4 = 660 \text{ A}$

For the air gap:

The flux density will be the same in the air gap as in the iron, i.e. 1.4 T . (This assumes no leakage or fringing occurring.)

$$\begin{aligned} \text{For air, } H &= \frac{B}{\mu_0} = \frac{1.4}{4\pi \times 10^{-7}} \\ &= 1\,114\,000 \text{ A/m} \end{aligned}$$

$$\begin{aligned} \text{Hence the mmf for the air gap} &= HI = 1\,114\,000 \times 2 \times 10^{-3} \\ &= 2228 \text{ A} \end{aligned}$$

$$\begin{aligned} \text{Total mmf to produce a flux of } 0.7 \text{ mWb} &= 660 + 2228 \\ &= \mathbf{2888 \text{ A}} \end{aligned}$$

A tabular method could have been used as shown below.

Part of circuit	Material	Φ (Wb)	A (m^2)	B (T)	H (A/m)	l (m)	mmf = HI (A)
Ring	Silicon iron	0.7×10^{-3}	5×10^{-4}	1.4	1650 (from graph)	0.4	660
Air-gap	Air	0.7×10^{-3}	5×10^{-4}	1.4	$\frac{1.4}{4\pi \times 10^{-7}}$ $= 1\,114\,000$	2×10^{-3}	2228

Total: **2888 A**

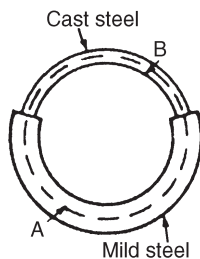


Figure 7.4

Problem 13. Figure 7.4 shows a ring formed with two different materials—cast steel and mild steel. The dimensions are:

	mean length	cross-sectional area
Mild steel	400 mm	500 mm^2
Cast steel	300 mm	312.5 mm^2

Find the total mmf required to cause a flux of $500 \mu\text{Wb}$ in the magnetic circuit. Determine also the total circuit reluctance.

A tabular solution is shown below.

Part of circuit	Material	Φ (Wb)	A (m^2)	B (T) (= Φ/A)	H (A/m) (from graphs p 78)	l (m)	mmf= HI (A)
A	Mild steel	500×10^{-6}	500×10^{-6}	1.0	1400	400×10^{-3}	560
B	Cast steel	500×10^{-6}	312.5×10^{-6}	1.6	4800	300×10^{-3}	1440
Total:							2000 A

$$\text{Total circuit reluctance } S = \frac{\text{mmf}}{\Phi} = \frac{2000}{500 \times 10^{-6}} = 4 \times 10^6/\text{H}$$

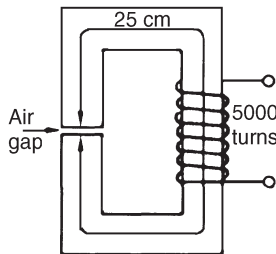


Figure 7.5

Problem 14. A section through a magnetic circuit of uniform cross-sectional area 2 cm^2 is shown in Figure 7.5. The cast steel core has a mean length of 25 cm . The air gap is 1 mm wide and the coil has 5000 turns. The $B-H$ curve for cast steel is shown on page 78. Determine the current in the coil to produce a flux density of 0.80 T in the air gap, assuming that all the flux passes through both parts of the magnetic circuit.

For the cast steel core, when $B = 0.80 \text{ T}$, $H = 750 \text{ A/m}$ (from page 78)

Reluctance of core $S_1 = \frac{l_1}{\mu_0 \mu_r A_1}$ and since $B = \mu_0 \mu_r H$,

$$\text{then } \mu_r = \frac{B}{\mu_0 H}. \text{ Thus } S_1 = \frac{l_1}{\mu_0 \left(\frac{B}{\mu_0 H} \right) A} = \frac{l_1 H}{BA} = \frac{(25 \times 10^{-2})(750)}{(0.8)(2 \times 10^{-4})} = 1\,172\,000/\text{H}$$

For the air gap: Reluctance, $S_2 = \frac{l_2}{\mu_0 \mu_r A_2} = \frac{l_2}{\mu_0 A_2}$
 (since $\mu_r = 1$ for air)
 $= \frac{1 \times 10^{-3}}{(4\pi \times 10^{-7})(2 \times 10^{-4})}$
 $= 3\,979\,000/\text{H}$

Total circuit reluctance $S = S_1 + S_2 = 1\,172\,000 + 3\,979\,000$
 $= 5\,151\,000/\text{H}$

$$\text{Flux } \Phi = BA = 0.80 \times 2 \times 10^{-4} = 1.6 \times 10^{-4} \text{ Wb}$$

$$S = \frac{\text{mmf}}{\Phi}, \text{ thus mmf} = S\Phi$$

$$\text{Hence } NI = S\Phi$$

$$\text{and current } I = \frac{S\Phi}{N} = \frac{(5\,151\,000)(1.6 \times 10^{-4})}{5000} = \mathbf{0.165 \text{ A}}$$

Further problems on composite series magnetic circuits may be found in Section 7.9, problems 15 to 19, page 86.

7.7 Comparison between electrical and magnetic quantities

Electrical circuit	Magnetic circuit
e.m.f. E (V)	mmf F_m (A)
current I (A)	flux Φ (Wb)
resistance R (Ω)	reluctance S (H^{-1})
$I = \frac{E}{R}$	$\Phi = \frac{\text{mmf}}{S}$
$R = \frac{\rho l}{A}$	$S = \frac{l}{\mu_0 \mu_r A}$

7.8 Hysteresis and hysteresis loss

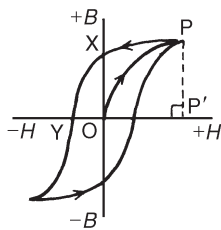


Figure 7.6

Hysteresis is the ‘lagging’ effect of flux density B whenever there are changes in the magnetic field strength H . When an initially unmagnetized ferromagnetic material is subjected to a varying magnetic field strength H , the flux density B produced in the material varies as shown in Figure 7.6, the arrows indicating the direction of the cycle. Figure 7.6 is known as a **hysteresis loop**.

From Figure 7.6, distance OX indicates the **residual flux density** or **remanence**, OY indicates the **coercive force**, and PP' is the **saturation flux density**.

Hysteresis results in a dissipation of energy which appears as a heating of the magnetic material. **The energy loss associated with hysteresis is proportional to the area of the hysteresis loop.**

The production of the hysteresis loop and hysteresis loss are explained in greater detail in Chapter 38, Section 3, page 692.

The area of a hysteresis loop varies with the type of material. The area, and thus the energy loss, is much greater for hard materials than for soft materials.

For AC-excited devices the hysteresis loop is repeated every cycle of alternating current. Thus a hysteresis loop with a large area (as with hard steel) is often unsuitable since the energy loss would be considerable. Silicon steel has a narrow hysteresis loop, and thus small hysteresis loss, and is suitable for transformer cores and rotating machine armatures.

7.9 Further problems on magnetic circuits

(Where appropriate, assume $\mu_0 = 4\pi \times 10^{-7}$ H/m)

Magnetic circuit quantities

- 1 What is the flux density in a magnetic field of cross-sectional area 20 cm^2 having a flux of 3 mWb ? [1.5 T]
- 2 Determine the total flux emerging from a magnetic pole face having dimensions 5 cm by 6 cm , if the flux density is 0.9 T . [2.7 mWb]
- 3 The maximum working flux density of a lifting electromagnet is 1.9 T and the effective area of a pole face is circular in cross-section. If the total magnetic flux produced is 611 mWb determine the radius of the pole face. [32 cm]
- 4 Find the magnetic field strength and the magnetomotive force needed to produce a flux density of 0.33 T in an air-gap of length 15 mm . [(a) $262\,600 \text{ A/m}$ (b) 3939 A]
- 5 An air-gap between two pole pieces is 20 mm in length and the area of the flux path across the gap is 5 cm^2 . If the flux required in the air-gap is 0.75 mWb find the mmf necessary. [23 870 A]
- 6 Find the magnetic field strength applied to a magnetic circuit of mean length 50 cm when a coil of 400 turns is applied to it carrying a current of 1.2 A . [960 A/m]
- 7 A solenoid 20 cm long is wound with 500 turns of wire. Find the current required to establish a magnetizing force of 2500 A/m inside the solenoid. [1 A]
- 8 A magnetic field strength of 5000 A/m is applied to a circular magnetic circuit of mean diameter 250 mm . If the coil has 500 turns find the current in the coil. [7.85 A]
- 9 Find the relative permeability of a piece of silicon iron if a flux density of 1.3 T is produced by a magnetic field strength of 700 A/m [1478]
- 10 Part of a magnetic circuit is made from steel of length 120 mm , cross-sectional area 15 cm^2 and relative permeability 800 . Calculate (a) the reluctance and (b) the absolute permeability of the steel. [(a) $79\,580 \text{ /H}$ (b) 1 mH/m]
- 11 A steel ring of mean diameter 120 mm is uniformly wound with 1500 turns of wire. When a current of 0.30 A is passed through the coil a flux density of 1.5 T is set up in the steel. Find the relative permeability of the steel under these conditions. [1000]
- 12 A mild steel closed magnetic circuit has a mean length of 75 mm and a cross-sectional area of 320.2 mm^2 . A current of 0.40 A flows in a coil wound uniformly around the circuit and the flux produced is $200 \text{ }\mu\text{Wb}$. If the relative permeability of the steel at this value of current is 400 find (a) the reluctance of the material and (b) the number of turns of the coil. [(a) $466\,000 \text{ /H}$ (b) 233]
- 13 A uniform ring of cast steel has a cross-sectional area of 5 cm^2 and a mean circumference of 15 cm . Find the current required in a coil

of 1200 turns wound on the ring to produce a flux of 0.8 mWb. (Use the magnetization curve for cast steel shown on page 78.) [0.60 A]

- 14 (a) A uniform mild steel ring has a diameter of 50 mm and a cross-sectional area of 1 cm^2 . Determine the mmf necessary to produce a flux of $50 \mu\text{Wb}$ in the ring. (Use the $B-H$ curve for mild steel shown on page 78.)
- (b) If a coil of 440 turns is wound uniformly around the ring in part (a) what current would be required to produce the flux?
[(a) 110 A (b) 0.25 A]

Composite series magnetic circuits

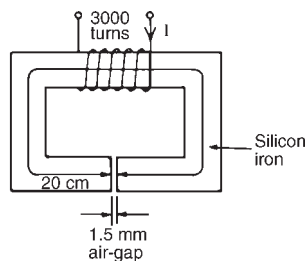


Figure 7.7

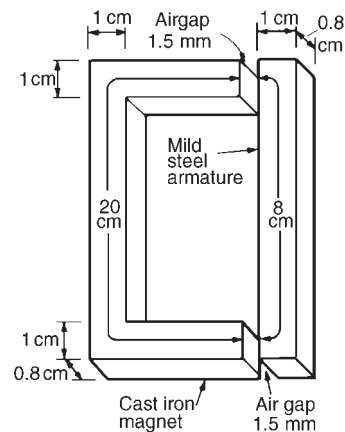


Figure 7.8

- 15 A magnetic circuit of cross-sectional area 0.4 cm^2 consists of one part 3 cm long, of material having relative permeability 1200, and a second part 2 cm long of material having relative permeability 750. With a 100 turn coil carrying 2 A, find the value of flux existing in the circuit.
[0.195 mWb]
- 16 (a) A cast steel ring has a cross-sectional area of 600 mm^2 and a radius of 25 mm. Determine the mmf necessary to establish a flux of 0.8 mWb in the ring. Use the $B-H$ curve for cast steel shown on page 78.
- (b) If a radial air gap 1.5 mm wide is cut in the ring of part (a) find the mmf now necessary to maintain the same flux in the ring.
[(a) 270 A (b) 1860 A]
- 17 For the magnetic circuit shown in Figure 7.7 find the current I in the coil needed to produce a flux of 0.45 mWb in the air-gap. The silicon iron magnetic circuit has a uniform cross-sectional area of 3 cm^2 and its magnetization curve is as shown on page 78.
[0.83 A]
- 18 A ring forming a magnetic circuit is made from two materials; one part is mild steel of mean length 25 cm and cross-sectional area 4 cm^2 , and the remainder is cast iron of mean length 20 cm and cross-sectional area 7.5 cm^2 . Use a tabular approach to determine the total mmf required to cause a flux of 0.30 mWb in the magnetic circuit. Find also the total reluctance of the circuit. Use the magnetization curves shown on page 78.
[550 A, $18.3 \times 10^5/\text{H}$]
- 19 Figure 7.8 shows the magnetic circuit of a relay. When each of the air gaps are 1.5 mm wide find the mmf required to produce a flux density of 0.75 T in the air gaps. Use the $B-H$ curves shown on page 78.
[2970 A]

8 Electromagnetism

At the end of this chapter you should be able to:

- understand that magnetic fields are produced by electric currents
- apply the screw rule to determine direction of magnetic field
- recognize that the magnetic field around a solenoid is similar to a magnet
- apply the screw rule or grip rule to a solenoid to determine magnetic field direction
- recognize and describe practical applications of an electromagnet, i.e. electric bell, relay, lifting magnet, telephone receiver
- appreciate factors upon which the force F on a current-carrying conductor depends
- perform calculations using $F = BIl$ and $F = BIl \sin \theta$
- recognize that a loudspeaker is a practical application of force F
- use Fleming's left-hand rule to pre-determine direction of force in a current-carrying conductor
- describe the principle of operation of a simple d.c. motor
- describe the principle of operation and construction of a moving coil instrument
- appreciate the force F on a charge in a magnetic field is given by $F = QvB$
- perform calculations using $F = QvB$

8.1 Magnetic field due to an electric current

Magnetic fields can be set up not only by permanent magnets, as shown in Chapter 7, but also by electric currents.

Let a piece of wire be arranged to pass vertically through a horizontal sheet of cardboard, on which is placed some iron filings, as shown in Figure 8.1(a).

If a current is now passed through the wire, then the iron filings will form a definite circular field pattern with the wire at the centre, when the cardboard is gently tapped. By placing a compass in different positions the lines of flux are seen to have a definite direction as shown in Figure 8.1(b). If the current direction is reversed, the direction of the lines of flux is also reversed. The effect on both the iron filings and the compass needle disappears when the current is switched off. The magnetic field is thus

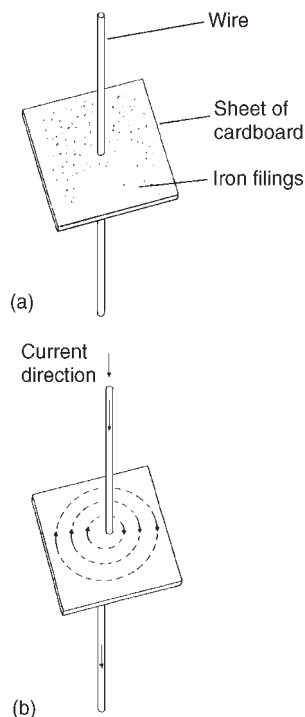


Figure 8.1

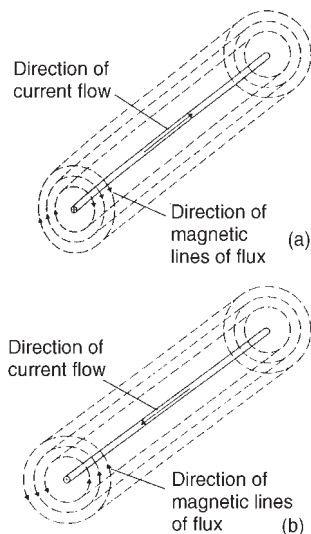


Figure 8.2

produced by the electric current. The magnetic flux produced has the same properties as the flux produced by a permanent magnet. If the current is increased the strength of the field increases and, as for the permanent magnet, the field strength decreases as we move away from the current-carrying conductor.

In Figure 8.1, the effect of only a small part of the magnetic field is shown.

If the whole length of the conductor is similarly investigated it is found that the magnetic field around a straight conductor is in the form of concentric cylinders as shown in Figure 8.2, the field direction depending on the direction of the current flow.

When dealing with magnetic fields formed by electric current it is usual to portray the effect as shown in Figure 8.3. The convention adopted is:

- (i) Current flowing away from the viewer, i.e. into the paper, is indicated by \odot . This may be thought of as the feathered end of the shaft of an arrow. See Figure 8.3(a).
- (ii) Current flowing towards the viewer, i.e. out of the paper, is indicated by \ominus . This may be thought of as the point of an arrow. See Figure 8.3(b).

The direction of the magnetic lines of flux is best remembered by the **screw rule**. This states that:

'If a normal right-hand thread screw is screwed along the conductor in the direction of the current, the direction of rotation of the screw is in the direction of the magnetic field.'

For example, with current flowing away from the viewer (Figure 8.3(a)) a right-hand thread screw driven into the paper has to be rotated clockwise. Hence the direction of the magnetic field is clockwise.

A magnetic field set up by a long coil, or **solenoid**, is shown in Figure 8.4(a) and is seen to be similar to that of a bar magnet. If the solenoid is wound on an iron bar, as shown in Figure 8.4(b), an even stronger magnetic field is produced, the iron becoming magnetized and behaving like a permanent magnet.

The direction of the magnetic field produced by the current I in the solenoid may be found by either of two methods, i.e. the screw rule or the grip rule.

- (a) **The screw rule** states that if a normal right-hand thread screw is placed along the axis of the solenoid and is screwed in the direction of the current it moves in the direction of the magnetic field **inside** the solenoid. The direction of the magnetic field **inside** the solenoid is from south to north. Thus in Figures 8.4(a) and (b) the north pole is to the right.
- (b) **The grip rule** states that if the coil is gripped with the **right** hand, with the fingers pointing in the direction of the current, then the thumb, outstretched parallel to the axis of the solenoid, points in the direction of the magnetic field **inside** the solenoid.

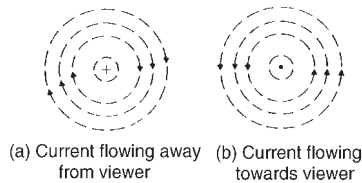


Figure 8.3

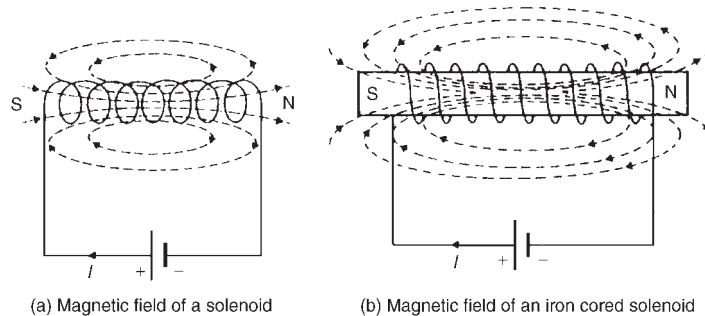


Figure 8.4

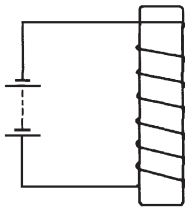


Figure 8.5

Problem 1. Figure 8.5 shows a coil of wire wound on an iron core connected to a battery. Sketch the magnetic field pattern associated with the current carrying coil and determine the polarity of the field.

The magnetic field associated with the solenoid in Figure 8.5 is similar to the field associated with a bar magnet and is as shown in Figure 8.6. The polarity of the field is determined either by the screw rule or by the grip rule. Thus the north pole is at the bottom and the south pole at the top.

8.2 Electromagnets

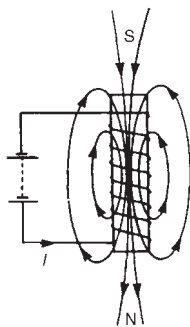


Figure 8.6

The solenoid is very important in electromagnetic theory since the magnetic field inside the solenoid is practically uniform for a particular current, and is also versatile, inasmuch that a variation of the current can alter the strength of the magnetic field. An electromagnet, based on the solenoid, provides the basis of many items of electrical equipment, examples of which include electric bells, relays, lifting magnets and telephone receivers.

(i) Electric bell

There are various types of electric bell, including the single-stroke bell, the trembler bell, the buzzer and a continuously ringing bell, but all depend on the attraction exerted by an electromagnet on a soft iron armature. A typical single stroke bell circuit is shown in Figure 8.7. When the push button is operated a current passes through the coil. Since the iron-cored coil is energized the soft iron armature is attracted to the electromagnet. The armature also carries a striker which hits the gong. When the circuit is broken the coil becomes demagnetized and the spring steel strip pulls the armature back to its original position. The striker will only operate when the push is operated.

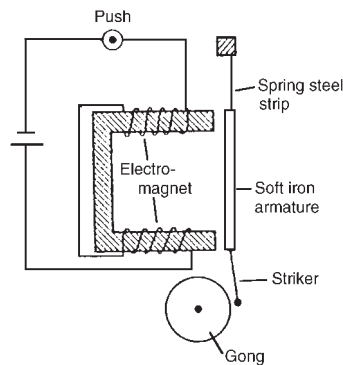


Figure 8.7

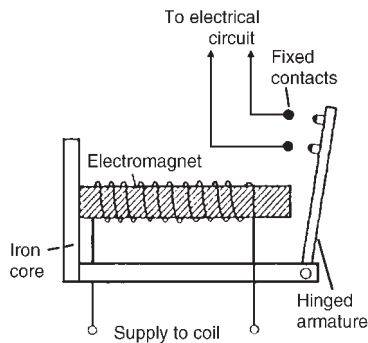


Figure 8.8

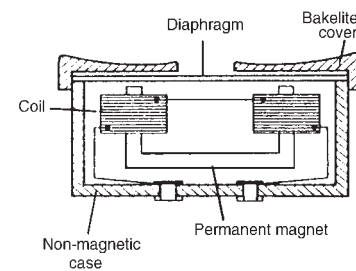
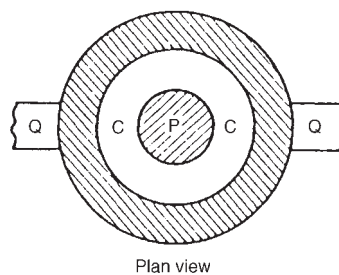
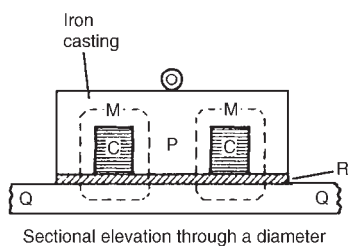


Figure 8.9

Figure 8.10

(ii) Relay

A relay is similar to an electric bell except that contacts are opened or closed by operation instead of a gong being struck. A typical simple relay is shown in Figure 8.8, which consists of a coil wound on a soft iron core. When the coil is energized the hinged soft iron armature is attracted to the electromagnet and pushes against two fixed contacts so that they are connected together, thus closing some other electrical circuit.

(iii) Lifting magnet

Lifting magnets, incorporating large electromagnets, are used in iron and steel works for lifting scrap metal. A typical robust lifting magnet, capable of exerting large attractive forces, is shown in the elevation and plan view of Figure 8.9 where a coil, C , is wound round a central core, P , of the iron casting. Over the face of the electromagnet is placed a protective non-magnetic sheet of material, R . The load, Q , which must be of magnetic material is lifted when the coils are energized, the magnetic flux paths, M , being shown by the broken lines.

(iv) Telephone receiver

Whereas a transmitter or microphone changes sound waves into corresponding electrical signals, a telephone receiver converts the electrical waves back into sound waves. A typical telephone receiver is shown in Figure 8.10 and consists of a permanent magnet with coils wound on its poles. A thin, flexible diaphragm of magnetic material is held in position near to the magnetic poles but not touching them. Variation in current from the transmitter varies the magnetic field and the diaphragm consequently vibrates. The vibration produces sound variations corresponding to those transmitted.

8.3 Force on a current-carrying conductor

If a current-carrying conductor is placed in a magnetic field produced by permanent magnets, then the fields due to the current-carrying conductor and the permanent magnets interact and cause a force to be exerted on

the conductor. The force on the current-carrying conductor in a magnetic field depends upon:

- (a) the flux density of the field, B teslas
- (b) the strength of the current, I amperes,
- (c) the length of the conductor perpendicular to the magnetic field, l metres, and
- (d) the directions of the field and the current.

When the magnetic field, the current and the conductor are mutually at right angles then:

$$\text{Force } F = BIl \text{ newtons}$$

When the conductor and the field are at an angle θ° to each other then:

$$\text{Force } F = BIl \sin \theta \text{ newtons}$$

Since when the magnetic field, current and conductor are mutually at right angles, $F = BIl$, the magnetic flux density B may be defined by $B = F/Il$, i.e. the flux density is 1 T if the force exerted on 1 m of a conductor when the conductor carries a current of 1 A is 1 N.

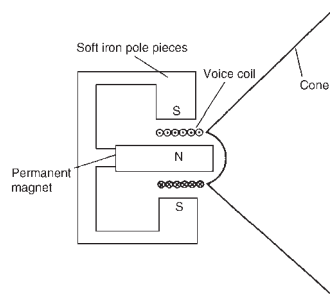


Figure 8.11

Loudspeaker

A simple application of the above force is the moving coil loudspeaker. The loudspeaker is used to convert electrical signals into sound waves.

Figure 8.11 shows a typical loudspeaker having a magnetic circuit comprising a permanent magnet and soft iron pole pieces so that a strong magnetic field is available in the short cylindrical airgap. A moving coil, called the voice or speech coil, is suspended from the end of a paper or plastic cone so that it lies in the gap. When an electric current flows through the coil it produces a force which tends to move the cone backwards and forwards according to the direction of the current. The cone acts as a piston, transferring this force to the air, and producing the required sound waves.

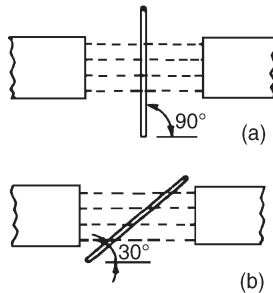


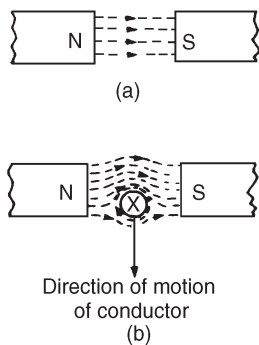
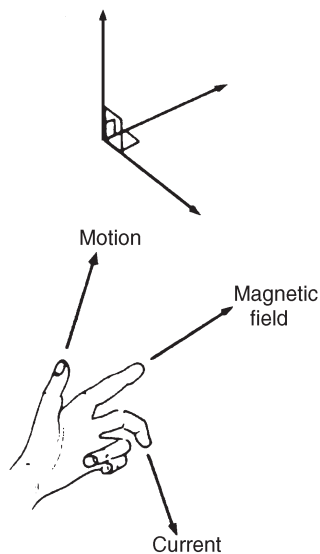
Figure 8.12

Problem 2. A conductor carries a current of 20 A and is at right-angles to a magnetic field having a flux density of 0.9 T. If the length of the conductor in the field is 30 cm, calculate the force acting on the conductor.

Determine also the value of the force if the conductor is inclined at an angle of 30° to the direction of the field.

$$B = 0.9 \text{ T}; I = 20 \text{ A}; l = 30 \text{ cm} = 0.30 \text{ m}$$

Force $F = BIl = (0.9)(20)(0.30)$ newtons when the conductor is at right-angles to the field, as shown in Figure 8.12(a), i.e. $F = 5.4 \text{ N}$

**Figure 8.13****Figure 8.14**

When the conductor is inclined at 30° to the field, as shown in Figure 8.12(b), then force $F = BIl \sin \theta$

$$= (0.9)(20)(0.30) \sin 30^\circ$$

i.e. $F = 2.7 \text{ N}$

If the current-carrying conductor shown in Figure 8.3(a) is placed in the magnetic field shown in Figure 8.13(a), then the two fields interact and cause a force to be exerted on the conductor as shown in Figure 8.13(b). The field is strengthened above the conductor and weakened below, thus tending to move the conductor downwards. This is the basic principle of operation of the electric motor (see Section 8.4) and the moving-coil instrument (see Section 8.5).

The direction of the force exerted on a conductor can be pre-determined by using **Fleming's left-hand rule** (often called the motor rule) which states:

Let the thumb, first finger and second finger of the left hand be extended such that they are all at right-angles to each other, (as shown in Figure 8.14). If the first finger points in the direction of the magnetic field, the second finger points in the direction of the current, then the thumb will point in the direction of the motion of the conductor.

Summarizing:

First finger - Field

SeCond finger - Current

ThuMb - Motion

Problem 3. Determine the current required in a 400 mm length of conductor of an electric motor, when the conductor is situated at right-angles to a magnetic field of flux density 1.2 T, if a force of 1.92 N is to be exerted on the conductor. If the conductor is vertical, the current flowing downwards and the direction of the magnetic field is from left to right, what is the direction of the force?

Force = 1.92 N; $l = 400 \text{ mm} = 0.40 \text{ m}$; $B = 1.2 \text{ T}$

Since $F = BIl$, then $I = \frac{F}{Bl}$

hence current $I = \frac{1.92}{(1.2)(0.4)} = 4 \text{ A}$

If the current flows downwards, the direction of its magnetic field due to the current alone will be clockwise when viewed from above. The lines of flux will reinforce (i.e. strengthen) the main magnetic field at the back of the conductor and will be in opposition in the front (i.e. weaken the field).

Hence the force on the conductor will be from back to front (i.e. toward the viewer). This direction may also have been deduced using Fleming's left-hand rule.

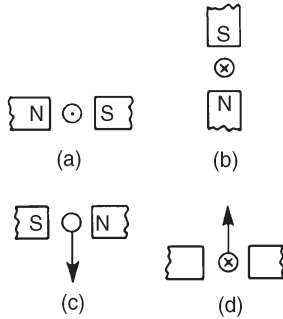


Figure 8.15

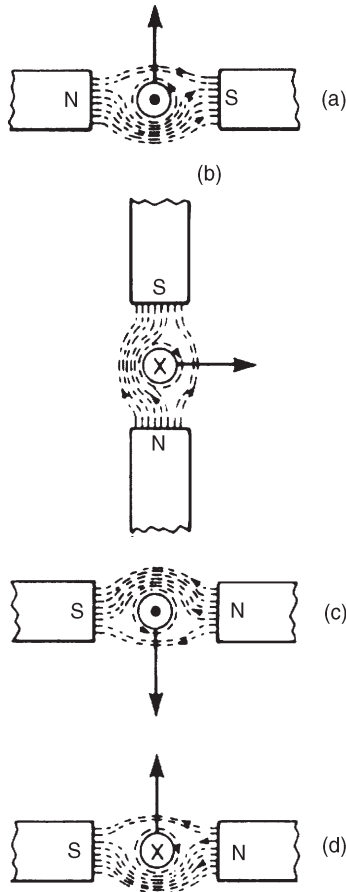


Figure 8.16

Problem 4. A conductor 350 mm long carries a current of 10 A and is at right-angles to a magnetic field lying between two circular pole faces each of radius 60 mm. If the total flux between the pole faces is 0.5 mWb, calculate the magnitude of the force exerted on the conductor.

$$l = 350 \text{ mm} = 0.35 \text{ m}; I = 10 \text{ A};$$

$$\text{Area of pole face } A = \pi r^2 = \pi(0.06)^2 \text{ m}^2;$$

$$\Phi = 0.5 \text{ mWb} = 0.5 \times 10^{-3} \text{ Wb}$$

$$\text{Force } F = BIl, \text{ and } B = \frac{\Phi}{A}$$

$$\text{hence force } F = \left(\frac{\Phi}{A} \right) Il = \frac{(0.5 \times 10^{-3})}{\pi(0.06)^2} (10)(0.35) \text{ newtons}$$

$$\text{i.e. force} = 0.155 \text{ N}$$

Problem 5. With reference to Figure 8.15 determine (a) the direction of the force on the conductor in Figure 8.15(a), (b) the direction of the force on the conductor in Figure 8.15(b), (c) the direction of the current in Figure 8.15(c), (d) the polarity of the magnetic system in Figure 8.15(d).

- The direction of the main magnetic field is from north to south, i.e. left to right. The current is flowing towards the viewer, and using the screw rule, the direction of the field is anticlockwise. Hence either by Fleming's left-hand rule, or by sketching the interacting magnetic field as shown in Figure 8.16(a), the direction of the force on the conductor is seen to be upward.
- Using a similar method to part (a) it is seen that the force on the conductor is to the right—see Figure 8.16(b).
- Using Fleming's left-hand rule, or by sketching as in Figure 8.16(c), it is seen that the current is toward the viewer, i.e. out of the paper.
- Similar to part (c), the polarity of the magnetic system is as shown in Figure 8.16(d).

Problem 6. A coil is wound on a rectangular former of width 24 mm and length 30 mm. The former is pivoted about an axis passing through the middle of the two shorter sides and is placed in a uniform magnetic field of flux density 0.8 T, the axis being perpendicular to the field. If the coil carries a current of 50 mA, determine the force on each coil side (a) for a single-turn coil, (b) for a coil wound with 300 turns.

- (a) Flux density $B = 0.8 \text{ T}$; length of conductor lying at right-angles to field $l = 30 \text{ mm} = 30 \times 10^{-3} \text{ m}$; current $I = 50 \text{ mA} = 50 \times 10^{-3} \text{ A}$
For a single-turn coil, force on each coil side

$$F = BIl = 0.8 \times 50 \times 10^{-3} \times 30 \times 10^{-3}$$

$$= 1.2 \times 10^{-3} \text{ N, or } 0.0012 \text{ N}$$

- (b) When there are 300 turns on the coil there are effectively 300 parallel conductors each carrying a current of 50 mA. Thus the total force produced by the current is 300 times that for a single-turn coil. Hence force on coil side $F = 300 BIl = 300 \times 0.0012 = 0.36 \text{ N}$

Further problems on the force on a current-carrying conductor may be found in Section 8.7, problems 1 to 6, page 98.

8.4 Principle of operation of a simple d.c. motor

A rectangular coil which is free to rotate about a fixed axis is shown placed inside a magnetic field produced by permanent magnets in Figure 8.17. A direct current is fed into the coil via carbon brushes bearing on a commutator, which consists of a metal ring split into two halves separated by insulation.

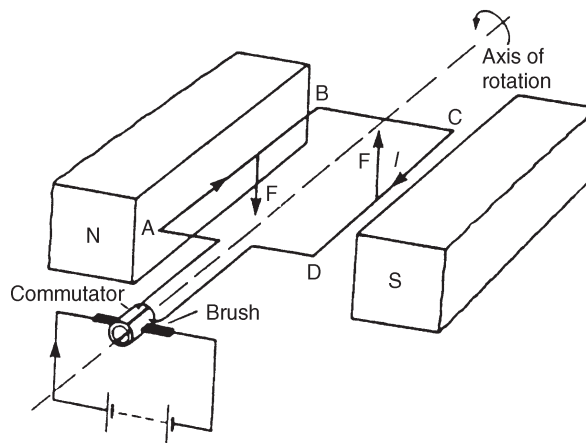


Figure 8.17

When current flows in the coil a magnetic field is set up around the coil which interacts with the magnetic field produced by the magnets. This causes a force F to be exerted on the current-carrying conductor which, by Fleming's left-hand rule, is downwards between points A and B and upward between C and D for the current direction shown. This causes a torque and the coil rotates anticlockwise. When the coil has turned through 90° from the position shown in Figure 8.17 the brushes connected to the

positive and negative terminals of the supply make contact with different halves of the commutator ring, thus reversing the direction of the current flow in the conductor. If the current is not reversed and the coil rotates past this position the forces acting on it change direction and it rotates in the opposite direction thus never making more than half a revolution. The current direction is reversed every time the coil swings through the vertical position and thus the coil rotates anti-clockwise for as long as the current flows. This is the principle of operation of a d.c. motor which is thus a device that takes in electrical energy and converts it into mechanical energy.

8.5 Principle of operation of a moving coil instrument

A moving-coil instrument operates on the motor principle. When a conductor carrying current is placed in a magnetic field, a force F is exerted on the conductor, given by $F = BIl$. If the flux density B is made constant (by using permanent magnets) and the conductor is a fixed length (say, a coil) then the force will depend only on the current flowing in the conductor.

In a moving-coil instrument a coil is placed centrally in the gap between shaped pole pieces as shown by the front elevation in Figure 8.18(a). (The airgap is kept as small as possible, although for clarity it is shown exaggerated in Figure 8.18). The coil is supported by steel pivots, resting in jewel bearings, on a cylindrical iron core. Current is led into and out of the coil by two phosphor bronze spiral hairsprings which are wound in opposite directions to minimize the effect of temperature change and to limit the coil swing (i.e. to **control** the movement) and return the movement to zero position when no current flows. Current flowing in the coil produces forces as shown in Fig 8.18(b), the directions being obtained by Fleming's left-hand rule. The two forces, F_A and F_B , produce a torque which will move the coil in a clockwise direction, i.e. move the pointer from left to right. Since force is proportional to current the scale is linear.

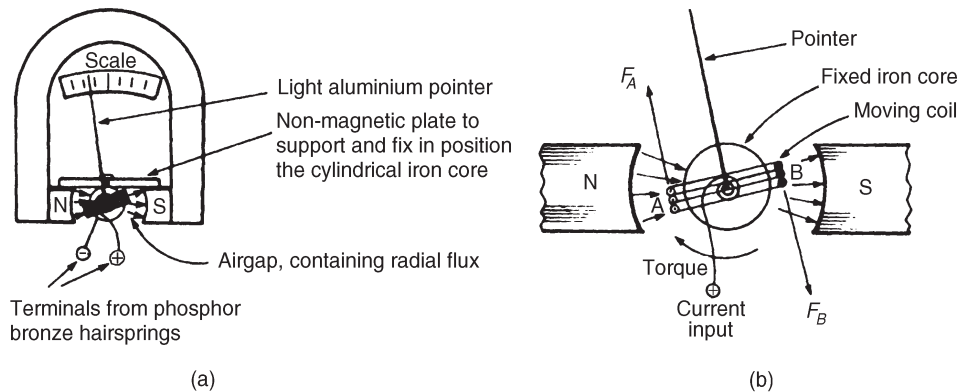


Figure 8.18

When the aluminium frame, on which the coil is wound, is rotated between the poles of the magnet, small currents (called eddy currents) are induced into the frame, and this provides automatically the necessary **damping** of the system due to the reluctance of the former to move within the magnetic field. The moving-coil instrument will measure only direct current or voltage and the terminals are marked positive and negative to ensure that the current passes through the coil in the correct direction to deflect the pointer 'up the scale'.

The range of this sensitive instrument is extended by using shunts and multipliers (see Chapter 10).

8.6 Force on a charge

When a charge of Q coulombs is moving at a velocity of v m/s in a magnetic field of flux density B teslas, the charge moving perpendicular to the field, then the magnitude of the force F exerted on the charge is given by:

$$F = QvB \text{ newtons}$$

Problem 17. An electron in a television tube has a charge of 1.6×10^{-19} coulombs and travels at 3×10^7 m/s perpendicular to a field of flux density $18.5 \mu\text{T}$. Determine the force exerted on the electron in the field.

From above, force $F = QvB$ newtons, where

Q = charge in coulombs = 1.6×10^{-19} C;

v = velocity of charge = 3×10^7 m/s;

and B = flux density = 18.5×10^{-6} T

$$\begin{aligned} \text{Hence force on electron } F &= 1.6 \times 10^{-19} \times 3 \times 10^7 \times 18.5 \times 10^{-6} \\ &= 1.6 \times 3 \times 18.5 \times 10^{-18} \\ &= 88.8 \times 10^{-18} = \mathbf{8.88 \times 10^{-17} \text{ N}} \end{aligned}$$

Further problems on the force on a charge may be found in Section 8.7 following, problems 7 and 8, page 99.

8.7 Further problems on electromagnetism

Force on a current-carrying conductor

- 1 A conductor carries a current of 70 A at right-angles to a magnetic field having a flux density of 1.5 T. If the length of the conductor in the field is 200 mm calculate the force acting on the conductor. What is the force when the conductor and field are at an angle of 45° ?

[21.0 N, 14.8 N]

- 2 Calculate the current required in a 240 mm length of conductor of a d.c. motor when the conductor is situated at right-angles to the magnetic field of flux density 1.25 T, if a force of 1.20 N is to be exerted on the conductor. [4.0 A]
- 3 A conductor 30 cm long is situated at right-angles to a magnetic field. Calculate the strength of the magnetic field if a current of 15 A in the conductor produces a force on it of 3.6 N. [0.80 T]
- 4 A conductor 300 mm long carries a current of 13 A and is at right-angles to a magnetic field between two circular pole faces, each of diameter 80 mm. If the total flux between the pole faces is 0.75 mWb calculate the force exerted on the conductor. [0.582 N]
- 5 (a) A 400 mm length of conductor carrying a current of 25 A is situated at right-angles to a magnetic field between two poles of an electric motor. The poles have a circular cross-section. If the force exerted on the conductor is 80 N and the total flux between the pole faces is 1.27 mWb, determine the diameter of a pole face.
(b) If the conductor in part (a) is vertical, the current flowing downwards and the direction of the magnetic field is from left to right, what is the direction of the 80 N force?
[(a) 14.2 mm (b) towards the viewer]
- 6 A coil is wound uniformly on a former having a width of 18 mm and a length of 25 mm. The former is pivoted about an axis passing through the middle of the two shorter sides and is placed in a uniform magnetic field of flux density 0.75 T, the axis being perpendicular to the field. If the coil carries a current of 120 mA, determine the force exerted on each coil side, (a) for a single-turn coil, (b) for a coil wound with 400 turns. [(a) 2.25×10^{-3} N (b) 0.9 N]

Force on a charge

- 7 Calculate the force exerted on a charge of 2×10^{-18} C travelling at 2×10^6 m/s perpendicular to a field of density 2×10^{-7} T. [8 $\times 10^{-19}$ N]
- 8 Determine the speed of a 10^{-19} C charge travelling perpendicular to a field of flux density 10^{-7} T, if the force on the charge is 10^{-20} N. [10^6 m/s]

9 Electromagnetic induction

At the end of this chapter you should be able to:

- understand how an e.m.f. may be induced in a conductor
- state Faraday's laws of electromagnetic induction
- state Lenz's law
- use Fleming's right-hand rule for relative directions
- appreciate that the induced e.m.f., $E = Blv$ or $E = Blv \sin \theta$
- calculate induced e.m.f. given B , l , v and θ and determine relative directions
- define inductance L and state its unit
- define mutual inductance
- appreciate that e.m.f. $E = -N \frac{d\Phi}{dt} = -L \frac{dI}{dt}$
- calculate induced e.m.f. given N , t , L , change of flux or change of current
- appreciate factors which affect the inductance of an inductor
- draw the circuit diagram symbols for inductors
- calculate the energy stored in an inductor using $W = \frac{1}{2}LI^2$ joules
- calculate inductance L of a coil, given $L = \frac{N\Phi}{I}$
- calculate mutual inductance using $E_2 = -M \frac{dI_1}{dt}$

9.1 Introduction to electromagnetic induction

When a conductor is moved across a magnetic field so as to cut through the lines of force (or flux), an electromotive force (e.m.f.) is produced in the conductor. If the conductor forms part of a closed circuit then the e.m.f. produced causes an electric current to flow round the circuit. Hence an e.m.f. (and thus current) is 'induced' in the conductor as a result of its movement across the magnetic field. This effect is known as '**electromagnetic induction**'.

Figure 9.1(a) shows a coil of wire connected to a centre-zero galvanometer, which is a sensitive ammeter with the zero-current position in the centre of the scale.

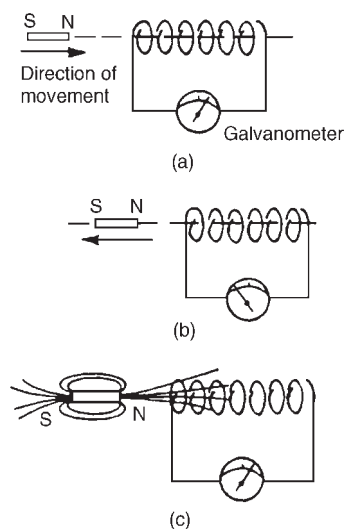


Figure 9.1

- When the magnet is moved at constant speed towards the coil (Figure 9.1(a)), a deflection is noted on the galvanometer showing that a current has been produced in the coil.
- When the magnet is moved at the same speed as in (a) but away from the coil the same deflection is noted but is in the opposite direction (see Figure 9.1(b)).
- When the magnet is held stationary, even within the coil, no deflection is recorded.
- When the coil is moved at the same speed as in (a) and the magnet held stationary the same galvanometer deflection is noted.
- When the relative speed is, say, doubled, the galvanometer deflection is doubled.
- When a stronger magnet is used, a greater galvanometer deflection is noted.
- When the number of turns of wire of the coil is increased, a greater galvanometer deflection is noted.

Figure 9.1(c) shows the magnetic field associated with the magnet. As the magnet is moved towards the coil, the magnetic flux of the magnet moves across, or cuts, the coil. **It is the relative movement of the magnetic flux and the coil that causes an e.m.f. and thus current, to be induced in the coil.** This effect is known as electromagnetic induction. The laws of electromagnetic induction stated in Section 9.2 evolved from experiments such as those described above.

9.2 Laws of electromagnetic induction

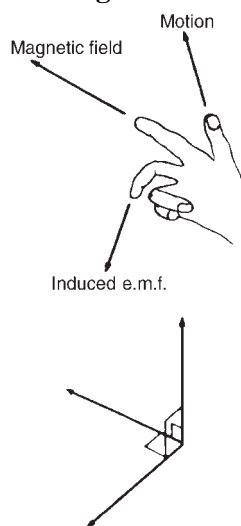


Figure 9.2

Faraday's laws of electromagnetic induction state:

- 'An induced e.m.f. is set up whenever the magnetic field linking that circuit changes.'
- 'The magnitude of the induced e.m.f. in any circuit is proportional to the rate of change of the magnetic flux linking the circuit.'

Lenz's law states:

'The direction of an induced e.m.f. is always such that it tends to set up a current opposing the motion or the change of flux responsible for inducing that e.m.f.'

An alternative method to Lenz's law of determining relative directions is given by **Fleming's Right-hand rule** (often called the geneRator rule) which states:

Let the thumb, first finger and second finger of the right hand be extended such that they are all at right angles to each other (as shown in Figure 9.2).

If the first finger points in the direction of the magnetic field, the thumb points in the direction of motion of the conductor relative to the magnetic field, then the second finger will point in the direction of the induced e.m.f.

Summarizing:

First finger — Field

ThuMb — Motion

SEcond finger — E.m.f.

In a generator, conductors forming an electric circuit are made to move through a magnetic field. By Faraday's law an e.m.f. is induced in the conductors and thus a source of e.m.f. is created. A generator converts mechanical energy into electrical energy. (The action of a simple a.c. generator is described in Chapter 14.) The induced e.m.f. E set up between the ends of the conductor shown in Figure 9.3 is given by:

$$E = Blv \text{ volts,}$$

where B , the flux density, is measured in teslas, l , the length of conductor in the magnetic field, is measured in metres, and v , the conductor velocity, is measured in metres per second.

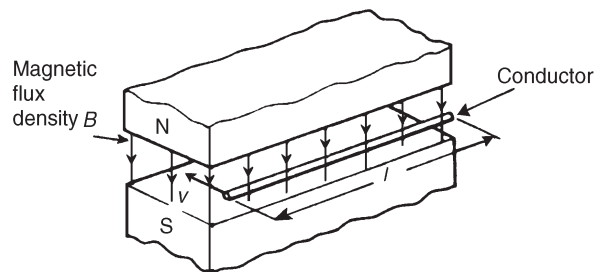


Figure 9.3

If the conductor moves at an angle θ° to the magnetic field (instead of at 90° as assumed above) then

$$E = Blv \sin \theta \text{ volts}$$

Problem 1. A conductor 300 mm long moves at a uniform speed of 4 m/s at right-angles to a uniform magnetic field of flux density 1.25 T. Determine the current flowing in the conductor when (a) its ends are open-circuited, (b) its ends are connected to a load of 20Ω resistance.

When a conductor moves in a magnetic field it will have an e.m.f. induced in it but this e.m.f. can only produce a current if there is a closed circuit.

$$\text{Induced e.m.f. } E = Blv = (1.25) \left(\frac{300}{1000} \right) (4) = 1.5 \text{ V}$$

- (a) If the ends of the conductor are open circuited no current will flow even though 1.5 V has been induced.
- (b) From Ohm's law, $I = \frac{E}{R} = \frac{1.5}{20} = \mathbf{0.075\text{ A}}$ or $\mathbf{75\text{ mA}}$

Problem 2. At what velocity must a conductor 75 mm long cut a magnetic field of flux density 0.6 T if an e.m.f. of 9 V is to be induced in it? Assume the conductor, the field and the direction of motion are mutually perpendicular.

Induced e.m.f. $E = Blv$, hence velocity $v = \frac{E}{Bl}$

$$\text{Hence } v = \frac{9}{(0.6)(75 \times 10^{-3})} = \frac{9 \times 10^3}{0.6 \times 75} = \mathbf{200\text{ m/s}}$$

Problem 3. A conductor moves with a velocity of 15 m/s at an angle of (a) 90° , (b) 60° and (c) 30° to a magnetic field produced between two square-faced poles of side length 2 cm. If the flux leaving a pole face is $5\text{ }\mu\text{Wb}$, find the magnitude of the induced e.m.f. in each case.

$v = 15\text{ m/s}$; length of conductor in magnetic field, $l = 2\text{ cm} = 0.02\text{ m}$;
 $A = 2 \times 2\text{ cm}^2 = 4 \times 10^{-4}\text{ m}^2$, $\Phi = 5 \times 10^{-6}\text{ Wb}$

$$\begin{aligned} \text{(a) } E_{90} &= Blv \sin 90^\circ = \left(\frac{\Phi}{A}\right) lv \sin 90^\circ = \frac{(5 \times 10^{-6})}{(4 \times 10^{-4})}(0.02)(15)(1) \\ &= \mathbf{3.75\text{ mV}} \end{aligned}$$

$$\text{(b) } E_{60} = Blv \sin 60^\circ = E_{90} \sin 60^\circ = 3.75 \sin 60^\circ = \mathbf{3.25\text{ mV}}$$

$$\text{(c) } E_{30} = Blv \sin 30^\circ = E_{90} \sin 30^\circ = 3.75 \sin 30^\circ = \mathbf{1.875\text{ mV}}$$

Problem 4. The wing span of a metal aeroplane is 36 m. If the aeroplane is flying at 400 km/h, determine the e.m.f. induced between its wing tips. Assume the vertical component of the earth's magnetic field is $40\text{ }\mu\text{T}$

Induced e.m.f. across wing tips, $E = Blv$

$B = 40\text{ }\mu\text{T} = 40 \times 10^{-6}\text{ T}$; $l = 36\text{ m}$

$$\begin{aligned} v &= 400 \frac{\text{km}}{\text{h}} \times 1000 \frac{\text{m}}{\text{km}} \times \frac{1\text{ h}}{60 \times 60\text{ s}} = \frac{(400)(1000)}{3600} \\ &= \frac{4000}{36}\text{ m/s} \end{aligned}$$

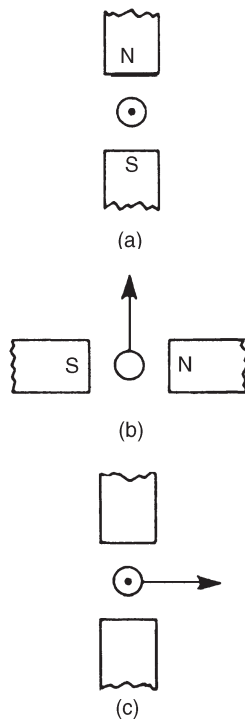


Figure 9.4

$$\begin{aligned} \text{Hence } E = Blv &= (40 \times 10^{-6})(36) \left(\frac{4000}{36} \right) \\ &= \mathbf{0.16 \text{ V}} \end{aligned}$$

Problem 5. The diagram shown in Figure 9.4 represents the generation of e.m.f.'s. Determine (i) the direction in which the conductor has to be moved in Figure 9.4(a), (ii) the direction of the induced e.m.f. in Figure 9.4(b), (iii) the polarity of the magnetic system in Figure 9.4(c).

The direction of the e.m.f., and thus the current due to the e.m.f. may be obtained by either Lenz's law or Fleming's Right-hand rule (i.e. GeneRator rule).

- (i) Using Lenz's law: The field due to the magnet and the field due to the current-carrying conductor are shown in Figure 9.5(a) and are seen to reinforce to the left of the conductor. Hence the force on the conductor is to the right. However Lenz's law states that the direction of the induced e.m.f. is always such as to oppose the effect producing it. **Thus the conductor will have to be moved to the left.**
- (ii) Using Fleming's right-hand rule:
First finger — Field, i.e. $N \rightarrow S$, or right to left;
Thumb — Motion, i.e. upwards;
SEcond finger — E.m.f., i.e. **towards the viewer or out of the paper**, as shown in Figure 9.5(b)
- (iii) The polarity of the magnetic system of Figure 9.4(c) is shown in Figure 9.5(c) and is obtained using Fleming's right-hand rule.

Further problems on induced e.m.f.'s may be found in Section 9.8, problems 1 to 5, page 109.

9.3 Inductance

Inductance is the name given to the property of a circuit whereby there is an e.m.f. induced into the circuit by the change of flux linkages produced by a current change.

When the e.m.f. is induced in the same circuit as that in which the current is changing, the property is called **self inductance, L**

When the e.m.f. is induced in a circuit by a change of flux due to current changing in an adjacent circuit, the property is called **mutual inductance, M** .

The unit of inductance is the **henry, H**.

'A circuit has an inductance of one henry when an e.m.f. of one volt is induced in it by a current changing at the rate of one ampere per second.'

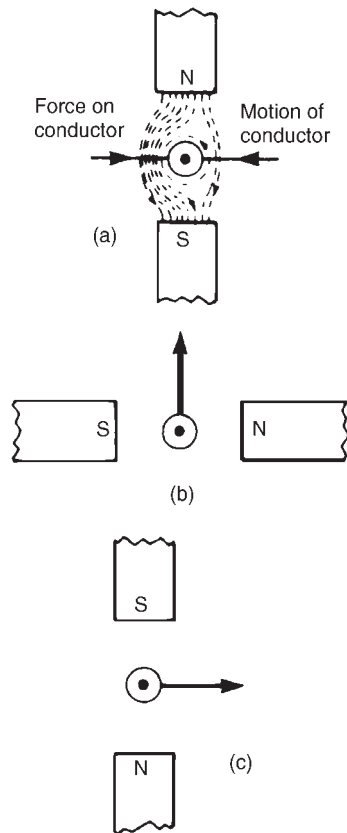


Figure 9.5

Induced e.m.f. in a coil of N turns,

$$E = -N \frac{d\Phi}{dt} \text{ volts,}$$

where $d\Phi$ is the change in flux in Webers, and dt is the time taken for the flux to change in seconds (i.e., $d\Phi/dt$ is the rate of change of flux).

Induced e.m.f. in a coil of inductance L henrys,

$$E = -L \frac{dI}{dt} \text{ volts,}$$

where dI is the change in current in amperes and dt is the time taken for the current to change in seconds (i.e., dI/dt is the rate of change of current). The minus sign in each of the above two equations remind us of its direction (given by Lenz's law).

Problem 6. Determine the e.m.f. induced in a coil of 200 turns when there is a change of flux of 25 mWb linking with it in 50 ms

$$\text{Induced e.m.f. } E = -N \frac{d\Phi}{dt} = -(200) \left(\frac{25 \times 10^{-3}}{50 \times 10^{-3}} \right) = \mathbf{-100 \text{ volts}}$$

Problem 7. A flux of 400 μWb passing through a 150-turn coil is reversed in 40 ms. Find the average e.m.f. induced.

Since the flux reverses, the flux changes from +400 μWb to -400 μWb , a total change of flux of 800 μWb

$$\begin{aligned} \text{Induced e.m.f. } E &= -N \frac{d\Phi}{dt} = -(150) \left(\frac{800 \times 10^{-6}}{40 \times 10^{-3}} \right) \\ &= - \left(\frac{150 \times 800 \times 10^3}{40 \times 10^6} \right) \end{aligned}$$

Hence **the average e.m.f. induced** $E = \mathbf{-3 \text{ volts}}$

Problem 8. Calculate the e.m.f. induced in a coil of inductance 12 H by a current changing at the rate of 4 A/s

$$\text{Induced e.m.f. } E = -L \frac{dI}{dt} = -(12)(4) = \mathbf{-48 \text{ volts}}$$

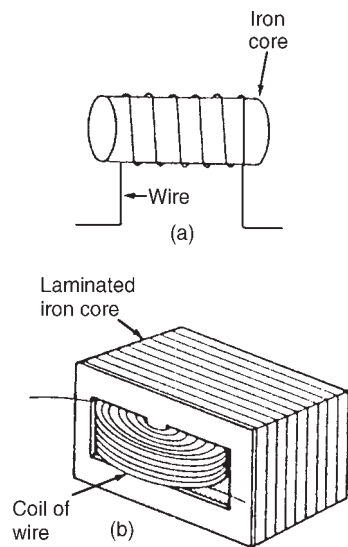


Figure 9.6

9.4 Inductors

A component called an inductor is used when the property of inductance is required in a circuit. The basic form of an inductor is simply a coil of wire.

Factors which affect the inductance of an inductor include:

- (i) the number of turns of wire—the more turns the higher the inductance
- (ii) the cross-sectional area of the coil of wire—the greater the cross-sectional area the higher the inductance
- (iii) the presence of a magnetic core—when the coil is wound on an iron core the same current sets up a more concentrated magnetic field and the inductance is increased
- (iv) the way the turns are arranged—a short thick coil of wire has a higher inductance than a long thin one.

Figure 9.7

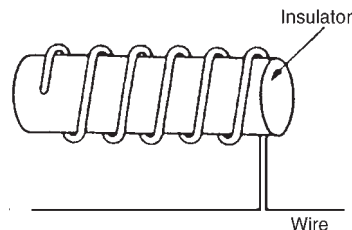
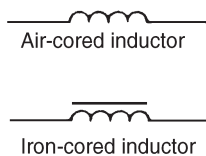


Figure 9.8

Problem 9. An e.m.f. of 1.5 kV is induced in a coil when a current of 4 A is reversed in 8 ms. Calculate the inductance of the coil.

Change in current, $dI = (4 - 0) = 4$ A; $dt = 8$ ms $= 8 \times 10^{-3}$ s;

$$\frac{dI}{dt} = \frac{4}{8 \times 10^{-3}} = \frac{4000}{8} = 500 \text{ A/s}; E = 1.5 \text{ kV} = 1500 \text{ V}$$

$$\text{Since } |E| = L \left(\frac{dI}{dt} \right), \text{ inductance, } L = \frac{|E|}{(dI/dt)} = \frac{1500}{500} = 3 \text{ H}$$

(Note that $|E|$ means the 'magnitude of E ', which disregards the minus sign)

Further problems on inductance may be found in Section 9.8, problems 6 to 9, page 110.

9.5 Energy stored An inductor possesses an ability to store energy. The energy stored, W , in the magnetic field of an inductor is given by:

$$W = \frac{1}{2}LI^2 \text{ joules}$$

Problem 10. An 8 H inductor has a current of 3 A flowing through it. How much energy is stored in the magnetic field of the inductor?

$$\text{Energy stored, } W = \frac{1}{2}LI^2 = \frac{1}{2}(8)(3)^2 = \mathbf{36 \text{ joules}}$$

Further problems on energy stored may be found in Section 9.8, problems 10 and 11, page 110.

9.6 Inductance of a coil

If a current changing from 0 to I amperes, produces a flux change from 0 to Φ Webers, then $dI = I$ and $d\Phi = \Phi$. Then, from Section 9.3, induced e.m.f. $E = N\Phi/t = LI/t$, from which

$$\text{inductance of coil, } L = \frac{N\Phi}{I} \text{ henrys}$$

Problem 11. Calculate the coil inductance when a current of 4 A in a coil of 800 turns produces a flux of 5 mWb linking with the coil.

$$\text{For a coil, inductance } L = \frac{N\Phi}{I} = \frac{(800)(5 \times 10^{-3})}{4} = \mathbf{1 \text{ H}}$$

Problem 12. A flux of 25 mWb links with a 1500 turn coil when a current of 3 A passes through the coil. Calculate (a) the inductance of the coil, (b) the energy stored in the magnetic field, and (c) the average e.m.f. induced if the current falls to zero in 150 ms.

$$\text{(a) Inductance, } L = \frac{N\Phi}{I} = \frac{(1500)(25 \times 10^{-3})}{3} = \mathbf{12.5 \text{ H}}$$

$$\text{(b) Energy stored, } W = \frac{1}{2}LI^2 = \frac{1}{2}(12.5)(3)^2 = \mathbf{56.25 \text{ J}}$$

$$\text{(c) Induced e.m.f., } E = -L \frac{dI}{dt} = -(12.5) \left(\frac{3 - 0}{150 \times 10^{-3}} \right) = \mathbf{-250 \text{ V}}$$

$$\begin{aligned} \text{(Alternatively, } E &= -N \left(\frac{d\Phi}{dt} \right) = -(1500) \left(\frac{25 \times 10^{-3}}{150 \times 10^{-3}} \right) \\ &= \mathbf{-250 \text{ V}} \end{aligned}$$

since if the current falls to zero so does the flux)

Problem 13. A 750 turn coil of inductance 3 H carries a current of 2 A. Calculate the flux linking the coil and the e.m.f. induced in the coil when the current collapses to zero in 20 ms

$$\begin{aligned} \text{Coil inductance, } L &= \frac{N\Phi}{I} \text{ from which, flux } \Phi = \frac{LI}{N} \\ &= \frac{(3)(2)}{750} = 8 \times 10^{-3} = \mathbf{8 \text{ mWb}} \end{aligned}$$

$$\text{Induced e.m.f. } E = -L \left(\frac{dI}{dt} \right) = -3 \left(\frac{2 - 0}{20 \times 10^{-3}} \right) = \mathbf{-300 \text{ V}}$$

$$\text{(Alternatively, } E = -N \frac{d\Phi}{dt} = -(750) \left(\frac{8 \times 10^{-3}}{20 \times 10^{-3}} \right) = \mathbf{-300 \text{ V}})$$

Further problems on the inductance of a coil may be found in Section 9.8, problems 12 to 18, page 110.

9.7 Mutual inductance

Mutually induced e.m.f. in the second coil,

$$E_2 = -M \frac{dI_1}{dt} \text{ volts,}$$

where M is the **mutual inductance** between two coils, in henrys, and dI_1/dt is the rate of change of current in the first coil.

The phenomenon of mutual inductance is used in **transformers** (see Chapter 20, page 315). Mutual inductance is developed further in Chapter 43 on magnetically coupled circuits (see page 841).

Problem 14. Calculate the mutual inductance between two coils when a current changing at 200 A/s in one coil induces an e.m.f. of 1.5 V in the other.

$$\text{Induced e.m.f. } |E_2| = M \frac{dI_1}{dt}, \text{ i.e., } 1.5 = M(200)$$

$$\text{Thus mutual inductance, } M = \frac{1.5}{200} = \mathbf{0.0075 \text{ H or } 7.5 \text{ mH}}$$

Problem 15. The mutual inductance between two coils is 18 mH. Calculate the steady rate of change of current in one coil to induce an e.m.f. of 0.72 V in the other.

$$\text{Induced e.m.f., } |E_2| = M \frac{dI_1}{dt}$$

$$\text{Hence rate of change of current, } \frac{dI_1}{dt} = \frac{|E_2|}{M} = \frac{0.72}{0.018} = \mathbf{40 \text{ A/s}}$$

Problem 16. Two coils have a mutual inductance of 0.2 H. If the current in one coil is changed from 10 A to 4 A in 10 ms, calculate (a) the average induced e.m.f. in the second coil, (b) the change of flux linked with the second coil if it is wound with 500 turns.

$$(a) \text{ Induced e.m.f. } E_2 = -M \frac{dI_1}{dt} = -(0.2) \left(\frac{10 - 4}{10 \times 10^{-3}} \right) = \mathbf{-120 \text{ V}}$$

$$(b) \text{ Induced e.m.f. } |E_2| = N \frac{d\Phi}{dt}, \text{ hence } d\Phi = \frac{|E_2| dt}{N}$$

$$\text{Thus the change of flux, } d\Phi = \frac{120(10 \times 10^{-3})}{500} = \mathbf{2.4 \text{ mWb}}$$

Further problems on mutual inductance may be found in Section 9.8 following, problems 19 to 22, page 111.

9.8 Further problems on electromagnetic induction

Induced e.m.f.

- A conductor of length 15 cm is moved at 750 mm/s at right-angles to a uniform flux density of 1.2 T. Determine the e.m.f. induced in the conductor. [0.135 V]
- Find the speed that a conductor of length 120 mm must be moved at right angles to a magnetic field of flux density 0.6 T to induce in it an e.m.f. of 1.8 V. [25 m/s]
- A 25 cm long conductor moves at a uniform speed of 8 m/s through a uniform magnetic field of flux density 1.2 T. Determine the current flowing in the conductor when (a) its ends are open-circuited, (b) its ends are connected to a load of 15 ohms resistance. [(a) 0 (b) 0.16 A]
- A car is travelling at 80 km/h. Assuming the back axle of the car is 1.76 m in length and the vertical component of the earth's magnetic field is 40 μT , find the e.m.f. generated in the axle due to motion. [1.56 mV]

- 5 A conductor moves with a velocity of 20 m/s at an angle of (a) 90° (b) 45° (c) 30° , to a magnetic field produced between two square-faced poles of side length 2.5 cm. If the flux on the pole face is 60 mWb, find the magnitude of the induced e.m.f. in each case.
[(a) 48 V (b) 33.9 V (c) 24 V]

Inductance

- 6 Find the e.m.f. induced in a coil of 200 turns when there is a change of flux of 30 mWb linking with it in 40 ms. [−150 V]
- 7 An e.m.f. of 25 V is induced in a coil of 300 turns when the flux linking with it changes by 12 mWb. Find the time, in milliseconds, in which the flux makes the change. [144 ms]
- 8 An ignition coil having 10 000 turns has an e.m.f. of 8 kV induced in it. What rate of change of flux is required for this to happen? [0.8 Wb/s]
- 9 A flux of 0.35 mWb passing through a 125-turn coil is reversed in 25 ms. Find the magnitude of the average e.m.f. induced. [3.5 V]

Energy stored

- 10 Calculate the value of the energy stored when a current of 30 mA is flowing in a coil of inductance 400 mH. [0.18 mJ]
- 11 The energy stored in the magnetic field of an inductor is 80 J when the current flowing in the inductor is 2 A. Calculate the inductance of the coil. [40 H]

Inductance of a coil

- 12 A flux of 30 mWb links with a 1200 turn coil when a current of 5 A is passing through the coil. Calculate (a) the inductance of the coil, (b) the energy stored in the magnetic field, and (c) the average e.m.f. induced if the current is reduced to zero in 0.20 s.
[(a) 7.2 H (b) 90 J (c) 180 V]
- 13 An e.m.f. of 2 kV is induced in a coil when a current of 5 A collapses uniformly to zero in 10 ms. Determine the inductance of the coil. [4 H]
- 14 An average e.m.f. of 60 V is induced in a coil of inductance 160 mH when a current of 7.5 A is reversed. Calculate the time taken for the current to reverse. [40 ms]
- 15 A coil of 2500 turns has a flux of 10 mWb linking with it when carrying a current of 2 A. Calculate the coil inductance and the e.m.f. induced in the coil when the current collapses to zero in 20 ms.
[12.5 H, 1.25 kV]

- 16 A coil is wound with 600 turns and has a self inductance of 2.5 H. What current must flow to set up a flux of 20 mWb?
[4.8 A]
- 17 When a current of 2 A flows in a coil, the flux linking with the coil is 80 μ Wb. If the coil inductance is 0.5 H, calculate the number of turns of the coil.
[12 500]
- 18 A steady current of 5 A when flowing in a coil of 1000 turns produces a magnetic flux of 500 μ Wb. Calculate the inductance of the coil. The current of 5 A is then reversed in 12.5 ms. Calculate the e.m.f. induced in the coil.
[0.1 H, 80 V]

Mutual inductance

- 19 The mutual inductance between two coils is 150 mH. Find the magnitude of the e.m.f. induced in one coil when the current in the other is increasing at a rate of 30 A/s.
[4.5 V]
- 20 Determine the mutual inductance between two coils when a current changing at 50 A/s in one coil induces an e.m.f. of 80 mV in the other.
[1.6 mH]
- 21 Two coils have a mutual inductance of 0.75 H. Calculate the magnitude of the e.m.f. induced in one coil when a current of 2.5 A in the other coil is reversed in 15 ms.
[250 V]
- 22 The mutual inductance between two coils is 240 mH. If the current in one coil changes from 15 A to 6 A in 12 ms, calculate (a) the average e.m.f. induced in the other coil, (b) the change of flux linked with the other coil if it is wound with 400 turns.
[(a) -180 V (b) 5.4 mWb]