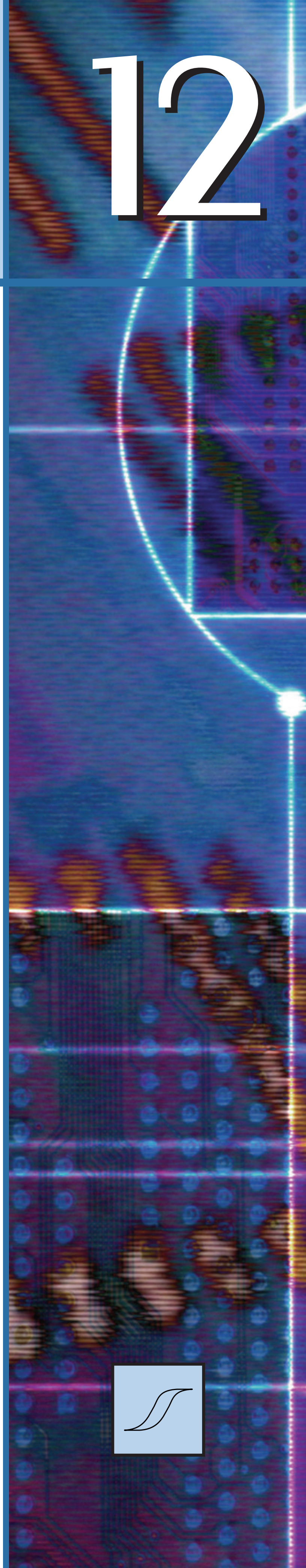
12 Magnetic Circuits



Objectives

- ***Become aware of the similarities between the analysis of magnetic circuits and electric circuits.***
- ***Develop a clear understanding of the important parameters of a magnetic circuit and how to find each quantity for a variety of magnetic circuit configurations.***
- ***Begin to appreciate why a clear understanding of magnetic circuit parameters is an important component in the design of electrical/electronic systems.***

12.1 INTRODUCTION

Magnetic and electromagnetic effects play an important role in the design of a wide variety of electrical/electronic systems in use today. Motors, generators, transformers, loudspeakers, relays, medical equipment and movements of all kinds depend on magnetic effects to function properly. The response and characteristics of each have an impact on the current and voltage levels of the system, the efficiency of the design, the resulting size, and many other important considerations.

Fortunately, there is a great deal of similarity between the analyses of electric circuits and magnetic circuits. The magnetic flux of magnetic circuits has properties very similar to the current of electric circuits. As shown in Fig. 11.15, it has a direction and a closed path. The magnitude of the established flux is a direct function of the applied **magnetomotive force** resulting in a duality with electric circuits where the resulting current is a function of the magnitude of the applied voltage. The flux established is also inversely related to the structural opposition of the magnetic path in the same way the current in a network is inversely related to the resistance of the network. All of these similarities are used throughout the analysis to clarify the approach.

One of the difficulties associated with studying magnetic circuits is that three different systems of units are commonly used in the industry. The manufacturer, application, and type of component all have an impact on which system is used. To the extent practical, the SI system is applied throughout the chapter. References to the CGS and English systems require the use of Appendix F.

12.2 MAGNETIC FIELD

The magnetic field distribution around a permanent magnet or **electromagnet** was covered in detail in Chapter 11. Recall that flux lines strive to be as short as possible and take the path with the highest permeability. The **flux density** is defined by Eq. 12.1 (Eq. 11.1 repeated here for convenience).

$$B = \frac{\Phi}{A} \qquad \begin{aligned} B &= \text{Wb/m}^2 = \text{teslas (T)} \\ \Phi &= \text{webers (Wb)} \\ A &= \text{m}^2 \end{aligned} \qquad \textbf{(12.1)}$$

The "pressure" on the system to establish magnetic lines of force is determined by the applied magnetomotive force which is directly related to the number of turns and current of the magnetizing coil as appearing in Eq. 12.2 (Eq. 11.3 repeated here for convenience).

$$\boxed{\mathscr{F} = NI} \qquad \begin{aligned} \mathscr{F} &= \text{ampere-turns (At)} \\ N &= \text{turns (t)} \\ I &= \text{amperes (A)} \end{aligned} \tag{12.2}$$

The level of magnetic flux established in a ferromagnetic core is a direction function of the permeability of the material. **Ferromagnetic materials** have a very high level of **permeability** while non-magnetic materials such as air and wood have very low levels. The ratio of the permeability of the material to that of air is called the **relative permeability** and is defined by Eq. 12.3 (Eq. 11.5 repeated here for convenience).

$$\boxed{\mu_r = \frac{\mu}{\mu_o}} \qquad \mu_o = 4\pi \times 10^{-7}\text{Wb/A}\cdot\text{m} \tag{12.3}$$

As mentioned in Chapter 11, the values of μ_r are not provided in a table format because the value is determined by the other quantities of the magnetic circuit. Change the magnetomotive force, and the relative permeability changes.

12.3 RELUCTANCE

The resistance of a material to the flow of charge (current) is determined for electric circuits by the equation

$$R = \rho\frac{l}{A} \qquad (\text{ohms}, \Omega)$$

The **reluctance** of a material to the setting up of magnetic flux lines in the material is determined by the following equation:

$$\boxed{\mathscr{R} = \frac{l}{\mu A}} \qquad (\text{rels, or At/Wb}) \tag{12.4}$$

where $\mathscr{R}$ is the reluctance, l is the length of the magnetic path, and A is the cross-sectional area. The t in the units At/Wb is the number of turns of the applied winding. More is said about ampere-turns (At) in the next section. Note that the resistance and reluctance are inversely proportional to the area, indicating that an increase in area results in a reduction in each and an *increase* in the desired result: current and flux. For an increase in length, the opposite is true, and the desired effect is reduced. The reluctance, however, is inversely proportional to the permeability, while the resistance is directly proportional to the resistivity. The larger the μ or the smaller the ρ, the smaller the reluctance and resistance, respectively. Obviously, therefore, materials with high permeability, such as the ferromagnetics, have very small reluctances and result in an increased measure of flux through the core. There is no widely accepted unit for reluctance, although the *rel* and the At/Wb are usually applied.

12.4 OHM'S LAW FOR MAGNETIC CIRCUITS

Recall the equation

$$\text{Effect} = \frac{\text{cause}}{\text{opposition}}$$

appearing in Chapter 4 to introduce Ohm's law for electric circuits. For magnetic circuits, the effect desired is the flux Φ. The cause is the **magnetomotive force (mmf)** $\mathscr{F}$, which is the external force (or "pressure") required to set up the **magnetic flux lines** within the magnetic material. The opposition to the setting up of the flux Φ is the reluctance $\mathscr{R}$.

Substituting, we have

$$\Phi = \frac{\mathscr{F}}{\mathscr{R}} \tag{12.5}$$

Since $\mathscr{F} = NI$, Eq. 12.5 clearly reveals that an increase in the number of turns or the current through the wire in Fig. 12.1 results in an increased "pressure" on the system to establish the flux lines through the core.

Although there is a great deal of similarity between electric and magnetic circuits, you must understand that the flux Φ is not a "flow" variable such as current in an electric circuit. Magnetic flux is established in the core through the alteration of the atomic structure of the core due to external pressure and is not a measure of the flow of some charged particles through the core.

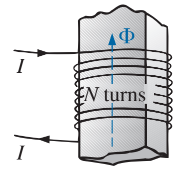


FIG. 12.1
Defining the components of a magnetomotive force.

12.5 MAGNETIZING FORCE

The magnetomotive force per unit length is called the **magnetizing force** (H). In equation form,

$$H = \frac{\mathscr{F}}{l} \qquad \text{(At/m)} \tag{12.6}$$

Substituting for the magnetomotive force results in

$$H = \frac{NI}{l} \qquad \text{(At/m)} \tag{12.7}$$

For the magnetic circuit in Fig. 12.2, if $NI = 40$ At and $l = 0.2$ m, then

$$H = \frac{NI}{l} = \frac{40 \text{ At}}{0.2 \text{ m}} = 200 \text{ At/m}$$

In words, the result indicates that there are 200 At of "pressure" per meter to establish flux in the core.

Note in Fig. 12.2 that the direction of the flux Φ can be determined by placing the fingers of your right hand in the direction of current around the core and noting the direction of the thumb. It is interesting to realize that *the magnetizing force is independent of the type of core material*—it is determined solely by the number of turns, the current, and the length of the core.

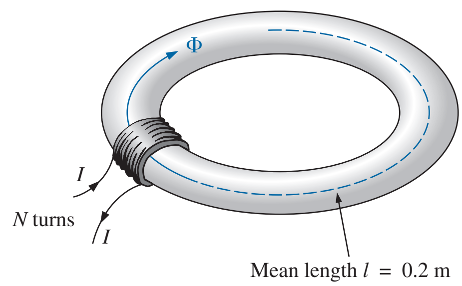


FIG. 12.2
Defining the magnetizing force of a magnetic circuit.

The applied magnetizing force has a pronounced effect on the resulting permeability of a magnetic material. As the magnetizing force increases, the permeability rises to a maximum and then drops to a minimum, as shown in Fig. 12.3 for three commonly employed magnetic materials.

The flux density and the magnetizing force are related by the following equation:

$$B = \mu H \tag{12.8}$$

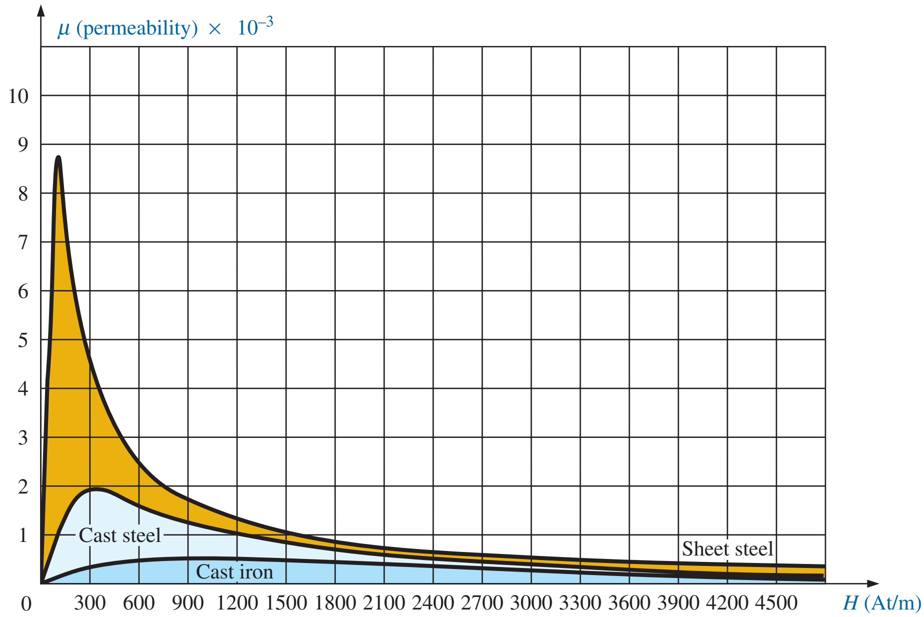


FIG. 12.3
Variation of μ with the magnetizing force.

This equation indicates that for a particular magnetizing force, the greater the permeability, the greater the induced flux density.

Since henries (H) and the magnetizing force (H) use the same capital letter, it must be pointed out that all units of measurement in the text, such as henries, use roman letters, such as H, whereas variables such as the magnetizing force use italic letters, such as H.

12.6 HYSTERESIS

A curve of the flux density B versus the magnetizing force H of a material is of particular importance to the engineer. Curves of this type can usually be found in manuals, descriptive pamphlets, and brochures published by manufacturers of magnetic materials. A typical B-H curve for a ferromagnetic material such as steel can be derived using the setup in Fig. 12.4.

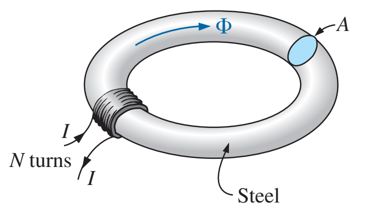


FIG. 12.4
Series magnetic circuit used to define the hysteresis curve.

The core is initially unmagnetized, and the current $I = 0$. If the current I is increased to some value above zero, the magnetizing force H increases to a value determined by

$$H\uparrow = \frac{NI\uparrow}{l}$$

The flux Φ and the flux density B ($B = \Phi/A$) also increase with the current I (or H). If the material has no residual magnetism, and the magnetizing force H is increased from zero to some value H_a, the B-H curve follows the path shown in Fig. 12.5 between o and a. If the magnetizing force H is increased until saturation (H_s) occurs, the curve continues as shown in the figure to point b. When saturation occurs, the flux density has, *for all practical purposes,* reached its maximum value. Any further increase in current through the coil increasing $H = NI/l$ results in a very small increase in flux density B.

If the magnetizing force is reduced to zero by letting I decrease to zero, the curve follows the path of the curve between b and c. The flux

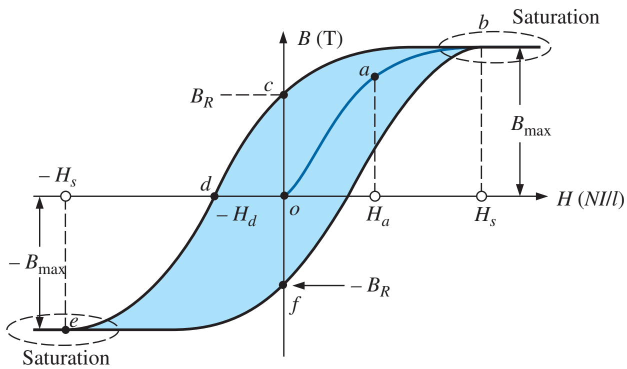


FIG. 12.5
Hysteresis curve.

density B_R, which remains when the magnetizing force is zero, is called the *residual flux density.* It is this residual flux density that makes it possible to create permanent magnets. If the coil is now removed from the core in Fig. 12.4, the core will still have the magnetic properties determined by the residual flux density, a measure of its "retentivity." If the current I is reversed, developing a magnetizing force, $-H$, the flux density B decreases with an increase in I. Eventually, the flux density will be zero when $-H_d$ (the portion of curve from c to d) is reached. The magnetizing force $-H_d$ required to "coerce" the flux density to reduce its level to zero is called the *coercive force,* a measure of the coercivity of the magnetic sample. As the force $-H$ is increased until saturation again occurs and is then reversed and brought back to zero, the path def results. If the magnetizing force is increased in the positive direction ($+H$), the curve traces the path shown from f to b. The entire curve represented by $bcdefb$ is called the **hysteresis** curve for the ferromagnetic material, from the Greek *hysterein,* meaning "to lag behind." The flux density B *lagged* behind the magnetizing force H during the entire plotting of the curve. When H was zero at c, B was not zero but had only begun to decline. Long after H had passed through zero and had become equal to $-H_d$ did the flux density B finally become equal to zero.

If the entire cycle is repeated, the curve obtained for the same core will be determined by the maximum H applied. Three hysteresis loops for the same material for maximum values of H less than the saturation value are shown in Fig. 12.6. In addition, the saturation curve is repeated for comparison purposes.

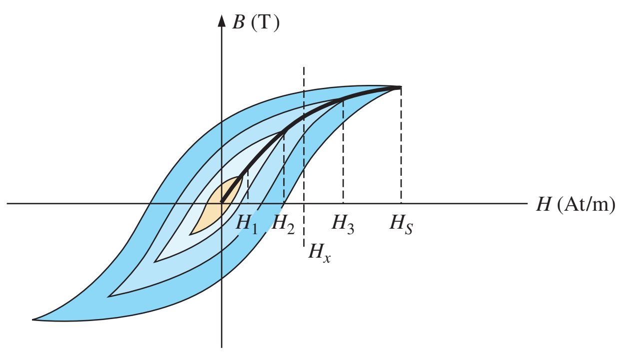


FIG. 12.6
Defining the normal magnetization curve.

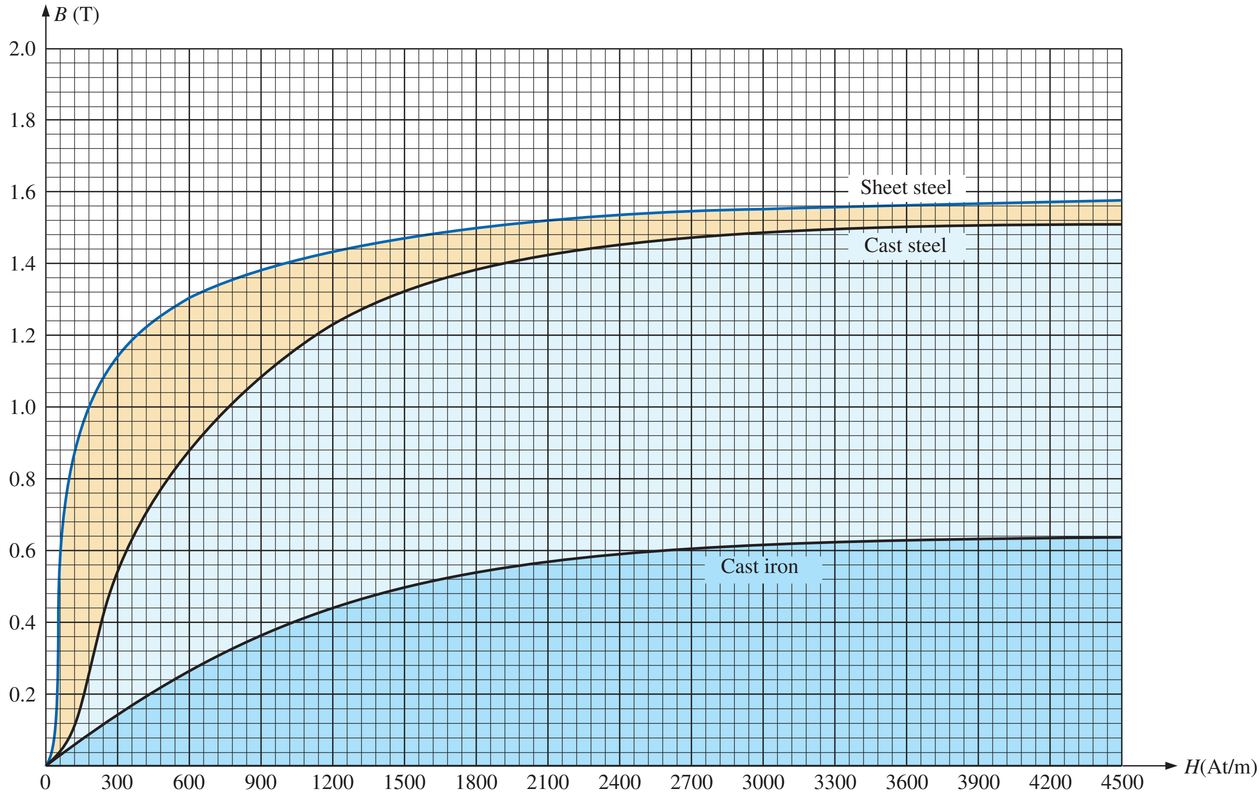


FIG. 12.7

Normal magnetization curve for three ferromagnetic materials.

Note from the various curves that for a particular value of H, say, H_x, the value of B can vary widely, as determined by the history of the core. In an effort to assign a particular value of B to each value of H, we compromise by connecting the tips of the hysteresis loops. The resulting curve, shown by the heavy, solid line in Fig. 12.6 and for various materials in Fig. 12.7, is called the *normal magnetization curve.* An expanded view of one region appears in Fig. 12.8.

A comparison of Figs. 12.3 and 12.7 shows that for the same value of H, the value of B is higher in Fig. 12.7 for the materials with the higher μ in Fig. 12.3. This is particularly obvious for low values of H. This correspondence between the two figures must exist since $B = \mu H$. In fact, if in Fig. 12.7 we find μ for each value of H using the equation $\mu = B/H$, we obtain the curves in Fig. 12.3.

It is interesting to note that the hysteresis curves in Fig. 12.6 have a *point symmetry* about the origin; that is, the inverted pattern to the left of the vertical axis is the same as that appearing to the right of the vertical axis. In addition, you will find that a further application of the same magnetizing forces to the sample results in the same plot. For a current I in $H = NI/l$ that moves between positive and negative maximums at a fixed rate, the same B-H curve results during each cycle. Such will be the case when we examine ac (sinusoidal) networks in the later chapters. The reversal of the field (Φ) due to the changing current direction results in a loss of energy that can best be described by first introducing the *domain theory of magnetism.*

Within each atom, the orbiting electrons (described in Chapter 2) are also spinning as they revolve around the nucleus. The atom, due to its

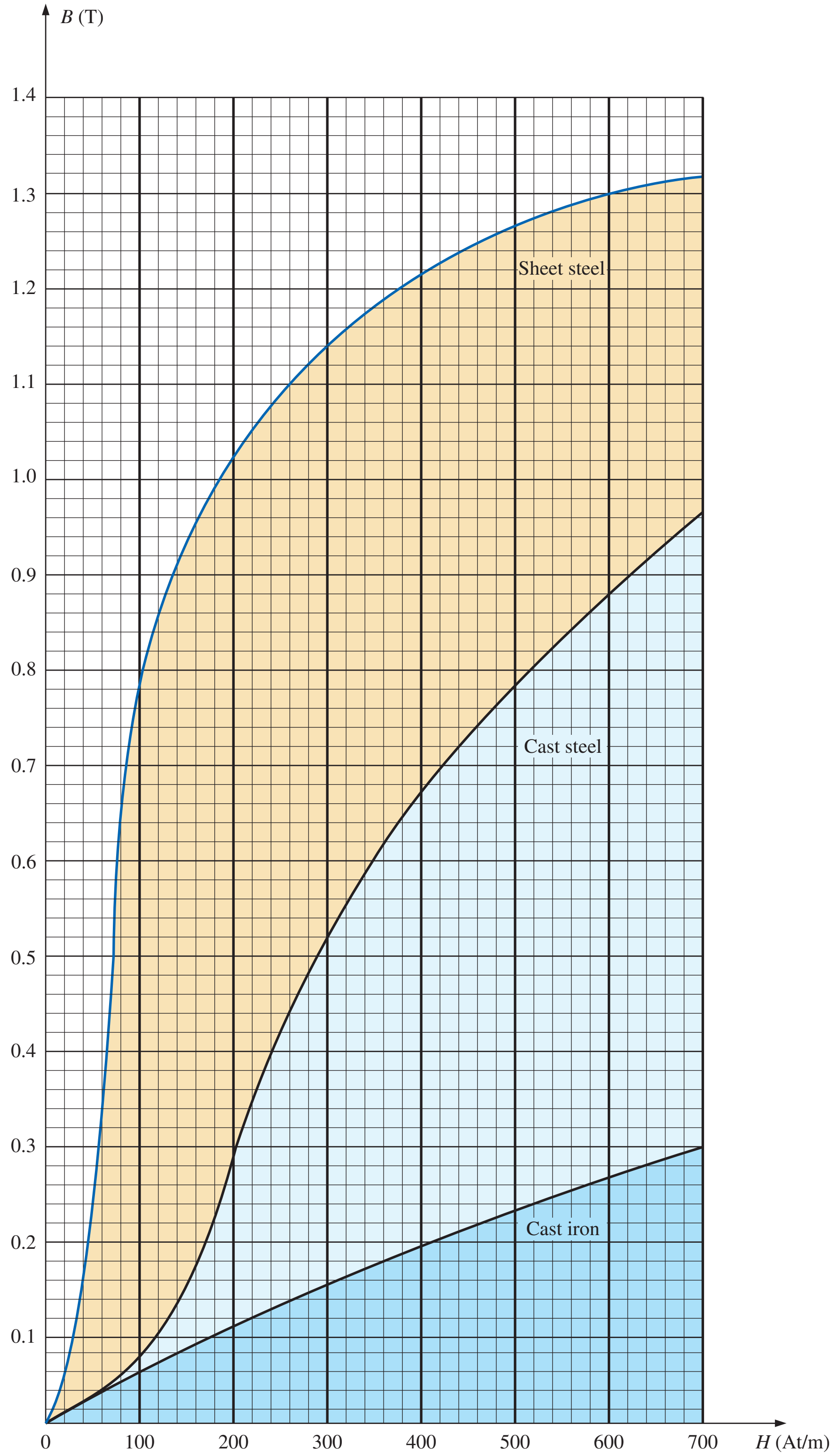


FIG. 12.8
Expanded view of Fig. 12.7 for the low magnetizing force region.

spinning electrons, has a magnetic field associated with it. In nonmagnetic materials, the net magnetic field is effectively zero since the magnetic fields due to the atoms of the material oppose each other. In magnetic materials such as iron and steel, however, the magnetic fields of groups of atoms numbering in the order of 10^{12} are aligned, forming very small bar magnets. This group of magnetically aligned atoms is called a **domain.** Each domain is a separate entity; that is, each domain is independent of the surrounding domains. For an unmagnetized sample of magnetic material, these domains appear in a random manner, such as shown in Fig. 12.9(a). The net magnetic field in any one direction is zero.

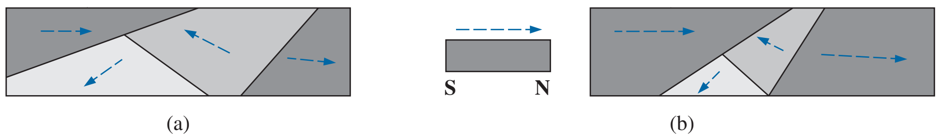


FIG. 12.9
Demonstrating the domain theory of magnetism.

When an external magnetizing force is applied, the domains that are nearly aligned with the applied field grow at the expense of the less favorably oriented domains, such as shown in Fig. 12.9(b). Eventually, if a sufficiently strong field is applied, all of the domains have the orientation of the applied magnetizing force, and any further increase in external field will not increase the strength of the magnetic flux through the core—a condition referred to as *saturation.* The elasticity of the above is evidenced by the fact that when the magnetizing force is removed, the alignment is lost to some measure, and the flux density drops to B_R. In other words, the removal of the magnetizing force results in the return of a number of misaligned domains within the core. The continued alignment of a number of the domains, however, accounts for our ability to create **permanent magnets.**

At a point just before saturation, the opposing unaligned domains are reduced to small cylinders of various shapes referred to as *bubbles.* These bubbles can be moved within the magnetic sample through the application of a *controlling* magnetic field. These magnetic bubbles form the basis of the recently designed bubble memory system for computers.

12.7 AMPÈRE'S CIRCUITAL LAW

As mentioned in the introduction to this chapter, there is a broad similarity between the analyses of electric and magnetic circuits. This has already been demonstrated to some extent for the quantities in Table 12.1.

TABLE 12.1

	Electric Circuits	Magnetic Circuits
Cause	E	$\mathscr{F}$
Effect	I	Φ
Opposition	R	$\mathscr{R}$

If we apply the "cause" analogy to Kirchhoff's voltage law ($\Sigma_{\circlearrowright} V = 0$), we obtain the following:

$$\Sigma_{\circlearrowright} \mathscr{F} = 0 \qquad \text{(for magnetic circuits)} \qquad \textbf{(12.9)}$$

which, in words, states that the algebraic sum of the rises and drops of the mmf around a closed loop of a magnetic circuit is equal to zero; that is, the sum of the rises in mmf equals the sum of the drops in mmf around a closed loop.

Eq. (12.9) is referred to as **Ampère's circuital law.** When it is applied to magnetic circuits, sources of mmf are expressed by the equation

$$\mathscr{F} = NI \qquad \text{(At)} \qquad \textbf{(12.10)}$$

The equation for the mmf drop across a portion of a magnetic circuit can be found by applying the relationships listed in Table 12.1; that is, for electric circuits,

$$V = IR$$

resulting in the following for magnetic circuits:

$$\mathscr{F} = \Phi\mathscr{R} \quad \text{(At)} \qquad \textbf{(12.11)}$$

where Φ is the flux passing through a section of the magnetic circuit and $\mathscr{R}$ is the reluctance of that section. The reluctance, however, is seldom calculated in the analysis of magnetic circuits. A more practical equation for the mmf drop is

$$\mathscr{F} = Hl \quad \text{(At)} \qquad \textbf{(12.12)}$$

as derived from Eq. (12.6), where H is the magnetizing force on a section of a magnetic circuit and l is the length of the section.

As an example of Eq. (12.9), consider the magnetic circuit appearing in Fig. 12.10 constructed of three different ferromagnetic materials.

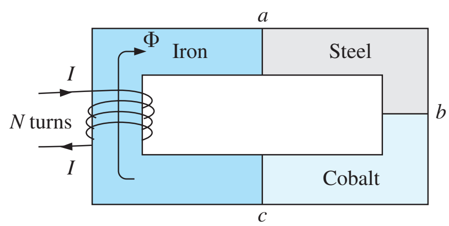


FIG. 12.10
Series magnetic circuit of three different materials.

Applying Ampère's circuital law, we have

$$\Sigma_{\circlearrowleft}\, \mathscr{F} = 0$$

$$\underbrace{+NI}_{\text{Rise}} - \underbrace{H_{ab}l_{ab}}_{\text{Drop}} - \underbrace{H_{bc}l_{bc}}_{\text{Drop}} - \underbrace{H_{ca}l_{ca}}_{\text{Drop}} = 0$$

$$\underbrace{NI}_{\substack{\text{Impressed}\\\text{mmf}}} = \underbrace{H_{ab}l_{ab} + H_{bc}l_{bc} + H_{ca}l_{ca}}_{\text{mmf drops}}$$

All the terms of the equation are known except the magnetizing force for each portion of the magnetic circuit, which can be found by using the *B-H* curve if the flux density *B* is known.

12.8 FLUX Φ

If we continue to apply the relationships described in the previous section to Kirchhoff's current law, we find that the sum of the fluxes entering a junction is equal to the sum of the fluxes leaving a junction; that is, for the circuit in Fig. 12.11,

$$\Phi_a = \Phi_b + \Phi_c \qquad \text{(at junction } a\text{)}$$

or

$$\Phi_b + \Phi_c = \Phi_a \qquad \text{(at junction } b\text{)}$$

both of which are equivalent.

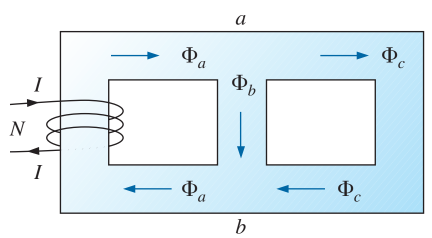


FIG. 12.11
Flux distribution of a series-parallel magnetic network.

12.9 SERIES MAGNETIC CIRCUITS: DETERMINING *NI*

We are now in a position to solve a few magnetic circuit problems, which are basically of two types. *In one type, Φ is given, and the impressed mmf NI must be computed.* This is the type of problem encountered in the design of motors, generators, and transformers. *In the other type, NI is given, and the flux Φ of the magnetic circuit must be found.* This type of problem is encountered primarily in the design of magnetic amplifiers and is more difficult since the approach is "hit or miss."

As indicated in earlier discussions, the value of μ varies from point to point along the magnetization curve. This eliminates the possibility of finding the reluctance of each "branch" or the "total reluctance" of a network, as was done for electric circuits where ρ had a fixed value for any applied current or voltage. If the total reluctance can be determined, Φ can then be determined using the Ohm's law analogy for magnetic circuits.

For magnetic circuits, the level of B or H is determined from the other using the B-H curve, and μ is seldom calculated unless asked for.

An approach frequently used in the analysis of magnetic circuits is the *table* method. Before a problem is analyzed in detail, a table is prepared listing in the far left column the various sections of the magnetic circuit (see Table 12.2). The columns on the right are reserved for the quantities to be found for each section. In this way, when you are solving a problem, you can keep track of what the next step should be and what is required to complete the problem. After a few examples, the usefulness of this method should become clear.

This section considers only *series* magnetic circuits in which the flux Φ is the same throughout. In each example, the magnitude of the magnetomotive force is to be determined.

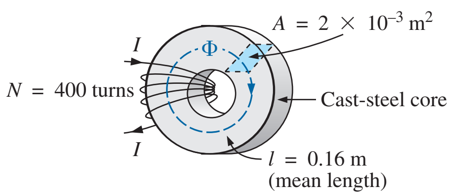
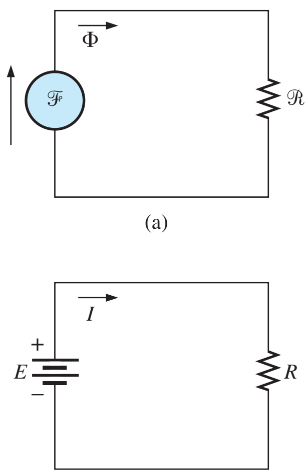


FIG. 12.12
Example 12.1.



(b)

FIG. 12.13
(a) Magnetic circuit equivalent and (b) electric circuit analogy.

EXAMPLE 12.1 For the series magnetic circuit in Fig. 12.12:

a. Find the value of I required to develop a magnetic flux of $\Phi = 4 \times 10^{-4}$ Wb.
b. Determine μ and μ_r for the material under these conditions.

Solutions: The magnetic circuit can be represented by the system shown in Fig. 12.13(a). The electric circuit analogy is shown in Fig. 12.13(b). Analogies of this type can be very helpful in the solution of magnetic circuits. Table 12.2 is for part (a) of this problem. The table is fairly trivial for this example, but it does define the quantities to be found.

a. The flux density B is

$$B = \frac{\Phi}{A} = \frac{4 \times 10^{-4}\ \text{Wb}}{2 \times 10^{-3}\ \text{m}^2} = 2 \times 10^{-1}\ \text{T} = 0.2\ \text{T}$$

Using the B-H curves in Fig. 12.8, we can determine the magnetizing force H:

$$H\ (\text{cast steel}) = 170\ \text{At/m}$$

Applying Ampère's circuital law yields

$$NI = Hl$$

and $$I = \frac{Hl}{N} = \frac{(170\ \text{At/m})(0.16\ \text{m})}{400\ \text{t}} = \mathbf{68\ mA}$$

(Recall that t represents turns.)

b. The permeability of the material can be found using Eq. (12.8):

$$\mu = \frac{B}{H} = \frac{0.2\ \text{T}}{170\ \text{At/m}} = \mathbf{1.176 \times 10^{-3}\ Wb/A \cdot m}$$

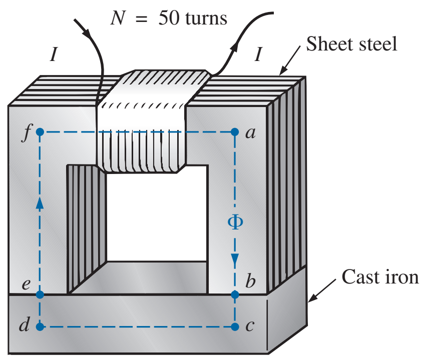
TABLE 12.2

Section	Φ (Wb)	A (m²)	B (T)	H (At/m)	l (m)	Hl (At)
One continuous section	4×10^{-4}	2×10^{-3}			0.16	

and the relative permeability is

$$\mu_r = \frac{\mu}{\mu_o} = \frac{1.176 \times 10^{-3}}{4\pi \times 10^{-7}} = \mathbf{935.83}$$

EXAMPLE 12.2 The electromagnet in Fig. 12.14 has picked up a section of cast iron. Determine the current *I* required to establish the indicated flux in the core.



$l_{ab} = l_{cd} = l_{ef} = l_{fa} = 4$ in.
$l_{bc} = l_{de} = 0.5$ in.
Area (throughout) = 1 in.^{2}
$\Phi = 3.5 \times 10^{-4}$ Wb

FIG. 12.14
Electromagnet for Example 12.2.

Solution: To be able to use Figs. 12.7 and 12.8, we must first convert to the metric system. However, since the area is the same throughout, we can determine the length for each material rather than work with the individual sections:

$$l_{efab} = 4 \text{ in.} + 4 \text{ in.} + 4 \text{ in.} = 12 \text{ in.}$$
$$l_{bcde} = 0.5 \text{ in.} + 4 \text{ in.} + 0.5 \text{ in.} = 5 \text{ in.}$$
$$12 \text{ in.}\left(\frac{1 \text{ m}}{39.37 \text{ in.}}\right) = 304.8 \times 10^{-3} \text{ m}$$
$$5 \text{ in.}\left(\frac{1 \text{ m}}{39.37 \text{ in.}}\right) = 127 \times 10^{-3} \text{ m}$$
$$1 \text{ in.}^2\left(\frac{1 \text{ m}}{39.37 \text{ in.}}\right)\left(\frac{1 \text{ m}}{39.37 \text{ in.}}\right) = 6.452 \times 10^{-4} \text{ m}^2$$

The information available from the *efab* and *bcde* specifications of the problem has been inserted in Table 12.3. When the problem has been completed, each space will contain some information. Sufficient data to complete the problem can be found if we fill in each column from left to right. As the various quantities are calculated, they will be placed in a similar table found at the end of the example.

TABLE 12.3

Section	Φ (Wb)	*A* (m^2)	*B* (T)	*H* (At/m)	*l* (m)	*Hl* (At)
efab	3.5×10^{-4}	6.452×10^{-4}			304.8×10^{-3}	
bcde	3.5×10^{-4}	6.452×10^{-4}			127×10^{-3}	

The flux density for each section is

$$B = \frac{\Phi}{A} = \frac{3.5 \times 10^{-4} \text{ Wb}}{6.452 \times 10^{-4} \text{ m}^2} = 0.542 \text{ T}$$

and the magnetizing force is

$$H \text{ (sheet steel, Fig. 12.8)} \cong 70 \text{ At/m}$$
$$H \text{ (cast iron, Fig. 12.7)} \cong 1600 \text{ At/m}$$

Note the extreme difference in magnetizing force for each material for the required flux density. In fact, when we apply Ampère's circuital law, we find that the sheet steel section can be ignored with a minimal error in the solution.

Determining *Hl* for each section yields

$$H_{efab}l_{efab} = (70 \text{ At/m})(304.8 \times 10^{-3} \text{ m}) = 21.34 \text{ At}$$
$$H_{bcde}l_{bcde} = (1600 \text{ At/m})(127 \times 10^{-3} \text{ m}) = 203.2 \text{ At}$$

Inserting the above data in Table 12.3 results in Table 12.4.

TABLE 12.4

Section	Φ (Wb)	A (m^2)	B (T)	H (At/m)	l (m)	Hl (At)
efab	3.5×10^{-4}	6.452×10^{-4}	0.542	70	304.8×10^{-3}	21.34
bcde	3.5×10^{-4}	6.452×10^{-4}	0.542	1600	127×10^{-3}	203.2

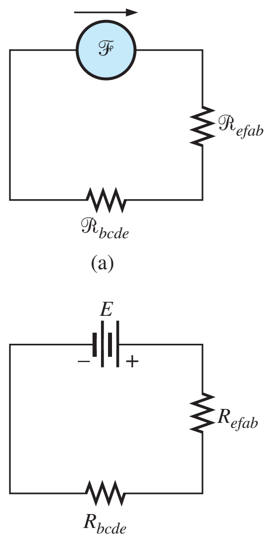


FIG. 12.15
(a) Magnetic circuit equivalent and (b) electric circuit analogy for the electromagnet in Fig. 12.14.

The magnetic circuit equivalent and the electric circuit analogy for the system in Fig. 12.14 appear in Fig. 12.15.

Applying Ampère's circuital law,

$$NI = H_{efab}l_{efab} + H_{bcde}l_{bcde}$$
$$= 21.34 \text{ At} + 203.2 \text{ At} = 224.54 \text{ At}$$

and $$(50 \text{ t})I = 224.54 \text{ At}$$

so that $$I = \frac{224.54 \text{ At}}{50 \text{ t}} = \mathbf{4.49\ A}$$

EXAMPLE 12.3 Determine the secondary current I_2 for the transformer in Fig. 12.16 if the resultant clockwise flux in the core is 1.5×10^{-5} Wb.

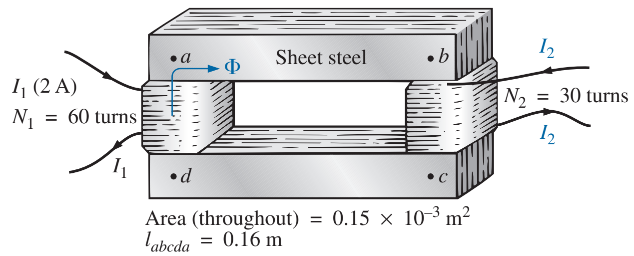


FIG. 12.16
Transformer for Example 12.3.

Solution: This is the first example with two magnetizing forces to consider. In the analogies in Fig. 12.17, note that the resulting flux of each is opposing, just as the two sources of voltage are opposing in the electric circuit analogy.

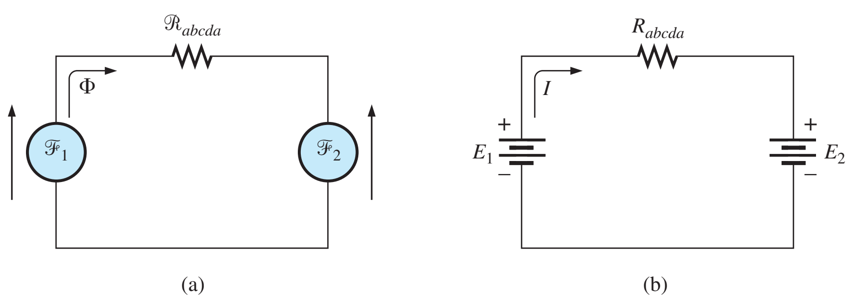


FIG. 12.17
(a) Magnetic circuit equivalent and (b) electric circuit analogy for the transformer in Fig. 12.16.

The *abcda* structural data appear in Table 12.5.

TABLE 12.5

Section	Φ (Wb)	A (m^2)	B (T)	H (At/m)	l (m)	Hl (At)
abcda	1.5×10^{-5}	0.15×10^{-3}			0.16	

The flux density throughout is

$$B = \frac{\Phi}{A} = \frac{1.5 \times 10^{-5}\ \text{Wb}}{0.15 \times 10^{-3}\ \text{m}^2} = 10 \times 10^{-2}\ \text{T} = 0.10\ \text{T}$$

and

$$H\ (\text{from Fig. 12.8}) \cong \frac{1}{5}(100\ \text{At/m}) = 20\ \text{At/m}$$

Applying Ampère's circuital law,

$$N_1I_1 - N_2I_2 = H_{abcda}l_{abcda}$$
$$(60\ \text{t})(2\ \text{A}) - (30\ \text{t})(I_2) = (20\ \text{At/m})(0.16\text{m})$$
$$120\ \text{At} - (30\ \text{t})I_2 = 3.2\ \text{At}$$

and
$$(30\ \text{t})I_2 = 120\ \text{At} - 3.2\ \text{At}$$

or
$$I_2 = \frac{116.8\ \text{At}}{30\ \text{t}} = \mathbf{3.89\ A}$$

For the analysis of most transformer systems, the equation $N_1I_1 = N_2I_2$ is used. This results in 4 A versus 3.89 A above. This difference is normally ignored, however, and the equation $N_1I_1 = N_2I_2$ considered exact.

Because of the nonlinearity of the *B-H* curve, *it is not possible to apply superposition to magnetic circuits;* that is, in Example 12.3, we cannot consider the effects of each source independently and then find the total effects by using superposition.

12.10 AIR GAPS

Before continuing with the illustrative examples, let us consider the effects that an air gap has on a magnetic circuit. Note the presence of air gaps in the magnetic circuits of the motor and meter in Fig. 11.15. The spreading of the flux lines outside the common area of the core for the air gap in Fig. 12.18(a) is known as *fringing*. For our purposes, we shall ignore this effect and assume the flux distribution to be as in Fig. 12.18(b).

The flux density of the air gap in Fig. 12.18(b) is given by

$$\boxed{B_g = \frac{\Phi_g}{A_g}} \qquad \textbf{(12.13)}$$

where, for our purposes,

$$\Phi_g = \Phi_{\text{core}}$$

and
$$A_g = A_{\text{core}}$$

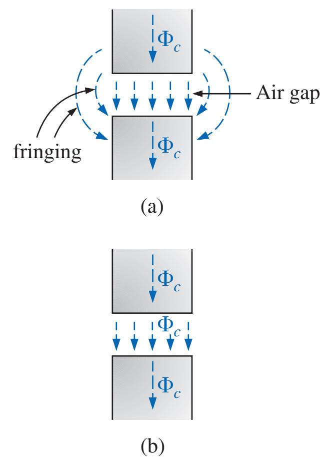


FIG. 12.18
Air gaps: (a) with fringing; (b) ideal.

For most practical applications, the permeability of air is taken to be equal to that of free space. The magnetizing force of the air gap is then determined by

$$H_g = \frac{B_g}{\mu_o} \quad \textbf{(12.14)}$$

and the mmf drop across the air gap is equal to $H_g L_g$. An equation for H_g is as follows:

$$H_g = \frac{B_g}{\mu_o} = \frac{B_g}{4\pi \times 10^{-7}}$$

and $$H_g = (7.96 \times 10^5)B_g \quad \text{(At/m)} \quad \textbf{(12.15)}$$

EXAMPLE 12.4 Find the value of I required to establish a magnetic flux of $\Phi = 0.75 \times 10^{-4}$ Wb in the series magnetic circuit in Fig. 12.19.

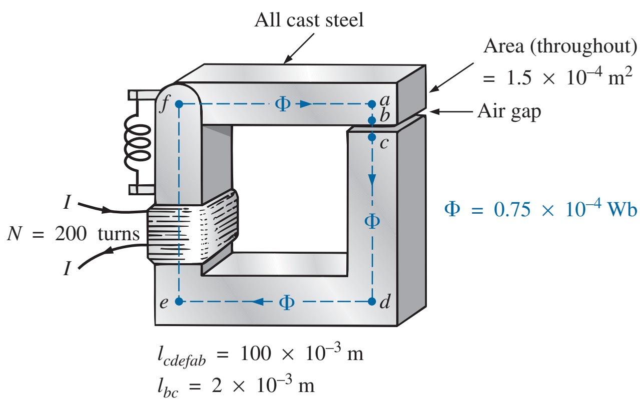


FIG. 12.19
Relay for Example 12.4.

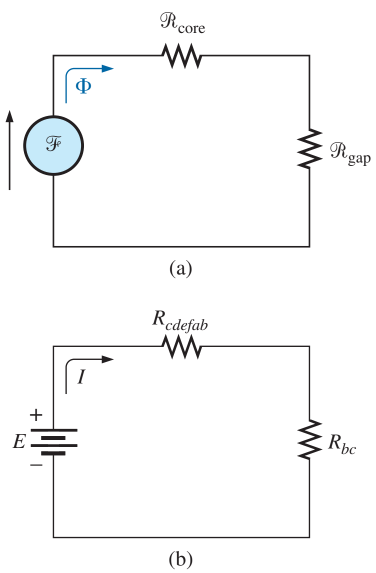


FIG. 12.20
(a) Magnetic circuit equivalent and (b) electric circuit analogy for the relay in Fig. 12.19.

Solution: An equivalent magnetic circuit and its electric circuit analogy are shown in Fig. 12.20.

The flux density for each section is

$$B = \frac{\Phi}{A} = \frac{0.75 \times 10^{-4}\ \text{Wb}}{1.5 \times 10^{-4}\ \text{m}^2} = 0.5\ \text{T}$$

From the *B-H* curves in Fig. 12.8,

$$H\ \text{(cast steel)} \cong 280\ \text{At/m}$$

Applying Eq. (12.15),

$$H_g = (7.96 \times 10^5)B_g = (7.96 \times 10^5)(0.5\ \text{T}) = 3.98 \times 10^5\ \text{At/m}$$

The mmf drops are

$$H_{core}l_{core} = (280\ \text{At/m})(100 \times 10^{-3}\ \text{m}) = 28\ \text{At}$$
$$H_g l_g = (3.98 \times 10^5\ \text{At/m})(2 \times 10^{-3}\ \text{m}) = 796\ \text{At}$$

Applying Ampère's circuital law,

$$\begin{aligned} NI &= H_{core}l_{core} + H_g l_g \\ &= 28\text{ At} + 796\text{ At} \\ (200\text{ t})I &= 824\text{ At} \\ I &= \mathbf{4.12\ A} \end{aligned}$$

Note from the above that the air gap requires the biggest share (by far) of the impressed *NI* because air is nonmagnetic.

12.11 SERIES-PARALLEL MAGNETIC CIRCUITS

As one might expect, the close analogies between electric and magnetic circuits eventually lead to series-parallel magnetic circuits similar in many respects to those encountered in Chapter 7. In fact, the electric circuit analogy will prove helpful in defining the procedure to follow toward a solution.

EXAMPLE 12.5 Determine the current *I* required to establish a flux of 1.5×10^{-4} Wb in the section of the core indicated in Fig. 12.21.

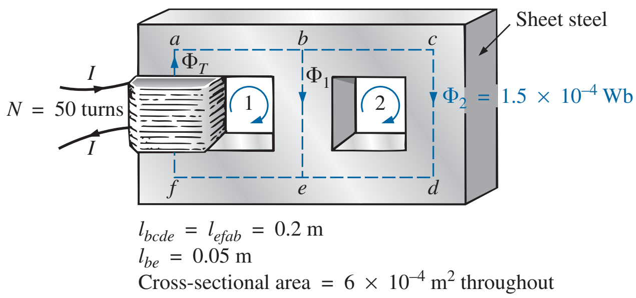


FIG. 12.21
Example 12.5.

Solution: The equivalent magnetic circuit and the electric circuit analogy appear in Fig. 12.22. We have

$$B_2 = \frac{\Phi_2}{A} = \frac{1.5 \times 10^{-4}\text{ Wb}}{6 \times 10^{-4}\text{ m}^2} = 0.25\text{ T}$$

From Fig. 12.8,

$$H_{bcde} \cong 40\text{ At/m}$$

Applying Ampère's circuital law around loop 2 in Figs. 12.21 and 12.22,

$$\begin{aligned} \Sigma_{\circlearrowleft}\, \mathcal{F} &= 0 \\ H_{be}l_{be} - H_{bcde}l_{bcde} &= 0 \\ H_{be}(0.05\text{ m}) - (40\text{ At/m})(0.2\text{m}) &= 0 \\ H_{be} = \frac{8\text{ At}}{0.05\text{ m}} &= 160\text{ At/m} \end{aligned}$$

From Fig. 12.8,

$$B_1 \cong 0.97\text{ T}$$

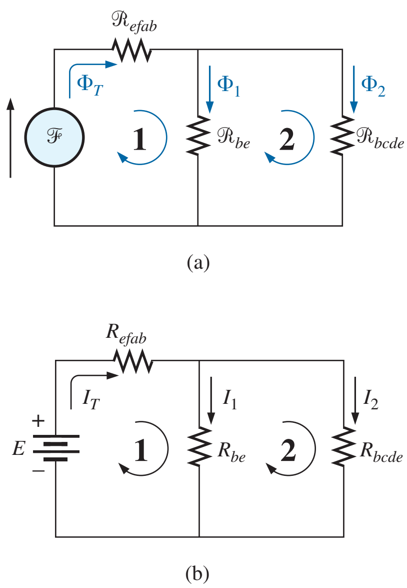


FIG. 12.22
(a) Magnetic circuit equivalent and (b) electric circuit analogy for the series-parallel system in Fig. 12.21.

and

$$\Phi_1 = B_1 A = (0.97\text{ T})(6 \times 10^{-4}\text{ m}^2) = 5.82 \times 10^{-4}\text{ Wb}$$

The results for *bcde, be,* and *efab* are entered in Table 12.6.

TABLE 12.6

Section	Φ (Wb)	A (m^2)	B (T)	H (At/m)	l (m)	Hl (At)
bcde	1.5×10^{-4}	6×10^{-4}	0.25	40	0.2	8
be	5.82×10^{-4}	6×10^{-4}	0.97	160	0.05	8
efab		6×10^{-4}			0.2	

Table 12.6 reveals that we must now turn our attention to section *efab:*

$$\Phi_T = \Phi_1 + \Phi_2 = 5.82 \times 10^{-4}\text{ Wb} + 1.5 \times 10^{-4}\text{ Wb}$$
$$= 7.32 \times 10^{-4}\text{ Wb}$$
$$B = \frac{\Phi_T}{A} = \frac{7.32 \times 10^{-4}\text{ Wb}}{6 \times 10^{-4}\text{ m}^2}$$
$$= 1.22\text{ T}$$

From Fig. 12.7,

$$H_{efab} \cong 400\text{ At}$$

Applying Ampère's circuital law,

$$+NI - H_{efab}l_{efab} - H_{be}l_{be} = 0$$
$$NI = (400\text{ At/m})(0.2\text{ m}) + (160\text{ At/m})(0.05\text{ m})$$
$$(50\text{ t})I = 80\text{ At} + 8\text{ At}$$
$$I = \frac{88\text{ At}}{50\text{ t}} = \mathbf{1.76\text{ A}}$$

To demonstrate that μ is sensitive to the magnetizing force H, the permeability of each section is determined as follows. For section *bcde,*

$$\mu = \frac{B}{H} = \frac{0.25\text{ T}}{40\text{ At/m}} = 6.25 \times 10^{-3}$$

and
$$\mu_r = \frac{\mu}{\mu_o} = \frac{6.25 \times 10^{-3}}{12.57 \times 10^{-7}} = \mathbf{4972.2}$$

For section *be,*

$$\mu = \frac{B}{H} = \frac{0.97\text{ T}}{160\text{ At/m}} = 6.06 \times 10^{-3}$$

and
$$\mu_r = \frac{\mu}{\mu_o} = \frac{6.06 \times 10^{-3}}{12.57 \times 10^{-7}} = \mathbf{4821}$$

For section *efab,*

$$\mu = \frac{B}{H} = \frac{1.22\text{ T}}{400\text{ At/m}} = 3.05 \times 10^{-3}$$

and
$$\mu_r = \frac{\mu}{\mu_o} = \frac{3.05 \times 10^{-3}}{12.57 \times 10^{-7}} = \mathbf{2426.41}$$

12.12 DETERMINING Φ

The examples of this section are of the second type, where NI is given and the flux Φ must be found. This is a relatively straightforward problem if only one magnetic section is involved. Then

$$H = \frac{NI}{l} \qquad H \rightarrow B \qquad (B\text{-}H \text{ curve})$$

and

$$\Phi = BA$$

For magnetic circuits with more than one section, there is no set order of steps that lead to an exact solution for every problem on the first attempt. In general, however, we proceed as follows. We must find the impressed mmf for a *calculated guess* of the flux Φ and then compare this with the specified value of mmf. We can then make adjustments to our guess to bring it closer to the actual value. For most applications, a value within ±5% of the actual Φ or specified NI is acceptable.

We can make a reasonable guess at the value of Φ if we realize that the maximum mmf drop appears across the material with the smallest permeability if the length and area of each material are the same. As shown in Example 12.4, if there is an air gap in the magnetic circuit, there will be a considerable drop in mmf across the gap. As a starting point for problems of this type, therefore, we shall assume that the total mmf (NI) is across the section with the lowest μ or greatest $\mathcal{R}$ (if the other physical dimensions are relatively similar). This assumption gives a value of Φ that will produce a calculated NI greater than the specified value. Then, after considering the results of our original assumption very carefully, we shall *cut* Φ and NI by introducing the effects (reluctance) of the other portions of the magnetic circuit and *try* the new solution. For obvious reasons, this approach is frequently called the *cut and try* method.

EXAMPLE 12.6 Calculate the magnetic flux Φ for the magnetic circuit in Fig. 12.23.

Solution: By Ampère's circuital law,

$$NI = H_{abcda} l_{abcda}$$

or

$$H_{abcda} = \frac{NI}{l_{abcda}} = \frac{(60\text{ t})(5\text{ A})}{0.3\text{ m}}$$

$$= \frac{300\text{ At}}{0.3\text{ m}} = 1000\text{ At/m}$$

and

$$B_{abcda} \text{ (from Fig. 12.7)} \cong 0.39\text{ T}$$

Since $B = \Phi/A$, we have

$$\Phi = BA = (0.39\text{ T})(2 \times 10^{-4}\text{ m}^2) = \mathbf{0.78 \times 10^{-4}\text{ Wb}}$$

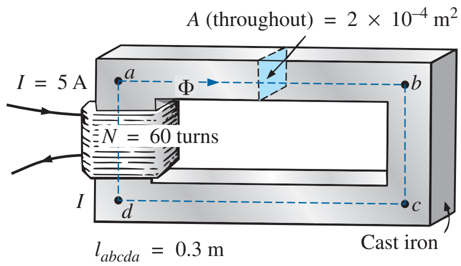


FIG. 12.23
Example 12.6.

EXAMPLE 12.7 Find the magnetic flux Φ for the series magnetic circuit in Fig. 12.24 for the specified impressed mmf.

Solution: Assuming that the total impressed mmf NI is across the air gap,

$$NI = H_g l_g$$

or

$$H_g = \frac{NI}{l_g} = \frac{400\text{ At}}{0.001\text{ m}} = 4 \times 10^5\text{ At/m}$$

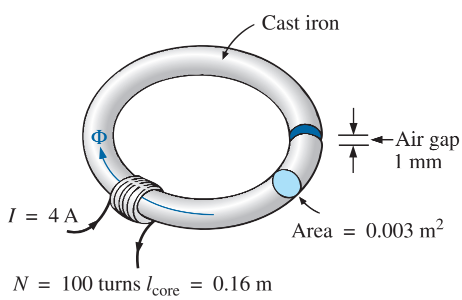


FIG. 12.24
Example 12.7.

and $$B_g = \mu_o H_g = (4\pi \times 10^{-7})(4 \times 10^5 \text{ At/m}) = 0.503 \text{ T}$$

The flux

$$\Phi_g = \Phi_{core} = B_g A = (0.503 \text{ T})(0.003 \text{ m}^2)$$
$$\Phi_{core} = 1.51 \times 10^{-3} \text{ Wb}$$

Using this value of Φ, we can find *NI*. The core and gap data are inserted in Table 12.7.

TABLE 12.7

Section	Φ (Wb)	*A* (m²)	*B* (T)	*H* (At/m)	*l* (m)	*Hl* (At)
Core	1.51×10^{-3}	0.003	0.503	1500 (*B-H* curve)	0.16	
Gap	1.51×10^{-3}	0.003	0.503	4×10^5	0.001	400

$$H_{core}l_{core} = (1500 \text{ At/m})(0.16 \text{ m}) = 240 \text{ At}$$

Applying Ampère's circuital law results in

$$NI = H_{core}l_{core} + H_g l_g = 240 \text{ At} + 400 \text{ At}$$
$$400 \text{ At} \neq 640 \text{ At}$$

Since we neglected the reluctance of all the magnetic paths but the air gap, the calculated value is greater than the specified value. We must therefore reduce this value by including the effect of these reluctances. Since approximately (640 At − 400 At)/640 At = 240 At/640 At ≅ 37.5% of our calculated value is above the desired value, let us reduce Φ by 30% and see how close we come to the impressed mmf of 400 At:

$$\Phi = (1 - 0.3)(1.51 \times 10^{-3} \text{ Wb}) = 1.057 \times 10^{-3} \text{ Wb}$$

See Table 12.8.

TABLE 12.8

Section	Φ (Wb)	*A* (m²)	*B* (T)	*H* (At/m)	*l* (m)	*Hl* (At)
Core	1.057×10^{-3}	0.003			0.16	
Gap	1.057×10^{-3}	0.003			0.001	

$$B = \frac{\Phi}{A} = \frac{1.057 \times 10^{-3} \text{ Wb}}{0.003 \text{ m}^3} \cong 0.352 \text{ T}$$

$$H_g l_g = (7.96 \times 10^5)B_g l_g = (7.96 \times 10^5)(0.352 \text{ T})(0.001 \text{ m}) \cong 280.19 \text{ At}$$

From the *B-H* curves,

$$H_{core} \cong 850 \text{ At/m}$$
$$H_{core}l_{core} = (850 \text{ At/m})(0.16 \text{ m}) = 136 \text{ At}$$

Applying Ampère's circuital law yields

$$NI = H_{core}l_{core} + H_g l_g = 136 \text{ At} + 280.19 \text{ At}$$

400 At = **416.19 At** (but within ±5% and therefore acceptable)

The solution is, therefore,

$$\Phi \cong \mathbf{1.057 \times 10^{-3}\ Wb}$$

12.13 APPLICATIONS

Speakers and Microphones

Electromagnetic effects are the moving force in the design of speakers such as the one shown in Fig. 12.25. The shape of the pulsating waveform of the input current is determined by the sound to be reproduced by the speaker at a high audio level. As the current peaks and returns to the valleys of the sound pattern, the strength of the electromagnet varies in exactly the same manner. This causes the cone of the speaker to vibrate at a frequency directly proportional to the pulsating input. The higher the pitch of the sound pattern, the higher the oscillating frequency between the peaks and valleys and the higher the frequency of vibration of the cone.

A second design used more frequently in more expensive speaker systems appears in Fig. 12.26. In this case, the permanent magnet is fixed, and the input is applied to a movable core within the magnet, as shown in the figure. High peaking currents at the input produce a strong flux pattern in the voice coil, causing it to be drawn well into the flux pattern of the permanent magnet. As occurred for the speaker in Fig. 12.25, the core then vibrates at a rate determined by the input and provides the audible sound.

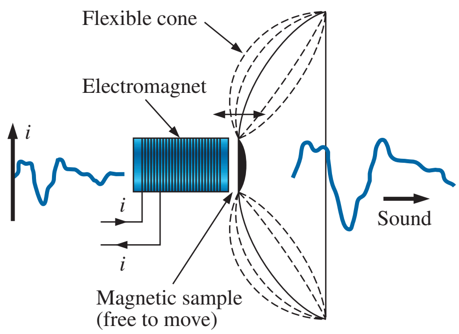


FIG. 12.25
Speaker.

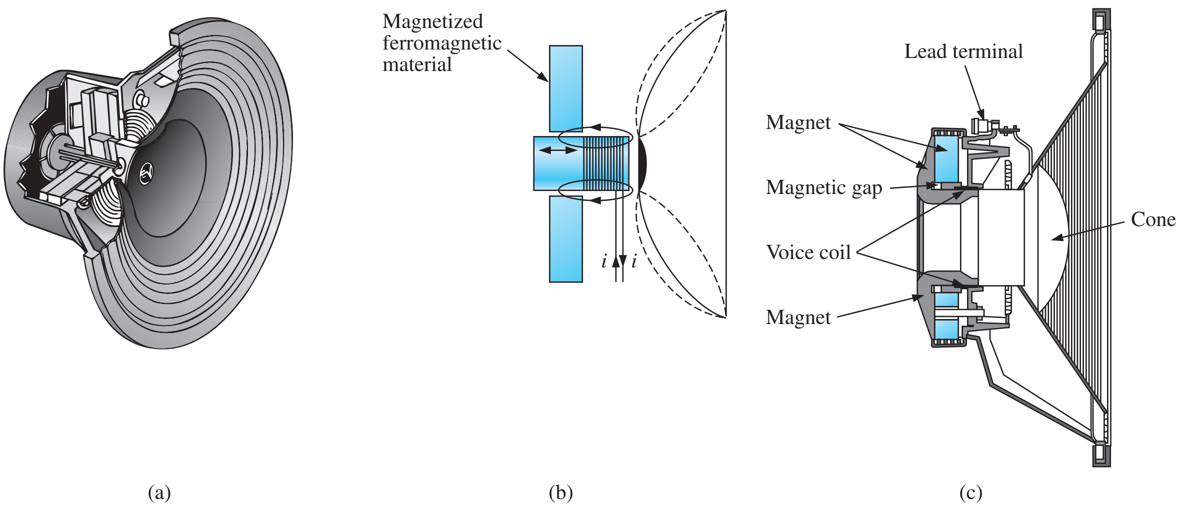


FIG. 12.26
Coaxial high-fidelity loudspeaker: (a) construction: (b) basic operation; (c) cross section of actual unit.
(Courtesy of Electro-Voice, Inc.)

Microphones also employ electromagnetic effects. The incoming sound causes the core and attached moving coil to move within the magnetic field of the permanent magnet. Through Faraday's law ($e = N\,d\phi/dt$), a voltage is induced across the movable coil proportional to the speed with which it is moving through the magnetic field. The resulting induced voltage pattern can then be amplified and reproduced at a much higher audio level through the use of speakers, as described earlier. Microphones of this type are the most frequently employed, although other types that use capacitive, carbon granular, and piezoelectric* effects are available. This particular design is commercially referred to as a *dynamic* microphone.

Hall Effect Sensor

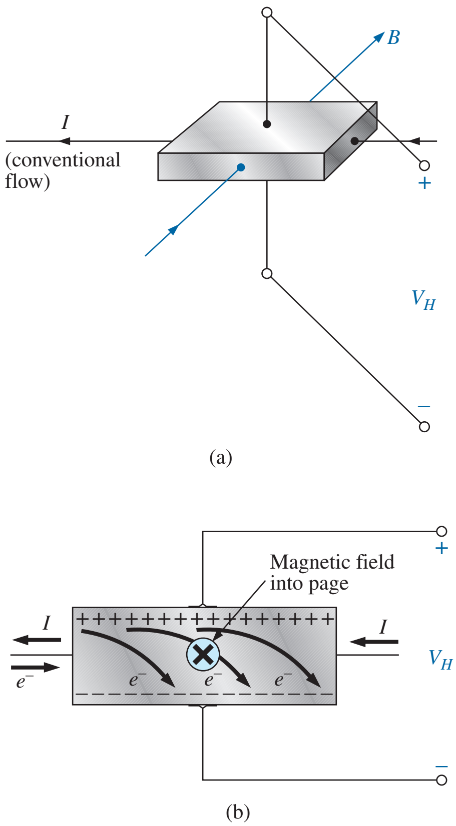


FIG. 12.27
Hall effect sensor: (a) orientation of controlling parameters; (b) effect on electron flow.

The Hall effect sensor is a semiconductor device that generates an output voltage when exposed to a magnetic field. The basic construction consists of a slab of semiconductor material through which a current is passed, as shown in Fig. 12.27(a). If a magnetic field is applied as shown in the figure perpendicular to the direction of the current, a voltage V_H is generated between the two terminals, as indicated in Fig. 12.27(a). The difference in potential is due to the separation of charge established by the Lorentz force first studied by Professor Hendrick Lorentz in the late 1800s. He found that electrons in a magnetic field are subjected to a force proportional to the velocity of the electrons through the field and the strength of the magnetic field. The direction of the force is determined by the left-hand rule. Simply place the index finger of your left hand in the direction of the magnetic field, with the second finger at right angles to the index finger in the direction of conventional current through the semiconductor material, as shown in Fig. 12.27(b). The thumb, if placed at right angles to the index finger, will indicate the direction of the force on the electrons. In Fig. 12.27(b), the force causes the electrons to accumulate in the bottom region of the semiconductor (connected to the negative terminal of the voltage V_H), leaving a net positive charge in the upper region of the material (connected to the positive terminal of V_H). The stronger the current or strength of the magnetic field, the greater the induced voltage V_H.

In essence, therefore, the Hall effect sensor can reveal the strength of a magnetic field or the level of current through a device if the other determining factor is held fixed. Two applications of the sensor are therefore apparent—to measure the strength of a magnetic field in the vicinity of a sensor (for an applied fixed current) and to measure the level of current through a sensor (with knowledge of the strength of the magnetic field linking the sensor). The gaussmeter in Fig. 11.14 uses a Hall effect sensor. Internal to the meter, a fixed current is passed through the sensor with the voltage V_H indicating the relative strength of the field. Through amplification, calibration, and proper scaling, the meter can display the relative strength in gauss.

The Hall effect sensor has a broad range of applications that are often quite interesting and innovative. The most widespread is as a trigger for an alarm system in large department stores, where theft is often a difficult problem. A magnetic strip attached to the merchandise sounds an alarm when a customer passes through the exit gates without paying for the product. The sensor, control current, and monitoring system are housed in the exit fence and react to the presence of the magnetic field as the product leaves the store. When the product is paid for, the cashier removes the strip or demagnetizes the strip by applying a magnetizing force that reduces the residual magnetism in the strip to essentially zero.

The Hall effect sensor is also used to indicate the speed of a bicycle on a digital display conveniently mounted on the handlebars. As shown in Fig. 12.28(a), the sensor is mounted on the frame of the bike, and a small

*Piezoelectricity is the generation of a small voltage by exerting pressure across certain crystals.

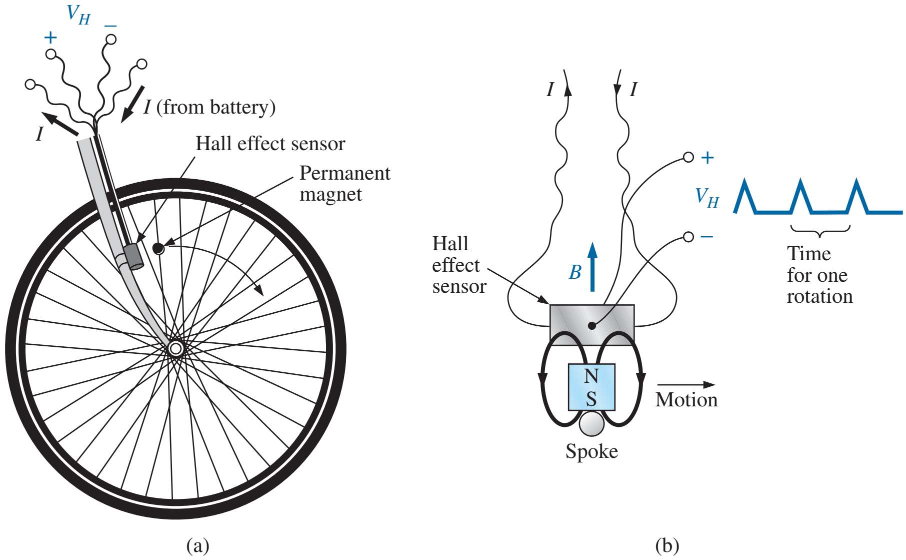


FIG. 12.28
Obtaining a speed indication for a bicycle using a Hall effect sensor: (a) mounting the components; (b) Hall effect response.

permanent magnet is mounted on a spoke of the front wheel. The magnet must be carefully mounted to be sure that it passes over the proper region of the sensor. When the magnet passes over the sensor, the flux pattern in Fig. 12.28(b) results, and a voltage with a sharp peak is developed by the sensor. Assuming a bicycle with a 26-in.-diameter wheel, the circumference will be about 82 in. Over 1 mi, the number of rotations is

$$5280\ \cancel{\text{ft}}\left(\frac{12\ \cancel{\text{in.}}}{1\ \cancel{\text{ft}}}\right)\left(\frac{1\ \text{rotation}}{82\ \cancel{\text{in.}}}\right) \cong 773\ \text{rotations}$$

If the bicycle is traveling at 20 mph, an output pulse occurs at a rate of 4.29 per second. It is interesting to note that at a speed of 20 mph, the wheel is rotating at more than 4 revolutions per second, and the total number of rotations over 20 mi is 15,460.

Magnetic Reed Switch

One of the most frequently employed switches in alarm systems is the *magnetic reed switch* shown in Fig. 12.29. As shown by the figure, there are two components of the reed switch—a permanent magnet embedded in one unit that is normally connected to the movable element (door, window, and so on) and a reed switch in the other unit that is connected to the electrical control circuit. The reed switch is constructed of two iron-alloy (ferromagnetic) reeds in a hermetically sealed capsule. The cantilevered ends of the two reeds do not touch but are in very close proximity to one another. In the absence of a magnetic field, the reeds remain separated. However, if a magnetic field is introduced, the reeds are drawn to each other because flux lines seek the path of least reluctance and, if possible, exercise every alternative to establish the path of least reluctance. It is similar to placing a ferromagnetic bar close to the ends of a U-shaped magnet. The bar is drawn to the poles of the magnet, establishing a magnetic flux path without air gaps and with minimum reluctance. In the open-circuit state, the resistance between reeds is in excess of 100 MΩ, while in the on state it drops to less than 1 Ω.

In Fig. 12.30 a reed switch has been placed on the fixed frame of a window and a magnet on the movable window unit. When the window is

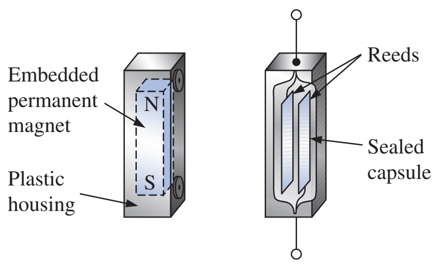


FIG. 12.29
Magnetic reed switch.

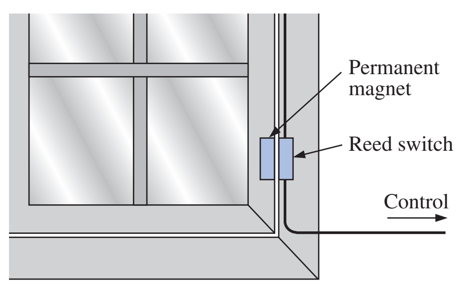


FIG. 12.30
Using a magnetic reed switch to monitor the state of a window.

FIG. 12.31
Magnetic resonance imaging equipment.
(Courtesy of Siemens Medical Systems, Inc.)

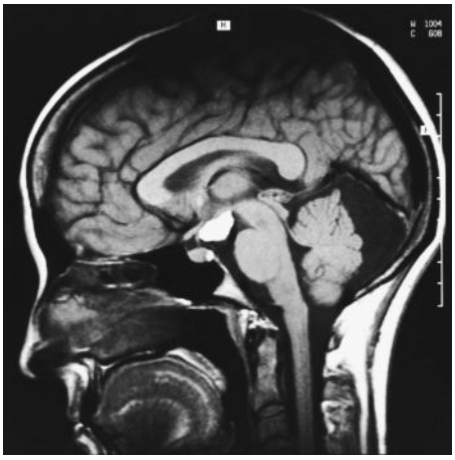


FIG. 12.32
Magnetic resonance image.
(Courtesy of Siemens Medical Systems, Inc.)

FIG. 12.33
Magnetic resonance imaging equipment (open variety).
(Courtesy of Siemens Medical Systems, Inc.)

closed as shown in Fig. 12.30, the magnet and reed switch are sufficiently close to establish contact between the reeds, and a current is established through the reed switch to the control panel. In the armed state, the alarm system accepts the resulting current flow as a normal secure response. If the window is opened, the magnet leaves the vicinity of the reed switch, and the switch opens. The current through the switch is interrupted, and the alarm reacts appropriately.

One of the distinct advantages of the magnetic reed switch is that the proper operation of any switch can be checked with a portable magnetic element. Simply bring the magnet to the switch and note the output response. There is no need to continually open and close windows and doors. In addition, the reed switch is hermetically enclosed so that oxidation and foreign objects cannot damage it, and the result is a unit that can last indefinitely. Magnetic reed switches are also available in other shapes and sizes, allowing them to be concealed from obvious view. One is a circular variety that can be set into the edge of a door and door jam, resulting in only two small visible disks when the door is open.

Magnetic Resonance Imaging

Magnetic resonance imaging (MRI) provides quality cross-sectional images of the body for medical diagnosis and treatment. MRI does not expose the patient to potentially hazardous X-rays or injected contrast materials such as those used to obtain computerized axial tomography (CAT) scans.

The three major components of an MRI system are a strong magnet, a table for transporting the patient into the circular hole in the magnet, and a control center, as shown in Fig. 12.31. The image is obtained by placing the patient in the tube to a precise depth depending on the cross section to be obtained and applying a strong magnetic field that causes the nuclei of certain atoms in the body to line up. Radio waves of different frequencies are then applied to the patient in the region of interest, and if the frequency of the wave matches the natural frequency of the atom, the nuclei is set into a state of resonance and absorbs energy from the applied signal. When the signal is removed, the nuclei release the acquired energy in the form of weak but detectable signals. The strength and duration of the energy emission vary from one tissue of the body to another. The weak signals are then amplified, digitized, and translated to provide a cross-sectional image such as the one shown in Fig. 12.32. For some patients the claustrophobic feeling they experience while in the circular tube is difficult to contend with. Today, however, a more open unit has been developed, as shown in Fig. 12.33, that has removed most of this discomfort.

Patients who have metallic implants or pacemakers or those who have worked in industrial environments where minute ferromagnetic particles may have become lodged in open, sensitive areas such as the eyes, nose, and so on, may have to use a CAT scan system because it does not employ magnetic effects. The attending physician is well trained in such areas of concern and will remove any unfounded fears or suggest alternative methods.

PROBLEMS

SECTION 12.2 Magnetic Field

1. Using Appendix F, fill in the blanks in the following table. Indicate the units for each quantity.

	Φ	B
SI	5×10^{-4} Wb	8×10^{-4} T
CGS	________	________
English	________	________

2. Repeat Problem 1 for the following table if area = 2 in.^{2}:

	Φ	B
SI	________	________
CGS	60,000 maxwells	________
English	________	________

3. For the electromagnet in Fig. 12.34:
a. Find the flux density in the core.
b. Sketch the magnetic flux lines and indicate their direction.
c. Indicate the north and south poles of the magnet.

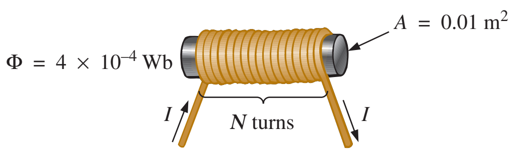


FIG. 12.34
Problem 3.

SECTION 12.3 Reluctance

4. Which section of Fig. 12.35—(a), (b), or (c)—has the largest reluctance to the setting up of flux lines through its longest dimension?

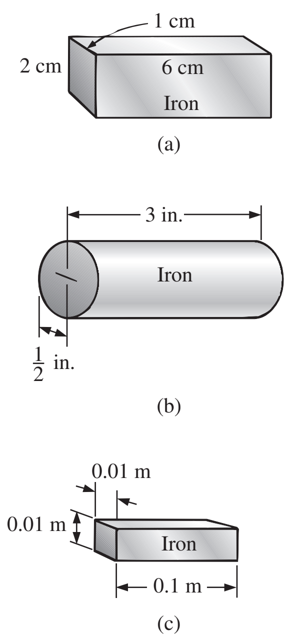


FIG. 12.35
Problem 4.

SECTION 12.4 Ohm's Law for Magnetic Circuits

5. Find the reluctance of a magnetic circuit if a magnetic flux $\Phi = 4.2 \times 10^{-4}$ Wb is established by an impressed mmf of 400 At.

6. Repeat Problem 5 for Φ = 72,000 maxwells and an impressed mmf of 120 gilberts.

SECTION 12.5 Magnetizing Force

7. Find the magnetizing force H for Problem 5 in SI units if the magnetic circuit is 6 in. long.

8. If a magnetizing force H of 600 At/m is applied to a magnetic circuit, a flux density B of 1200×10^{-4} Wb/m^2 is established. Find the permeability μ of a material that will produce twice the original flux density for the same magnetizing force.

SECTIONS 12.6–12.9 Hysteresis through Series Magnetic Circuits

9. For the series magnetic circuit in Fig. 12.36, determine the current I necessary to establish the indicated flux.

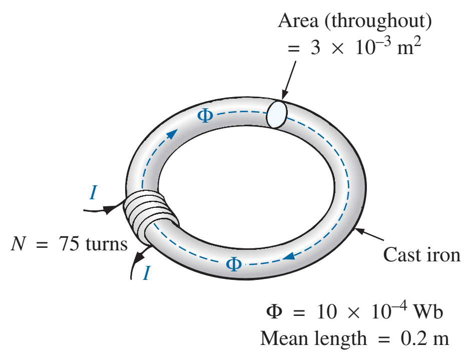


FIG. 12.36
Problem 9.

10. Find the current necessary to establish a flux of $\Phi = 3 \times 10^{-4}$ Wb in the series magnetic circuit in Fig 12.37.

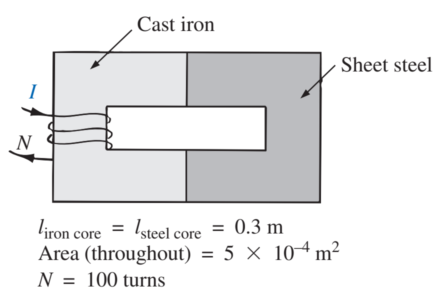


FIG. 12.37
Problem 10.

11. **a.** Find the number of turns N_1 required to establish a flux $\Phi = 12 \times 10^{-4}$ Wb in the magnetic circuit in Fig. 12.38.
b. Find the permeability μ of the material.

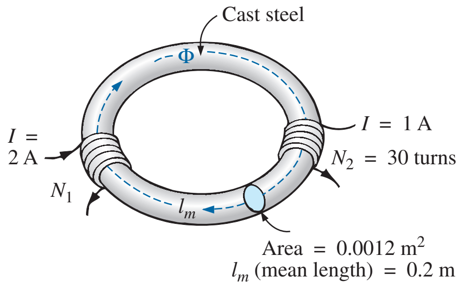


FIG. 12.38
Problem 11.

12. **a.** Find the mmf (NI) required to establish a flux Φ = 80,000 lines in the magnetic circuit in Fig. 12.39.
b. Find the permeability of each material.

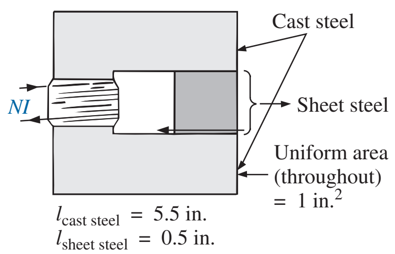


FIG. 12.39
Problem 12.

***13.** For the series magnetic circuit in Fig. 12.40 with two impressed sources of magnetic “pressure,” determine the current I. Each applied mmf establishes a flux pattern in the clockwise direction.

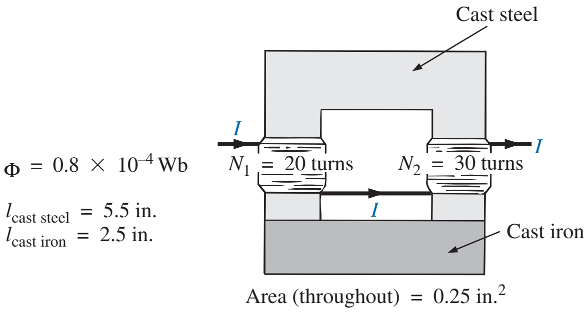


FIG. 12.40
Problem 13.

SECTION 12.10 Air Gaps

14. **a.** Find the current I required to establish a flux $\Phi = 2.4 \times 10^{-4}$ Wb in the magnetic circuit in Fig. 12.41.
b. Compare the mmf drop across the air gap to that across the rest of the magnetic circuit. Discuss your results using the value of μ for each material.

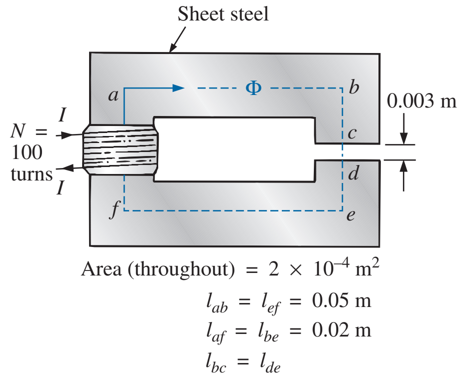


FIG. 12.41
Problem 14.

***15.** The force carried by the plunger of the door chime in Fig. 12.42 is determined by

$$f = \frac{1}{2} NI \frac{d\phi}{dx} \qquad \text{(newtons)}$$

where $d\phi/dx$ is the rate of change of flux linking the coil as the core is drawn into the coil. The greatest rate of change of flux occurs when the core is ¼ to ¾ the way through. In this region, if Φ changes from 0.5×10^{-4} Wb to 8×10^{-4} Wb, what is the force carried by the plunger?

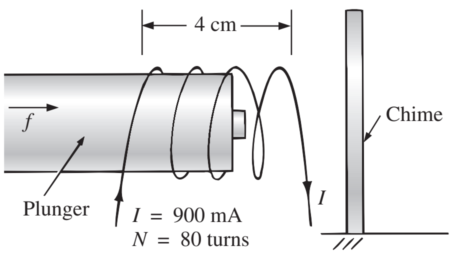


FIG. 12.42
Door chime for Problem 15.

16. Determine the current I_1 required to establish a flux of $\Phi = 2 \times 10^{-4}$ Wb in the magnetic circuit in Fig. 12.43.

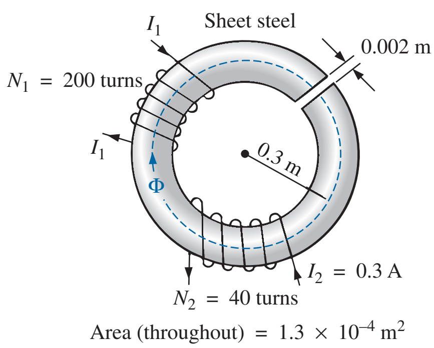


FIG. 12.43
Problem 16.

*17. **a.** A flux of 0.2×10^{-4} Wb will establish sufficient attractive force for the armature of the relay in Fig. 12.44 to close the contacts. Determine the required current to establish this flux level if we assume that the total mmf drop is across the air gap.

b. The force exerted on the armature is determined by the equation

$$F(\text{newtons}) = \frac{1}{2} \cdot \frac{B_g^2 A}{\mu_o}$$

where B_g is the flux density within the air gap and A is the common area of the air gap. Find the force in newtons exerted when the flux Φ specified in part (a) is established.

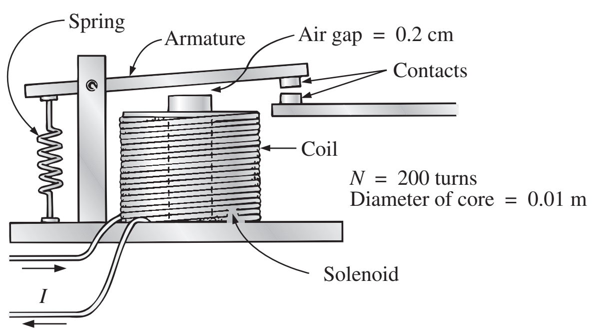


FIG. 12.44
Relay for Problem 17.

SECTION 12.11 Series-Parallel Magnetic Circuits

*18. For the series-parallel magnetic circuit in Fig. 12.45, find the value of I required to establish a flux in the gap of $\Phi_g = 2 \times 10^{-4}$ Wb.

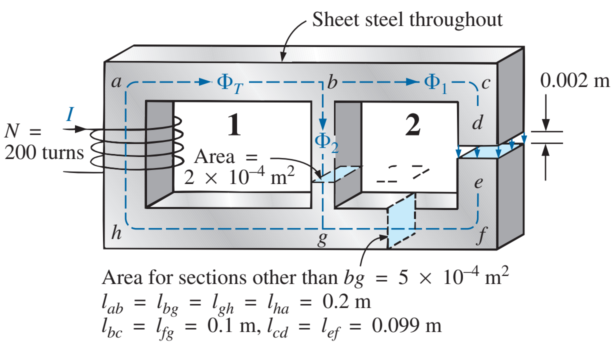


FIG. 12.45
Problem 18.

SECTION 12.12 Determining Φ

19. Find the magnetic flux Φ established in the series magnetic circuit in Fig. 12.46.

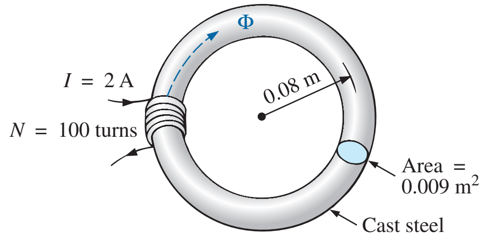


FIG. 12.46
Problem 19.

*20. Determine the magnetic flux Φ established in the series magnetic circuit in Fig. 12.47.

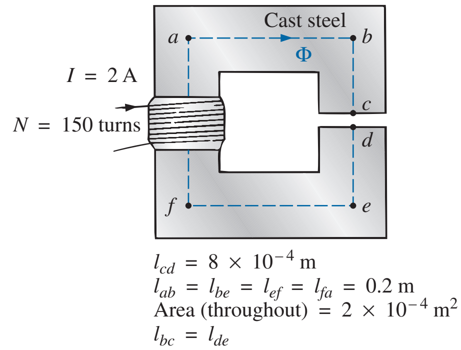


FIG. 12.47
Problem 20.

*21. Note how closely the B-H curve of cast steel in Fig. 12.7 matches the curve for the voltage across a capacitor as it charges from zero volts to its final value.

a. Using the equation for the charging voltage as a guide, write an equation for B as a function of H [$B = f(H)$] for cast steel.

b. Test the resulting equation at $H = 900$ At/m, 1800 At/m, and 2700 At/m.

c. Using the equation of part (a), derive an equation for H in terms of B [$H = f(B)$].

d. Test the resulting equation at $B = 1$ T and $B = 1.4$ T.

e. Using the result of part (c), perform the analysis of Example 12.1, and compare the results for the current I.

GLOSSARY

Ampère's circuital law A law establishing the fact that the algebraic sum of the rises and drops of the mmf around a closed loop of a magnetic circuit is equal to zero.

Domain A group of magnetically aligned atoms.

Electromagnetism Magnetic effects introduced by the flow of charge or current.

Ferromagnetic materials Materials having permeabilities hundreds and thousands of times greater than that of free space.

Flux density (B) A measure of the flux per unit area perpendicular to a magnetic flux path. It is measured in teslas (T) or webers per square meter (Wb/m^2).

Hysteresis The lagging effect between the flux density of a material and the magnetizing force applied.

Magnetic flux lines Lines of a continuous nature that reveal the strength and direction of a magnetic field.

Magnetizing force (*H*) A measure of the magnetomotive force per unit length of a magnetic circuit.

Magnetomotive force (mmf) ($\mathscr{F}$) The "pressure" required to establish magnetic flux in a ferromagnetic material. It is measured in ampere-turns (At).

Permanent magnet A material such as steel or iron that will remain magnetized for long periods of time without the aid of external means.

Permeability (μ) A measure of the ease with which magnetic flux can be established in a material. It is measured in Wb/Am.

Relative permeability (μ_r) The ratio of the permeability of a material to that of free space.

Reluctance ($\mathscr{R}$) A quantity determined by the physical characteristics of a material that will provide an indication of the "reluctance" of that material to the setting up of magnetic flux lines in the material. It is measured in rels or At/Wb.