MEC 3351 – Strength of Materials I

Dr C.G. Chizyuka
(Office 102)

Course Content:

- Direct Stress
- Shear Stress
- Shear Force and Bending Moment
- Theory of Bending Stresses
- Torsion
- Deflection of Straight Beams
- Shear Stress distribution in Beams
- Impact
- Thin Cylinders and Thin Spheres
- Springs
- Struts

<u>Assessment</u>

Assignments	- 5%
Labs	- 15%
Tests	- 20%
Final Exam	- 60%

Prescribed Books

1. Ryder G.H (1974), " Strength of Materials".
2. Timoshenko S. and Young, "Elements of Strength of Materials".

Recommended Books

1. Popov E.P, "Mechanics of Materials".
2. Gere J.M et al., "Mechanics of Materials".

Direct Stress and Strains

- In strength of materials, **internal effects** and **deformation** of bodies caused by externally applied forces are studied.
- Stress – resisting force per unit area.

$$\sigma = \frac{F}{A}, \frac{N}{m^2} \dots \dots \dots . (1)$$

Where σ - Stress intensity

F – Load applied

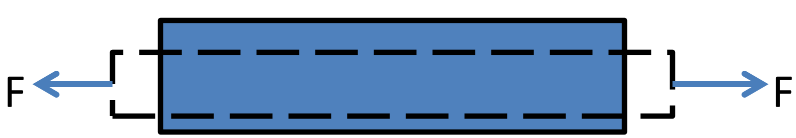
A - Area of x-section of loaded section

- Strain – measure of the deformation produced in a member by the load.
- Defined as change in length per unit length.

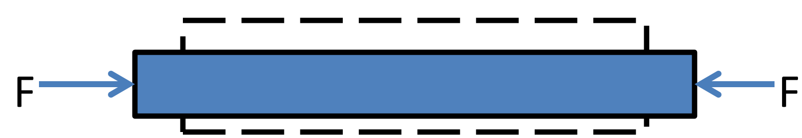
$$e = \frac{\delta l}{l}, (No\ units) \dots \dots \dots . (2)$$

Where δl – change in length of l

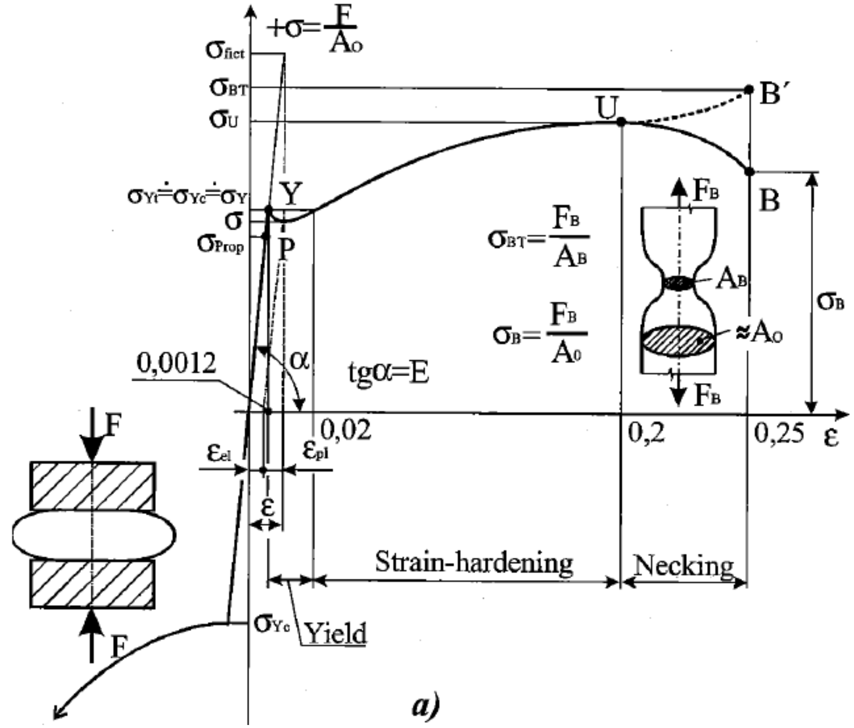
l – Original length



+ve



-ve



- Origin to Point P – Elastic range. (load proportional to resultant extension)
- Stress developed by limiting load – Elastic Limit
- Stress corresponding to point Y – Yield point stress
- P – B – Plastic range.
- U – point of maximum stress – Ultimate strength.

Hooke's Law

- For a material loaded within the elastic limit the stress is proportional to the strain.

$$\frac{Stress}{Strain} = Constant\ , E - Young's\ Modulus$$

$E = \frac{\sigma}{e} \ldots\ldots\ldots.(3)$ – Stress required to produce unit strain,

- Change in length of body due to application of load on it:
  - Consider a wire subjected to a pull of load *F*.
  - Let
    - *l* – Original length of the wire
    - *A* – X-section of wire
    - δl – change in length caused by applied load
    - *e* – Strain in wire due to applied load
    - σ - stress intensity in wire due to applied load.

From Hooke's law we have:

$$E = \frac{\sigma}{e} \ldots\ldots\ldots.(3)$$

Substituting for *e* from (2) and *σ* from (1) we have

$$\frac{\delta l}{l} = \frac{F}{AE} \quad or \quad \delta l = \frac{Fl}{AE} \ldots\ldots\ldots\ldots.(4)$$

Factor of Safety:

- In order to avoid permanent deformation in structures, it is usual to adopt working stress, σ_w well below the limit of proportionality.
- Adopt working stress as a fraction of ultimate stress, σ_{ult}.

$$\sigma_w = \frac{\sigma_{ult}}{n} \qquad or \qquad n = \frac{\sigma_{ult}}{\sigma_w}$$

Where n – factor of safety and may be defined as the ratio of the ultimate to the working stress adopted. Depends upon:

- homogeneity of material and
- accuracy with which stress can be evaluated.

E.g. A steel rod of 20mm dia and 500cm long is subjected to axial pull of 3000kg. Determine (i) the intensity of stress; (ii) the strain; (iii) the elongation of the rod. Take E=2.1 x 10^6 kg/cm^2.

Given: Diameter of rod = 20mm = 2.0cm; Rod length = 500cm; Load = 3000kg; Take E=2.1 x 10^6.

X-section area is $$A = \frac{\pi.2^2}{4} = 3.14\,cm^2$$

(i) Intensity of stress is $$\sigma = \frac{F}{A} = \frac{3000}{3.14} = \mathbf{955.41\,kg/cm^2}$$

(ii) Strain $$e = \frac{\sigma}{E} = \frac{955.41}{2.1*10^6} = \mathbf{0.000455}$$

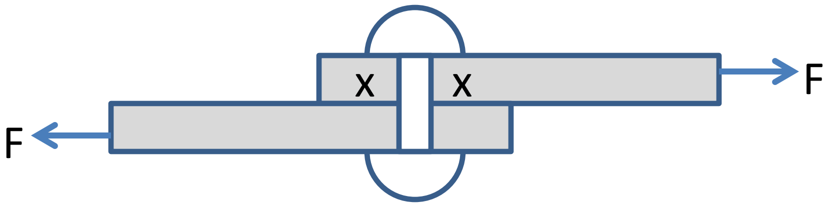
(iii) Elongation $$\delta l = \frac{Fl}{AE} = \frac{3000*500}{3.14*2.1*10^6} = \mathbf{0.2275\,cm.}$$

Shear Stress

Consider two riveted plates carrying tensile loading F. rivet may shear along x-x.

If *d* – diameter of rivet,

Rivet x-area at x-x:



$$A = \frac{\pi d^2}{4}$$

Shear stress:

$$\tau = \frac{F}{A} = \frac{4F}{\pi d^2} \dots\dots\dots\dots(6)$$

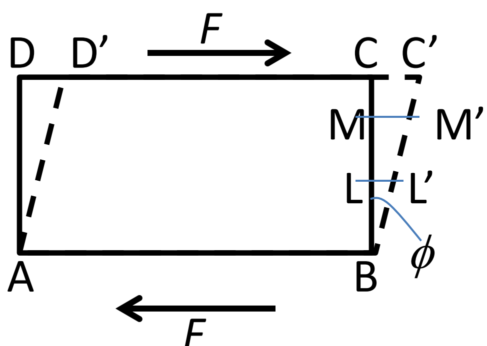
Note: - That applied force is tangential to the resisting area, and therefore shear stress is called **Tangential Stress**.

- Tensile or compressive stresses are caused by forces perpendicular to resisting area. Stresses – **Direct** or **normal stresses**

Shear Strain

Causes relative displacement of material in the direction of the force.

Consider block ABCD distorted to ABC’D’ under action of tangential force F on DC.



Deformation:

LL’ is in height BL

MM’ in height BM

CC’ in height BC

Shear stain is the deformation caused by shear force per unit length, therefore

$$Shear\ strain = \frac{LL'}{BL} = \frac{MM'}{BM} = \frac{CC'}{BC} = \tan\phi = \phi, (in\ radians)$$

$$(Since\ \phi\ is\ always\ very\ small, therefore\ \tan\phi = \phi)$$

Modulus of Rigidity

For elastic materials, it is found that within certain limits shear strain is directly proportional to the shear stress producing it.

$$ratio \ \ \frac{Shear\ stress}{Shear\ strain} = G, -Modulus\ of\ Rigidity\ (or\ Shear\ Modulus)$$

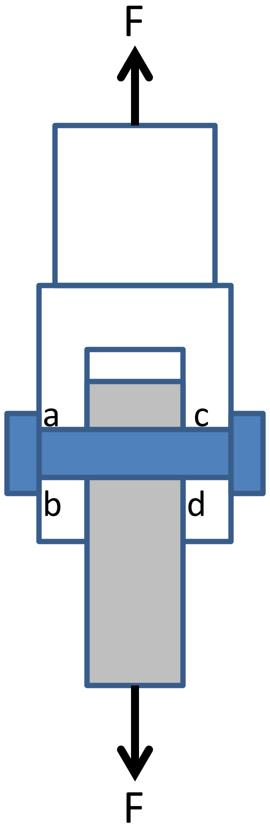
$$G = \frac{\tau}{\phi} = \frac{F/A}{\Delta x/l} \ \ , N/mm^2$$

Modulus of Rigidity – Shear stress needed to produce unit shear strain.

A tie rod made up of two parts is to carry a load of 9900 kg. determine a proper diameter for the connecting pin if the allowable working stress in shear is 700kg/sq-cm.

Let diameter of pin be *d*. Since the pin is in double shear,

$$\tau = \frac{F}{2A} = \frac{F}{2\frac{\pi d^2}{4}} = \frac{2d}{\pi d^2} =$$



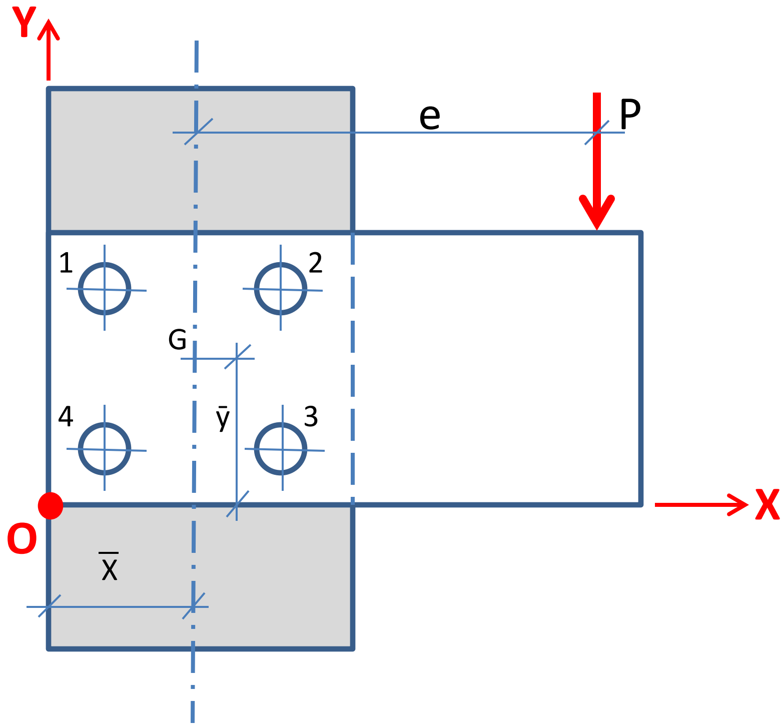
But it is given that [τ]=700kg/sq-cm and F=9900kg.

Therefore,

$$700 = \frac{2 * 9900}{\pi d^2}; \quad \boldsymbol{d = 3.0cm}$$

Eccentric Loaded Riveted Joint

When the line of action of the load does not pass through the centroid of the rivet system results in the rivets not equally loaded.



Let,

P – Eccentric load on the joint

e – Eccentricity of the load

1. The centre of gravity G of the rivet system

Let

x_1, x_2, x_3 , ….., x_n – Distance of rivet from OY

y_1, y_2, y_3 , ….., y_n – Distance of rivet from OX

n – Number of rivets

We know that

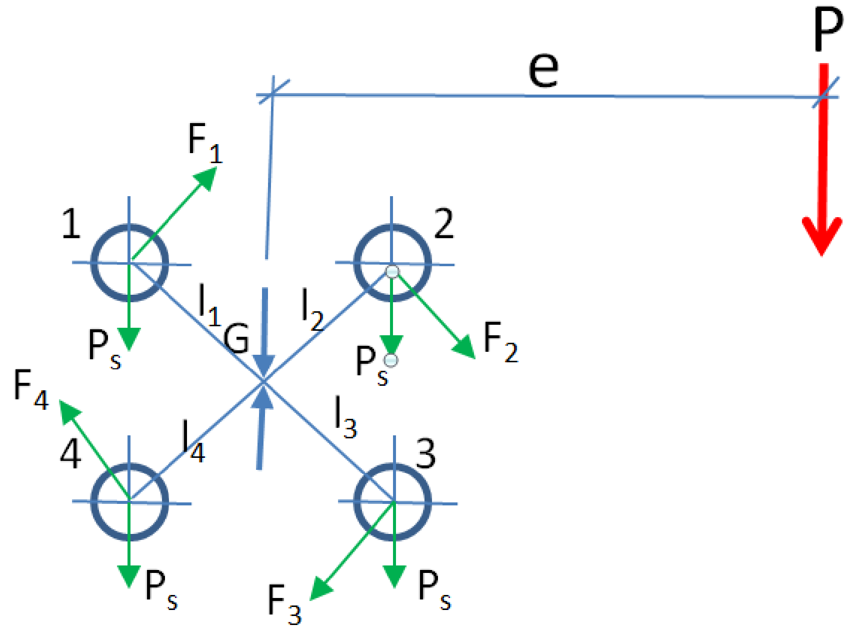
$$\bar{x} = \frac{x_1 + x_2 + \ldots x_n}{n}$$

$$\bar{y} = \frac{y_1 + y_2 + \ldots y_n}{n}$$

2) Introduce two forces P_1 and P_2 at the centre of gravity "G" of the rivet system. These forces are equal and opposite to P.

3) Assuming that all rivets are of the same size, the effect of P_1 = P is to produce a direct shear load on each rivet

where $P_s = \frac{P}{n}$ acting parallel to P



4) Due to the turning moment, secondary shear load on rivets are produced.

a) The secondary shear load is proportional to the radial distance of rivet under consideration from the centre of gravity of the system.

b) The direction of the same is perpendicular to the line joining G to the centre of rivet.

Let

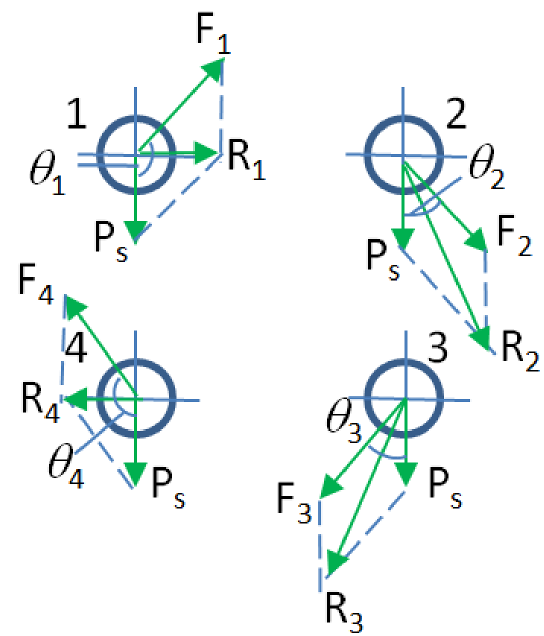
F_1 , F_2 , F_3 , F_4 – Secondary shear loads on rivets 1, 2, 3,…n.

l_1 , l_2 , l_3 , l_4 – Radial distances of the rivet from centre of system gravity G.

From (a) $F_1 \propto l_1$, $F_2 \propto l_2$ ....

Or $$\frac{F_1}{l_1} = \frac{F_2}{l_2} = \frac{F_3}{l_3} = \cdots$$

$$\therefore \quad F_2 = F_1 \frac{l_2}{l_1} \text{ , and } F_3 = F_1 \frac{l_3}{l_1}$$



$$\sum M = 0$$

$$P * e = F_1 \; l_1 + F_2 \; l_2 + F_3 \; l_3 + \; \ldots$$

$F_1 \rightarrow F_2 \rightarrow F_3$

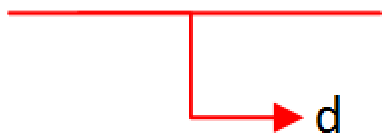
$$= F_1 \, l_1 + F_1 \frac{l_2}{l_1} * \; l_2 + F_1 \frac{l_3}{l_1} * \; l_3 + \; \ldots$$

$$R = \sqrt{{P_s}^2 + F^2 + 2P_S * F * \cos\theta}$$

θ - angle between the primary (P_s) and secondary shear load (F_i)

Max resultant shear load

$$R = \frac{\pi}{4} * d^2 * \tau$$



Crushing Stress

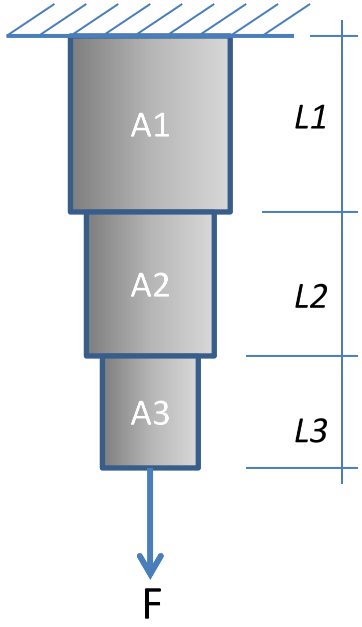
$$\sigma_c = \frac{Max.Load}{Crush\ Area} = \frac{R_{Max}}{d * t} \leq [\sigma_c]$$

Shear Stress

$$\sigma_c = \frac{Max.Load}{Shear\ Area} = \frac{R_{Max}}{\frac{\pi}{4} * d^2} \leq [\sigma_s]$$

Elongation of Bar of Varying Cross-Sections

Consider a bar of three sections of varying x-section areas and lengths subjected to an axial pull of *F*. If δl be respective changes in lengths of 3 sections, then we have



$$\delta l_1 = \frac{F l_1}{A_1 E} \qquad \delta l_2 = \frac{F l_2}{A_2 E} \qquad \delta l_3 = \frac{F l_3}{A_3 E}$$

where *E* – modulus of elasticity of bar material.

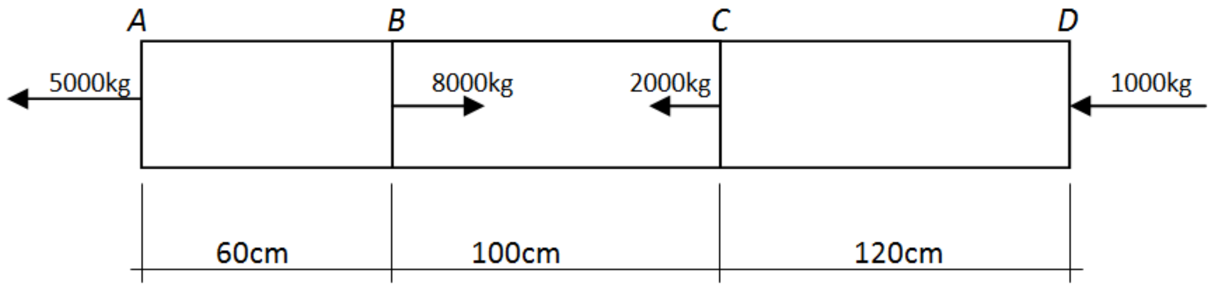
Now change in length δl of entire bar is

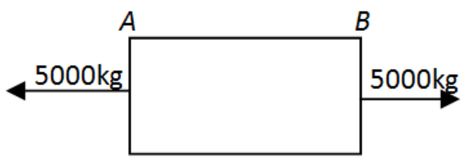
$$\delta l = \delta l_1 + \delta l_2 + \delta l_3 = \frac{F l_1}{A_1 E} + \frac{F l_2}{A_2 E} + \frac{F l_3}{A_3 E}$$

$$= \frac{F}{E}\left(\frac{l_1}{A_1} + \frac{l_2}{A_2} + \frac{l_3}{A_3}\right)$$

Principal of Superposition

For an elastic body acted upon by several forces, it is possible to determine the internal stresses and strains due to each force independently and then obtaining the resultant effect by algebraic summation of the individual effects.

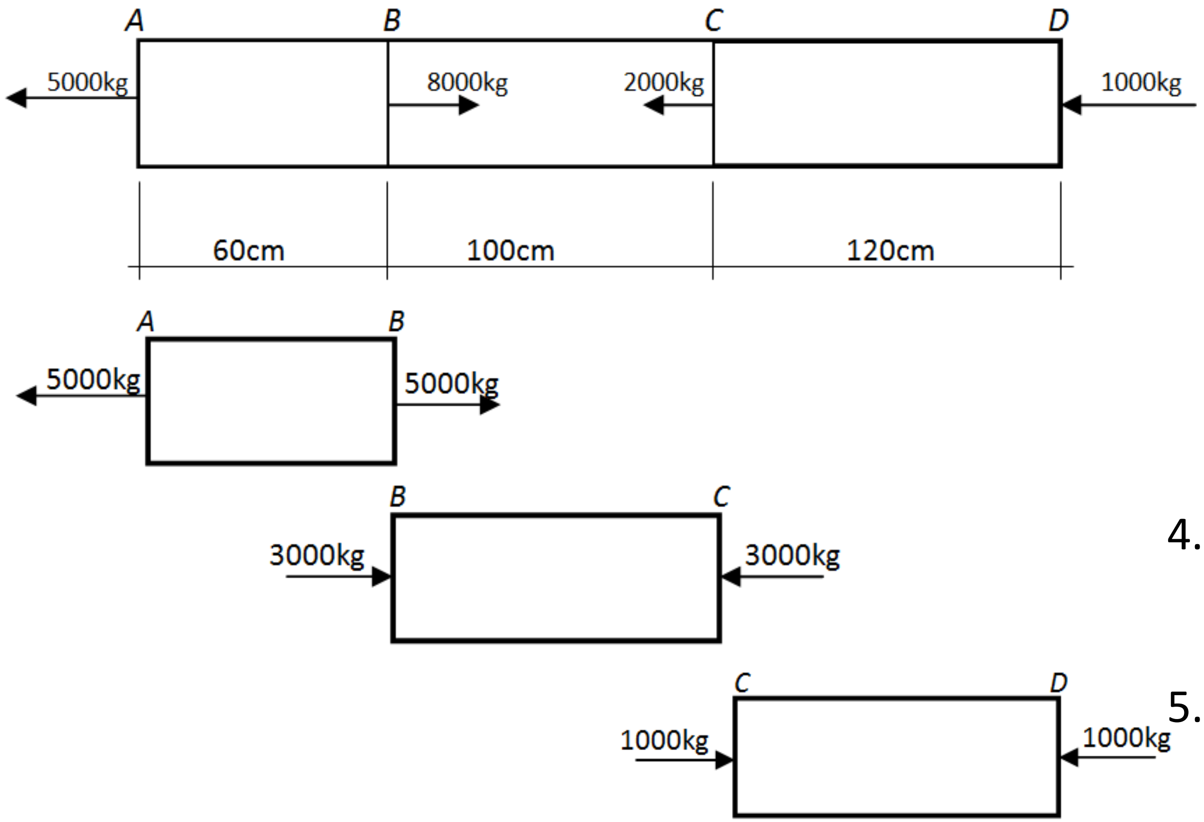




1. Consider all forces left of normal section *A* for *AB*. Net force is 5000kg tensile.

2. All forces right of normal section *B* for *AB*. Net force are (8000-2000-1000)= tensile.

Principal of Superposition (Cont.)

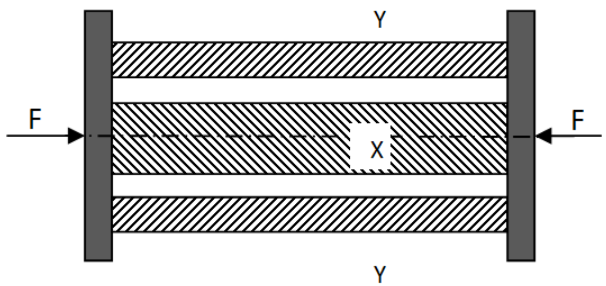


All forces left of normal section *B* for *BC* are (8000-5000)= 3000kg (compressive).

4. All forces to the right of *C* are (2000+1000)=3000kg.
5. All forces left of normal section *C* for *CD* are (8000-5000-2000)=1000kg.
6. Forces to the right of normal section *D* are 1000kg (compressive).

Compound Bars Subjected to Tension/Compression

Consider a rod *X* encased in a tube *Y* of same initial lengths made of different materials with both their ends firmly fixed together. Let the bar be subjected to compressive load *F*. **<u>δ*l*?</u>**



Let

l – Initial common length of both *X* and *Y*

Ex – Modulus of elasticity of material of rod *X*

Ey – Modulus of elasticity of material of tube *Y*

Ax - X-section area of rod X

Ay - X-section area of tube Y

Fx,y – Loads carried by rod and tube resp.

For equilibrium

$$F = F_x + F_y \ldots\ldots\ldots\ldots\ldots\ldots. (1)$$

Strain is equal in both tube and rod, therefore

$$e = \frac{F_x}{A_x E_x} = \frac{F_y}{A_y E_y} \ldots\ldots\ldots\ldots. (2)$$

Or

$$F_x = \frac{F_y A_x E_x}{A_y E_y}$$

Substituting *Fx* in equation (1)

$$F = \frac{F_y A_x E_x}{A_y E_y} + F_y = F_y\left(\frac{A_x E_x + A_y E_y}{A_y E_y}\right)$$

Or

$$F_y = \frac{F A_y E_y}{A_x E_x + A_y E_y}$$

Similarly

$$F_x = \frac{F A_x E_x}{A_x E_x + A_y E_y}$$

$$\left.\begin{aligned} F_y &= \frac{F A_y E_y}{A_x E_x + A_y E_y} \\ F_x &= \frac{F A_x E_x}{A_x E_x + A_y E_y} \end{aligned}\right] \ldots\ldots\ldots\ldots.(3)$$

Substituting *Fx* or *Fy* in equation (2)

$$e = \frac{F A_x E_x}{A_x E_x\left(A_x E_x + A_y E_y\right)} = \frac{F}{A_x E_x + A_y E_y}$$

Change δl in length of the composite section is

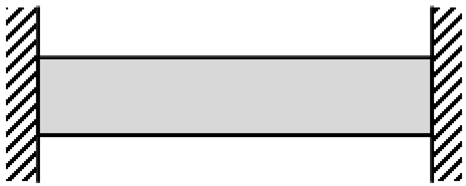
$$\delta l = el$$

Or

$$\delta l = \frac{Fl}{A_x E_x + A_y E_y} \ldots \ldots \ldots \ldots \ldots (4)$$

Temperature Stresses

If material expansion or contraction is wholly or partially resisted, stresses are set up in the body.



Let
l_o – Original length of bar
t – Rise in temperature
α - Coefficient of expansion of rod material

Length of bar l_t after t ℃ rise in temperature, had the bar been free to expand would have been:

$$l_t = l_o(1 + \alpha t) \dots\dots\dots\dots\dots (1)$$

Or

$$l_t - l_o = l_o \alpha t \dots\dots\dots\dots (2)$$

If no end restraint, bar length increase would have been: $l_t - l_o$

∴ Restraints have caused strain of: $e = \dfrac{l_t - l_o}{l_o}$

Or

$$e = \frac{l_o \alpha t}{l_o}$$

Or

$$e = \alpha t \dots\dots\dots\dots\dots.(3)$$

But from Hooke's law, we know that

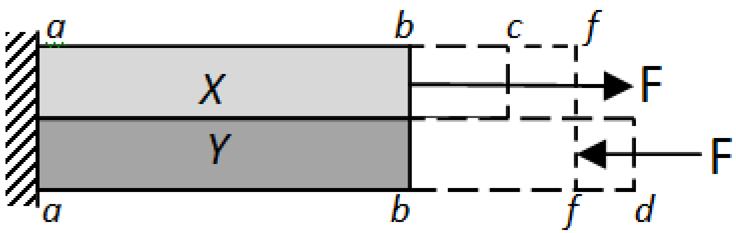
$$\frac{\sigma}{e} = E \qquad \text{or} \qquad \sigma = eE$$

Substitute for strain from equation (3)

$$\sigma = E\alpha t \dots\dots\dots\dots\dots\dots\dots\dots\dots(4)$$

Compound/ Composite Rods

Lets consider



Let l_o – Original length of both rods

$A_{x,y}$ – *Resp.* X-section areas for rods X and Y

$\alpha_{x,y}$ – Resp.Coefficients of expansion for rods X and Y ($>\alpha_x$)

t – Rise in temperature.

Rods X and Y will be subjected to tensile and compressive forces F respectively.

Let ***e*** – Common strain in the two rods

$e_{x,y}$ – Respective Strains in rods X and Y if they were free to expand

$E_{x,y}$ - Moduli of elasticity rods X and Y resp.

Now we have

$$e = \frac{bf}{l} \qquad e_x = \frac{bc}{l} = \frac{bf - cf}{l} = \frac{bf}{l} - \frac{cf}{l}$$

Or

$$e_x = e - \frac{F}{A_x E_x} \dots\dots\dots\dots\dots\dots\dots..(i)$$

Similarly, $$e_y = \frac{bd}{l} = \frac{bf + fd}{l} = \frac{bf}{l} + \frac{fd}{l}$$

Or $$e_y = e + \frac{F}{A_y E_y} \ldots\ldots\ldots\ldots\ldots\ldots\ldots..(ii)$$

Substituting equation (i) from (ii)

$$e_y - e_x = \frac{F}{A_y E_y} + \frac{F}{A_x E_x} = F\frac{A_x E_x + A_y E_y}{A_x E_x A_y E_y}$$

But $$e_y = \alpha_y t \quad \text{and} \quad e_x = \alpha_x t$$

$\therefore$ $$(\alpha_y - \alpha_x)t = F\frac{A_x E_x + A_y E_y}{A_x E_x A_y E_y}$$

Or $$F = \frac{t(\alpha_y - \alpha_x) A_x E_x A_y E_y}{A_x E_x + A_y E_y} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

From equation (i) above:

$$e - \frac{F}{A_x E_x} = e_x = \alpha_x t$$

Or

$$e = \alpha_x t + \frac{F}{A_x E_x}$$

Substituting for $\frac{F}{A_x E_x}$ we get:

$$e = t * \frac{A_x E_x \alpha_x + A_y E_y \alpha_y}{A_x E_x + A_y E_y} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(2)$$

Example: A rod is 2m long at a temperature of 10°C. Find the expansion of the rod when the temperature is raised to 80°C. If this expansion is prevented, find the stress in the material of the rod. Take $E=1x10^6$ kg/cm^2 and $\alpha=0.000012$ /°C.

Given: l_o =2m; t=80-10=70degC; α=0.000012 /^oC

Let l_t = length at 80degC

$$l_t = l_o(1 + \alpha t)$$

$\therefore$ Increase in length due to rise in temperature is

$$\delta l = l_t - l_o = l_o \alpha t$$

$$= 2 * 0.000012 * 70 = \mathbf{0.168cm}$$

If expansion is prevented, then strain caused

$$e = \frac{\delta l}{l_o} =$$

$$= \frac{0.168}{2 * 100} = 0.00084$$

Stress caused in the material

$$\sigma = eE \quad = 0.00084 * 1 * 10^6 = 840\ kg/cm^2$$