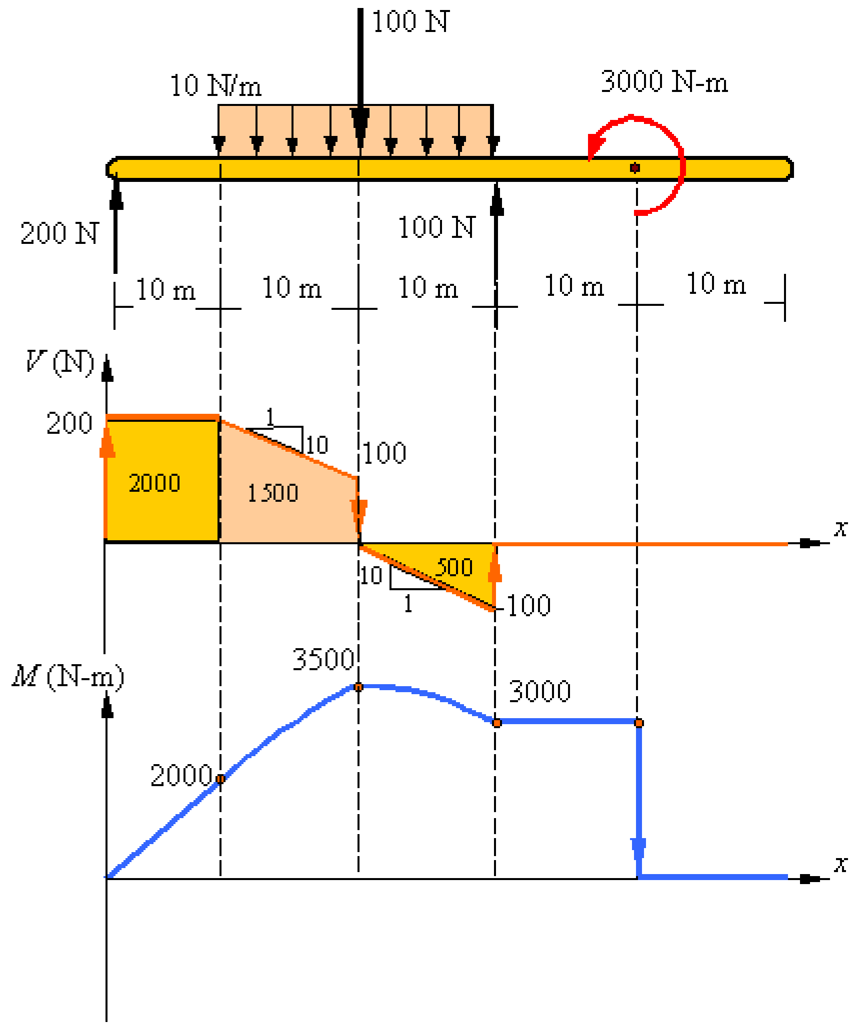
Shear Force and Bending Moment

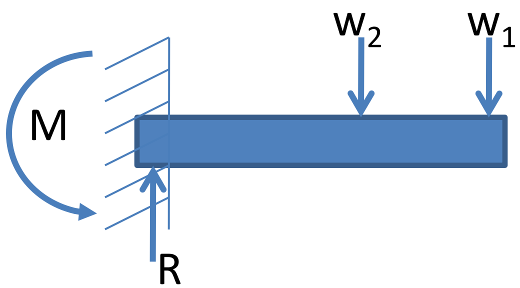


Beam – a horizontal bar carrying external forces that are inclined to its axis.

Freely/simply supported beam is one that rests on "knife edges" ie the supports that provide only the vertical reaction and no restraint at end from rotating to any slope at the supports.

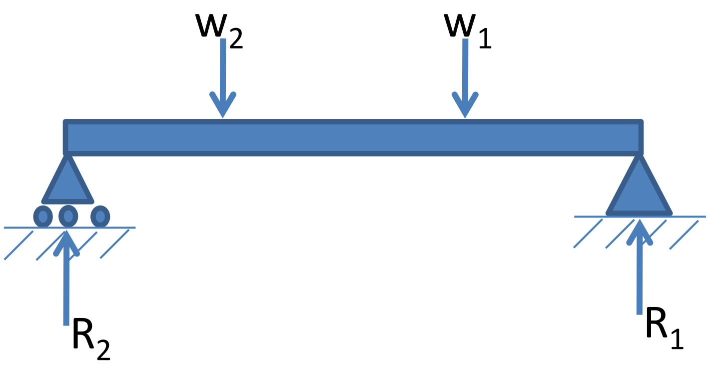
Modes of Supports

a) Cantilever Beam



- Sufficient restraint to prevent rotation of the end

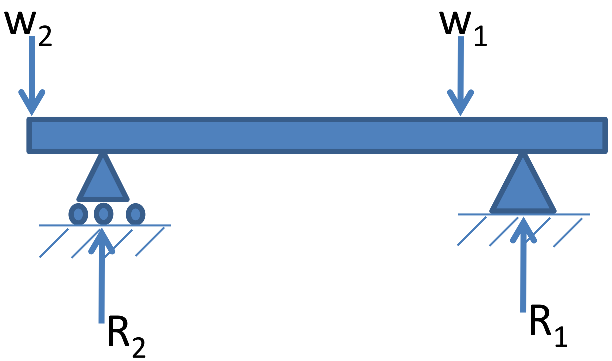
b) Simply Supported Beam



- Supported by hinged reaction at one end and roller support at the other

- Not restrained otherwise.

c) Overhanging Beam

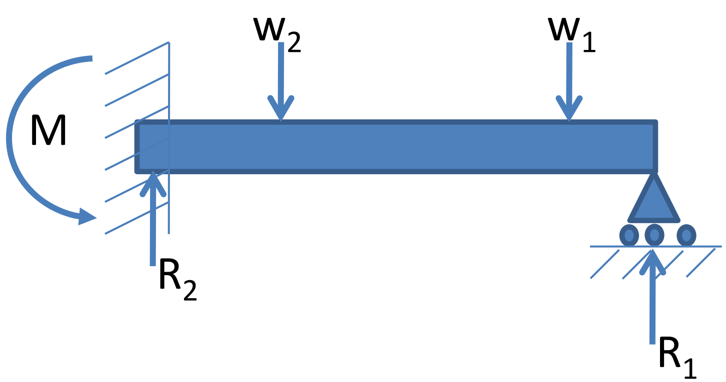


- Supported by a hinge and roller reaction
- Either or both ends extend beyond supports

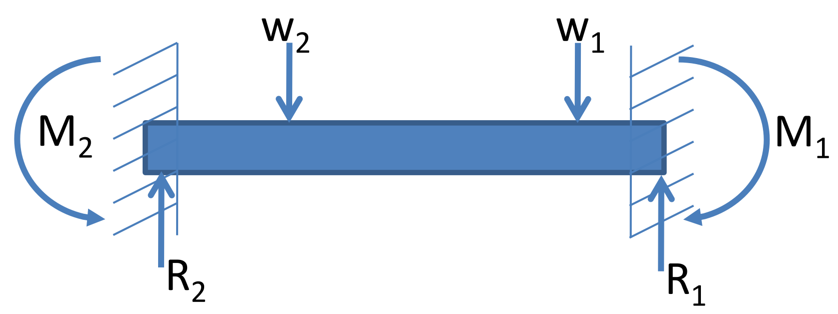
Support reactions in all the above cases a, b, c can be determined by equations of static equilibrium.

<u>Others</u>

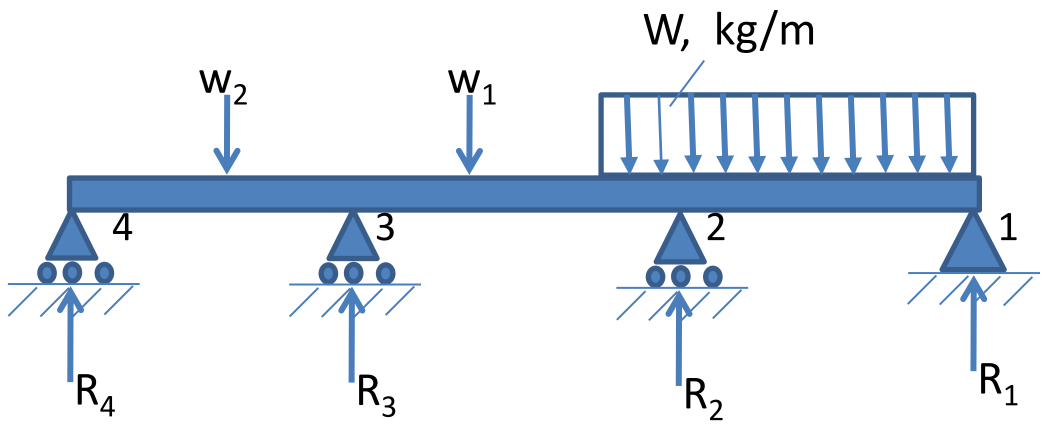
d) Propped Beam



e) Fixed or Restrained Beam



f) Continuous Beam



Support reaction in cases d, e, f can not be determined by equations of static equilibrium.

∴ Statically indeterminate.

Type of Loads

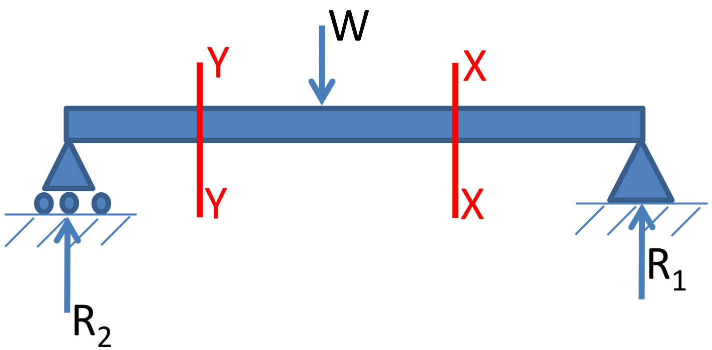
a) **Point Load (Concentrated load)** – is one acting over so small a distance that it can be assumed to act at a point.

b) **Distributed Load** – distributed over a part or entire length of beam.

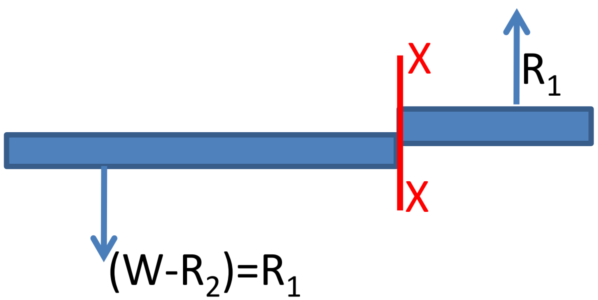
1) Spread uniformly (u.d.l, kg per metre).

2) Non-uniformly (called by its pattern of non-uniformity, e.g triangular distributed).

Consider a simply supported beam carrying a point load W. let support reactions be R_1 and R_2.



- The beam on the right of section X-X is subjected to an upward force R_1 and portion to the left of X-X is subjected to a downward force $(W-R_2)$.



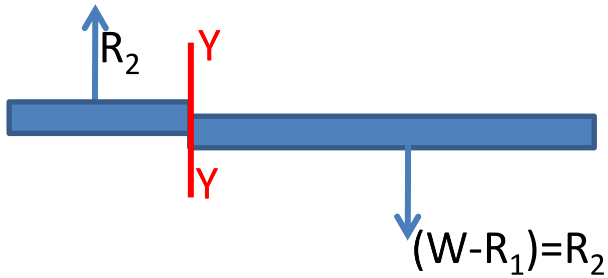
Shear force a X-X

+ve shear
(Right-up-left-down)

But $R_1 + R_2 = W \quad \therefore \; W - R_2 = R_1$

R_1 is thus shearing the beam at section X-X.

Similarly



Shear force at Y-Y

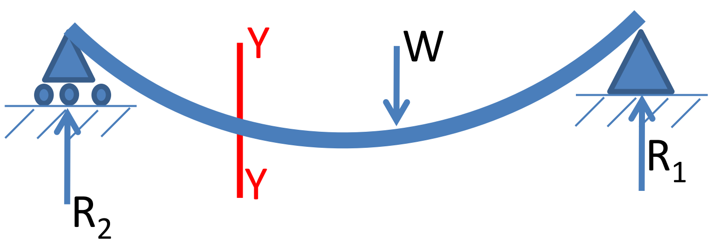
-ve Shear
(Left-up-Right-down)

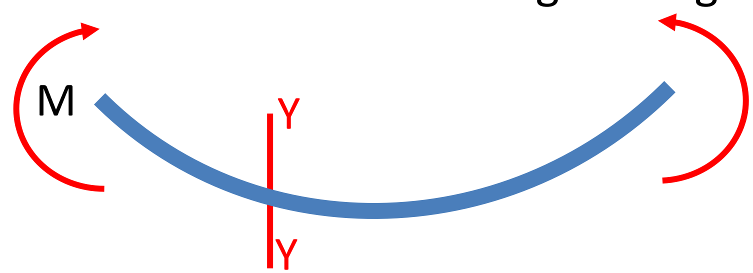
$\therefore$ At section Y-Y the beam is subjected to a shear force of R_2

- Shearing force at a section of a beam is the algebraic sum of the forces on either side of that section.

$$\sum F_{left} = 0 \qquad \text{and} \qquad \sum F_{right} = 0$$

- Consider a beam (simply supported) bent as a result of load W along its length

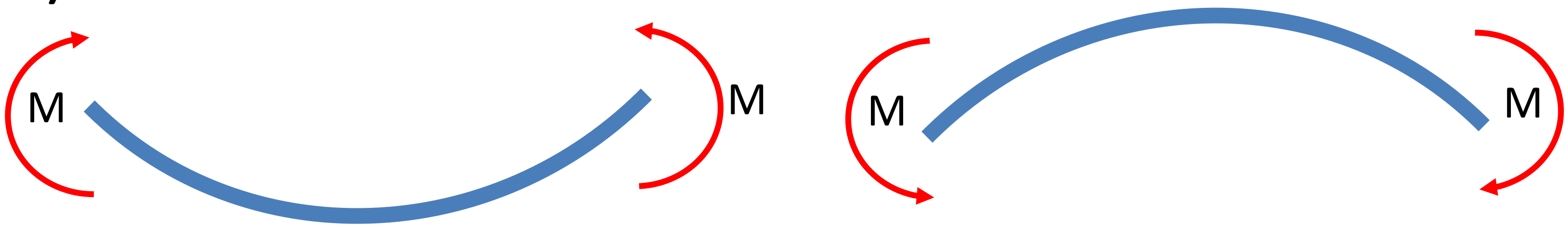




- For equilibrium, the algebraic sum of the moments due to forces to the right of this section should be equal to the moments in anti-clockwise direct

The moment causing this bending is called the **Bending Moment**.

a)



+ve B.M (Sagging)

-ve B.M (Hogging)

Calculation of S.F and B.M at any Section

1) In simply supported beams, first determine the support reaction(s).
2) Choose to start either from the right end or left end of beam. (In the case of cantilever always start from free end).
3) For S.F at any section, find ΣF (including R) acting on a chosen portion of beam or the portion of the cantilever between section and free end.
4) For B.M , find ΣM due to all forces acting on chosen portion.
5) Plot calculated values of S.F (and B.M) as ordinates to some scale. (Take +ve values above, and –ve below beam axis.)
6) Indicate important values, and sections where the values are max & zero.

Note

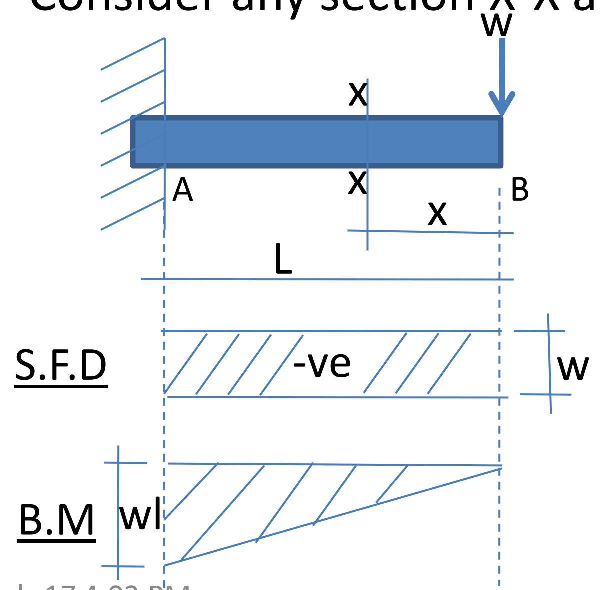
- For point loads, the S.F diagram has straight horizontal line & B.M diagram has inclined straight line.
- For u.d.l, the S.F diagram has inclined line (mostly) & B.M diagram has curve.

Shear Force & Bending Moment Diagrams.

1) Cantilevers

a) Point Load at the Free End.

Consider any section X-X at any distance x from free end.



S.F at X-X is

$$F_x = -W$$

Note:

- Left-up-right down ∴ -ve
- S.F is independent of x

-B.M. at X-X is

$$M_x = -Wx \qquad \text{..............} \text{ (i)}$$

Shear Force & Bending Moment Diagrams.

1) Cantilevers

a) Point Load at the Free End.

Note:

- The force causes hogging moment at section x-x $\therefore$ -ve M.
- B.M at free end

$$x = 0 \quad \therefore \quad M_B = 0$$

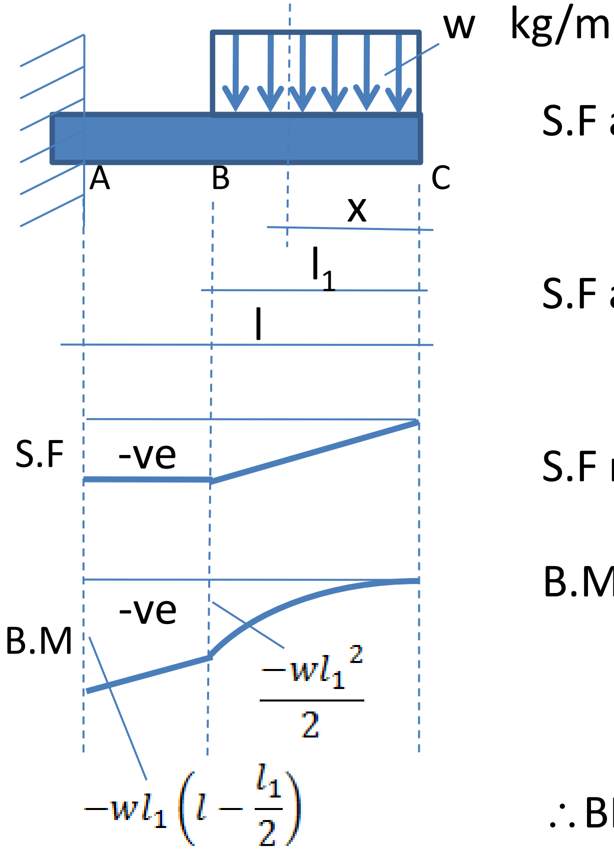
- B.M at fixed end

$$x = L \quad \therefore \quad M_A = -WL$$

- Eqn (i) is of first degree order $\therefore$ straight line where it changes from 0 at B to (-WL) at A.

b) Uniformly Distributed Load (u.d.l)

Consider a section at a distance x from C.



S.F at the section x

$F_x = -wX$; $F_C = 0$ at C

S.F at B where $x = l_1$

$F_B = -wl_1$

S.F remains constant from B to A.

B.M at the section x

$$M_x = -wx * \frac{x}{2} = \frac{-wx^2}{2}$$

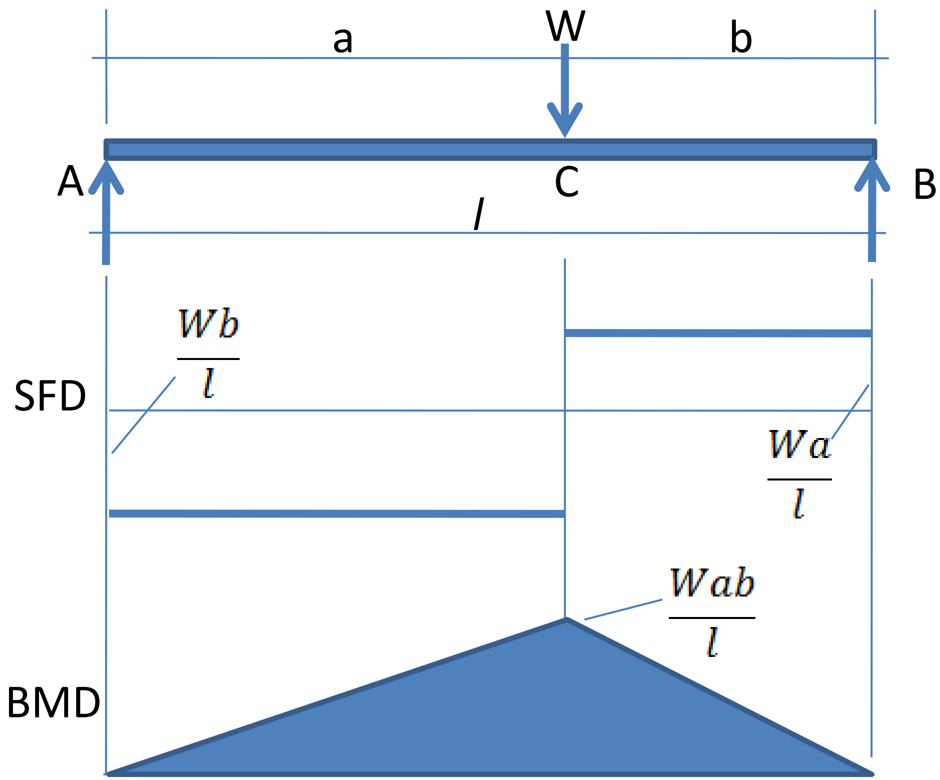
∴ BM at C : $M_C = 0$

BM at B : $M_B = \frac{-wl_1^{\,2}}{2}$

BM at A: $M_A = -wl_1\left(l - \frac{l_1}{2}\right)$

Beams Simply supported at the two ends

Let the beam *AB* of span *l* carry a load W at a distance a from A and b from B.



Take moments about A to determine support reactions:

$$R_B * l = Wa \quad \therefore \quad R_B = \frac{Wa}{l}$$

But $R_A + R_B = W$

$$\therefore \ R_A = W - \frac{Wa}{l} = \frac{W(l-a)}{l} = \frac{Wb}{l}$$

SF just left of B is: $+\frac{Wa}{l} = Const$ up to C.

SF just left of C is:

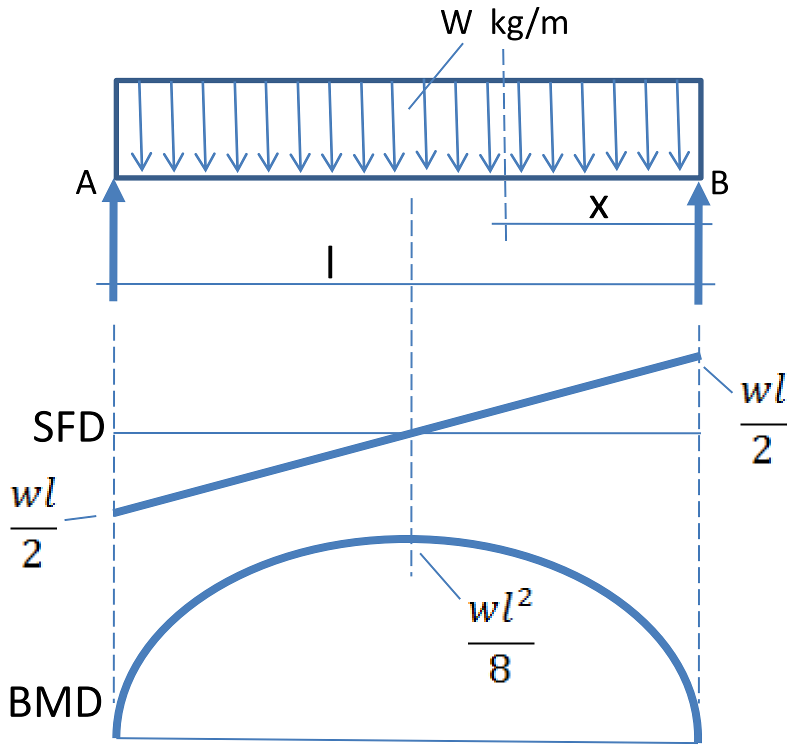
$$\left(+\frac{Wa}{l} - W\right) = \left[\frac{-W(l-a)}{l}\right] = -\frac{Wb}{l}$$

BM at C is : $M_C = \frac{Wa}{l}b = \frac{Wab}{l}$

Note: MB is max at C where SF changes sign.

BM at the supports A & B is zero (since the beam is simply supported).

U.d.l Over the entire span



By symmetry, each support is equal to

$$R_A = R_B = \frac{wl}{2}$$

SF at a section of distance x from B is

$$F_x = +\frac{wl}{2} - wx$$

$$\therefore F_B = +\frac{wl}{2} \qquad (x = 0)$$

At the mid span $\left(x = \frac{l}{2}\right)$

$$F_{Mid} = \frac{wl}{2} - \frac{wl}{2} = 0$$

$$F_A = \frac{wl}{2} - wl = -\frac{wl}{2} \qquad where\ x = l$$

BM at the section is

$$M_x = +\frac{wl}{2}x - \frac{wx^2}{2} \dots\dots\dots\dots (i)$$

$\therefore$ BM at B where x=0 is

$$M_B = +\frac{wl}{2} * 0 - \frac{w0^2}{2} = 0$$

$$M_x = +\frac{wl}{2}x - \frac{wx^2}{2} \dots\dots\dots\dots (i)$$

For the BM to be max, put $\frac{dM_x}{dx}$ for eqn (i) above equal to zero

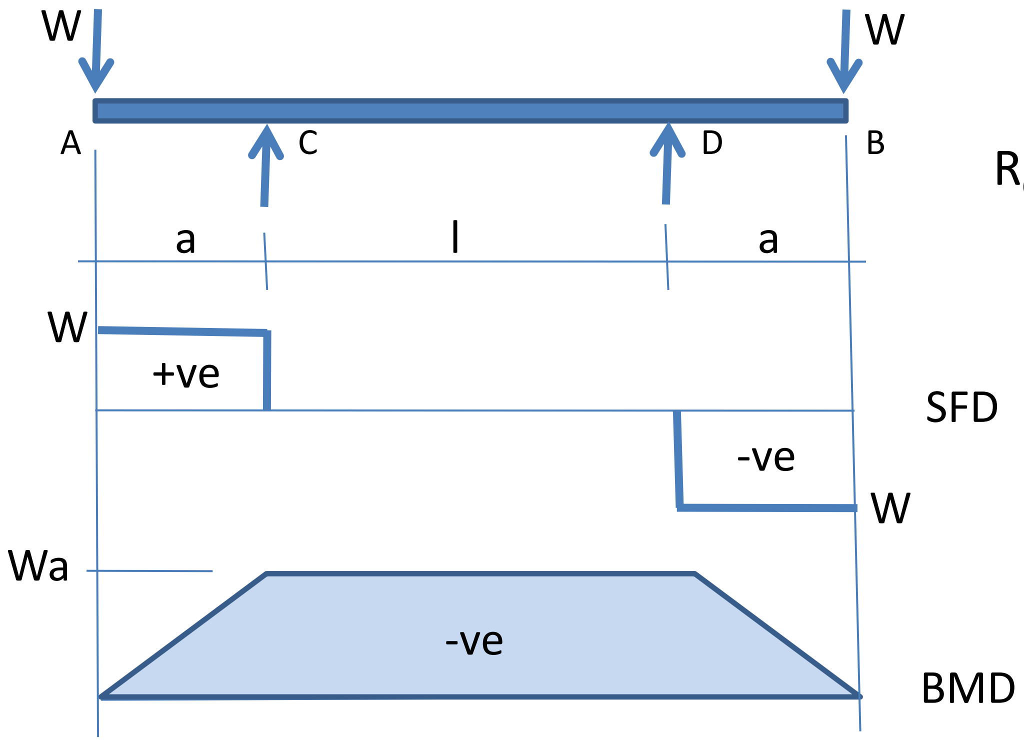
Slope

$$\therefore \quad \frac{dM_x}{dx} = +\frac{wl}{2} - wx = 0$$

$x = \frac{l}{2}$ for BM to be max (note: SF=0 at the same point)

$$M_{max} = +\frac{wl}{2} * \frac{l}{2} - \frac{w}{2} * \left(\frac{l}{2}\right)^2 = +\frac{wl^2}{8}$$

Beams with overhangs



$R_C = R_D = w$

SF just to the left of B is –w and is constant up to D.

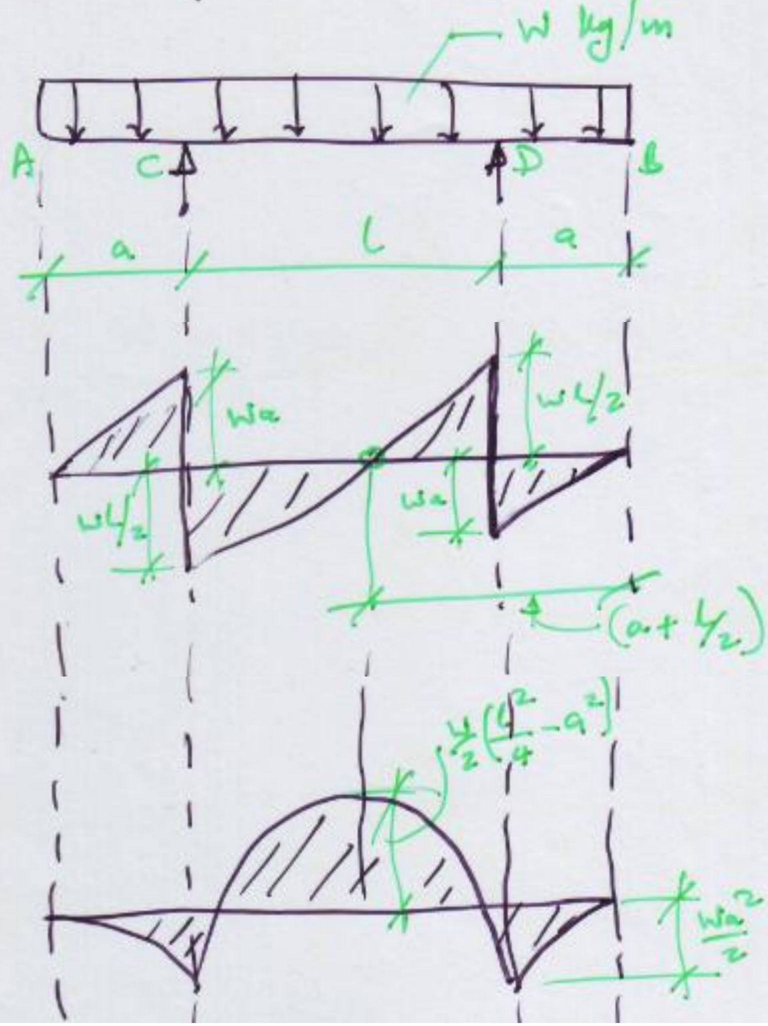
SF just to the left of D is (-w+w)=0 and is constant up to C.

SF just to the left of C is (-w+w+w) = w and is constant up to A.

BM at D is M_D = -wa.

BM at C is M_C = -w(a+l)+wl = -wa.

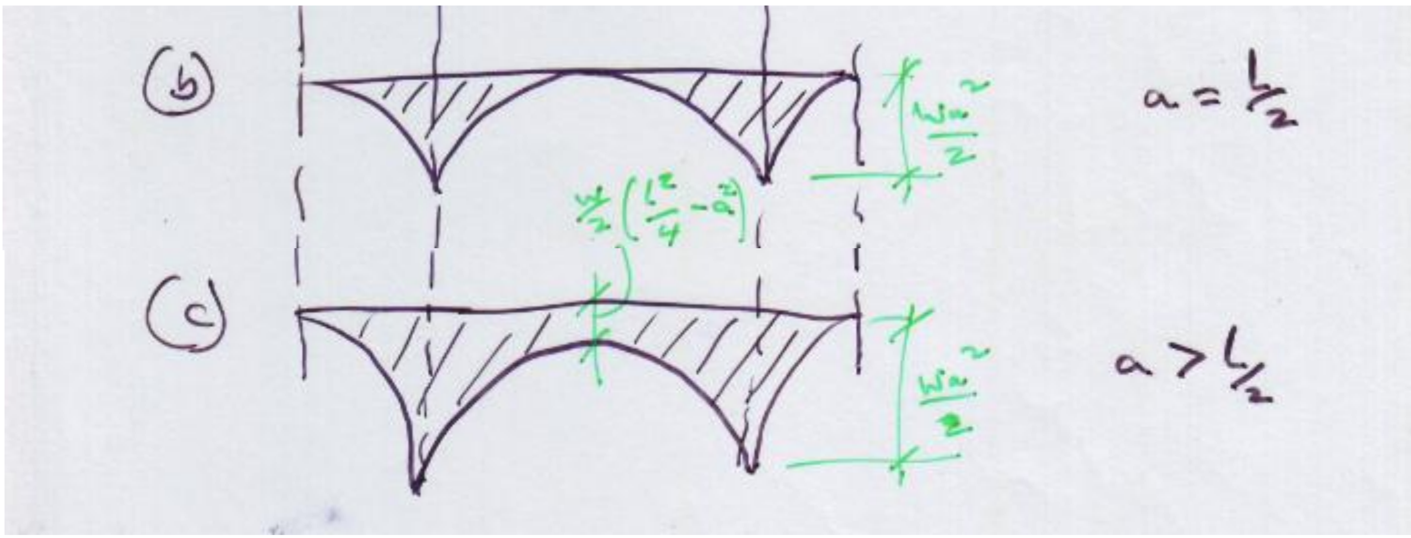
Beams with equal overhangs & carrying u.d.l over the entire span.



(a)

When

$a < \frac{L}{2}$


(b)
a = L/2
(c)
a > L/2

Since the beam is symmetrical load both the reactions are equal, each being $\frac{1}{2} \cdot w(l+2a) = R_C = R_D$.

SF just to the right of D is $= -wa$

—— " —— left of D is $= -wa + \frac{1}{2}w(l+2a)$
$= +\frac{wl}{2}$

—— " —— right of C is $= -w(l+a) + \frac{1}{2} \cdot w \cdot (l+2a) = -\frac{wl}{2}$

—— " —— left — " — is $= -w(l+a) + 2 \cdot \frac{1}{2} w(l+2a)$
$= +wa$

The SF at D is $+\frac{wl}{2}$ & $-\frac{wl}{2}$ at C

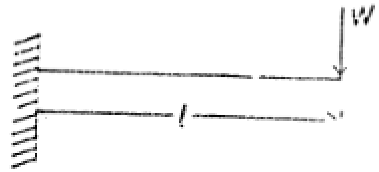
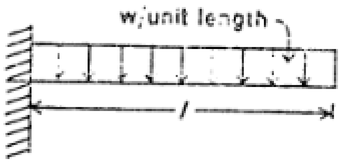
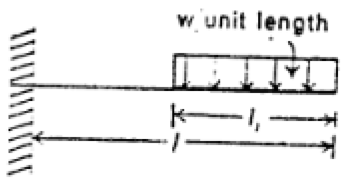
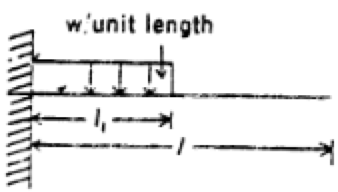
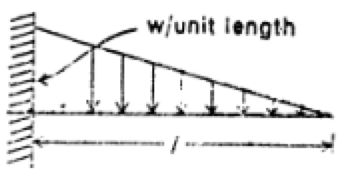
∴ Zero at mid part of C & D

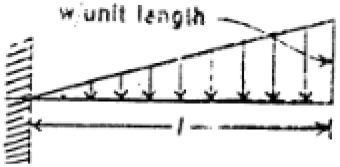
B.M at D is $-wa \cdot \frac{a}{2} = -\frac{wa^2}{2}$

BM at point of zero shear, i.e., at distance $\left(a+\frac{l}{2}\right)$ from B is max

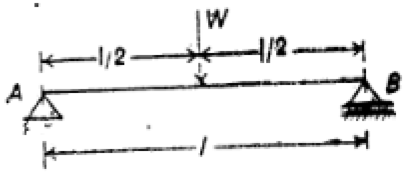
TYPICAL STANDARD CASES OF CANTILEVERS AND BEAMS

Cantilevers. *(S.F. and B.M. are both maximum at fixed end of a cantilever).*

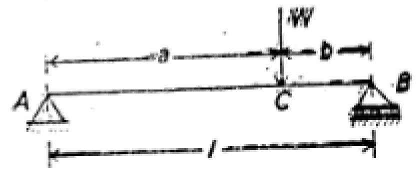
S. No.	*Diagram*	*Shear*	*Bending moment*
1.		$F_{max} = -W$	$M_{max} = -Wl$
2.		$F_{max} = -wl$	$M_{max} = -\frac{wl^2}{2}$
3.		$F_{max} = -wl_1$	$M_{max} = -wl_1\left(l - \frac{l_1}{2}\right)$
4.		$F_{max} = -wl_1$	$M_{max} = -\frac{wl_1^2}{2}$
5.		$F_{max} = -\frac{wl}{2}$	$M_{max} = -\frac{wl^2}{6}$

6. 

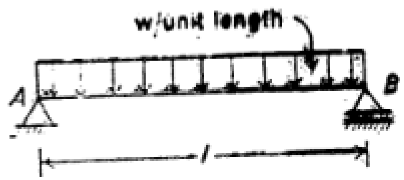
$F_{max} = -\frac{wl}{2}$ $M_{max} = -\frac{wl^2}{3}$

7. 

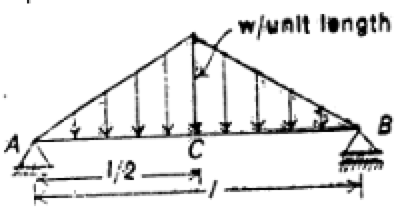
$F_{max} = \frac{W}{2}$ $M_{max} = +\frac{Wl}{4}$ at mid span.

8. 

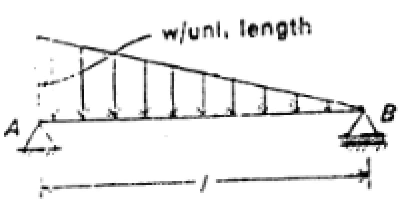
$F_{max} = \frac{Wa}{l}$ from B to C. $M_{max} = +\frac{Wab}{l}$ at C.

9. 

$F_{max} = \frac{wl}{2}$ at A and B. $M_{max} = +\frac{wl^2}{8}$ at $\frac{l}{2}$ from A

10. 

$F_{max} = -\frac{wl}{4}$ at A and B. $M_{max} = +\frac{wl^2}{12}$ at C.

11. 

$F_{max} = -\frac{wl}{3}$ at A. $M_{max} = \frac{wl^2}{9\sqrt{3}}$ at $\frac{l}{\sqrt{3}}$ from L.