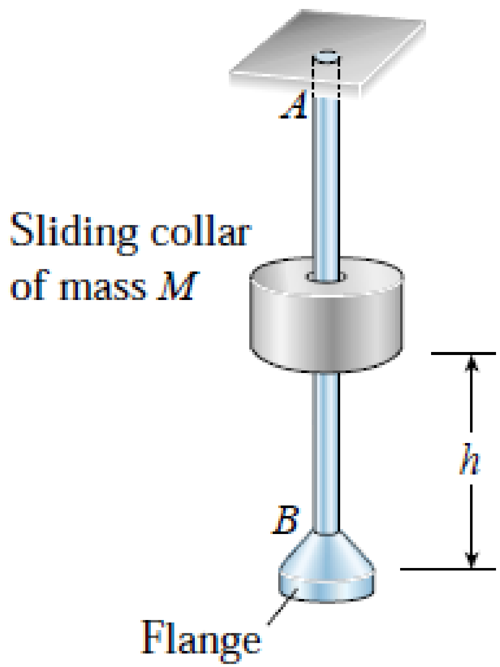
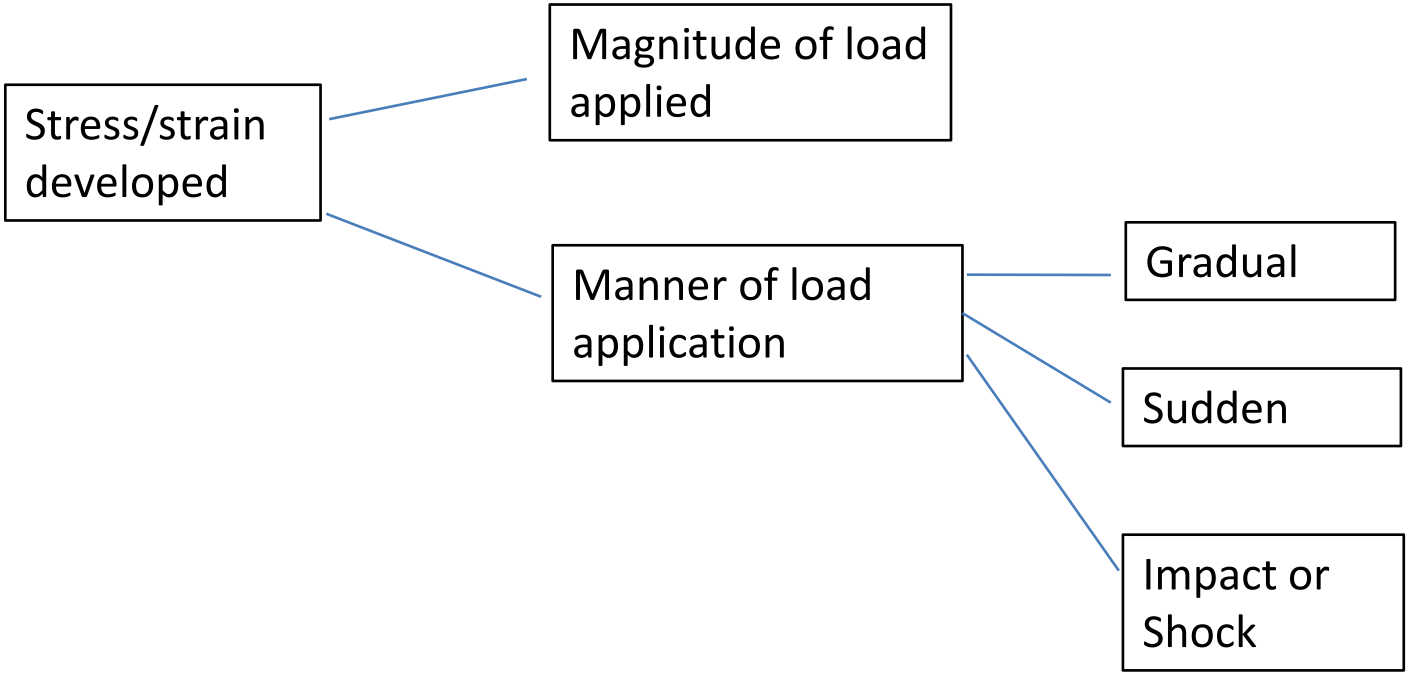
Impact



Resilience – Strain energy stored in a specimen when strained within elastic limit.

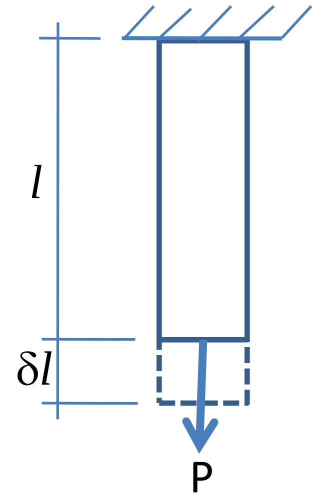
Proof Resilience – The maximum energy stored at the elastic limit.

Modulus of Resilience – Proof resilience per unit volume.



Gradually Applied Loads

Let a vertical bar of length l and x-sectional area A be rigidly held at one end and carry a gradually applied load P.



If

Resultant extension – δl

Resultant stress in the bar - σ

Therefore axial load P $P = \sigma A$

Now energy stored in the bar is

$$U = \text{Work done by gradually applied axial load P}$$
$$= {}^*\text{Average load x elongation}$$
$$= \frac{P}{2}\delta l = \frac{1}{2}(\sigma A)\frac{Pl}{AE}$$
$$= \frac{1}{2}(\sigma A)\frac{\sigma l}{E} = \frac{\sigma^2}{2E}Al$$

Note
* Load is gradually increased from 0 to P, hence average.

Or

$$U = \frac{\sigma^2}{2E} \text{ x Volume of bar} \qquad \ldots(1)$$

If the value of stress σ at the elastic limit is σ_e then proof resilience is

$$U_P = \frac{\sigma_e^{\ 2}}{2E} \text{ x Volume of bar}$$

Modulus of resilience

$$= \frac{\sigma_e^{\ 2}}{2E} \qquad \ldots(2)$$

Sudden Applied Loads

Let

Instantaneous Elongation $-\,\delta l$

Instantaneous Stress $-\,\sigma$

Then equating the strain energy in the bar to the work done by the applied load we get:

$$U = \text{Work done by load}$$

$$\therefore \qquad **\left(\frac{\sigma}{2}A\right)\delta l = P\,\delta l$$

$$or \qquad \sigma = \frac{2P}{A}$$

** Stress increases with increase in stretch from 0 to δl , hence average stress

Instantaneous stress developed in a bar subjected to suddenly applied load is thus twice the stress produced by the same load applied gradually.

Therefore, Instantaneous elongation:

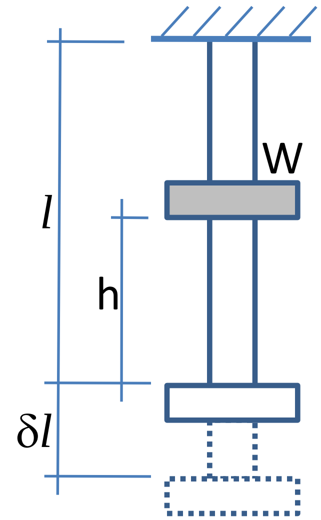
$$\delta l = \frac{\sigma}{E} l = \frac{2Pl}{AE}$$

Instantaneous elongation produced in a bar subjected to sudden applied load is thus twice that produced by the load applied gradually.

Stresses due to Impact Loading

Let

- W – Dropping Weight
- A – Bar cross-sectional area
- δl – Maximum elongation of bar
- σ – Corresponding stress
- P – Equivalent static or gradually applied load which could produce same elongation δl.



Strain energy in the bar at this instant $= \dfrac{P\delta l}{2}$

Also work done by the weight $= W\left(h + \delta l\right)$

But work done by the weight = Strain energy stored in the bar

therefore

$$W\left(h+\delta l\right) = \frac{P\delta l}{2}$$

But $$\delta l = \frac{Pl}{AE}$$

Therefore
$$W\left(h + \frac{Pl}{AE}\right) = \frac{P}{2}\frac{Pl}{AE}$$

$$Wh + \frac{WPl}{AE} = \frac{P^2 l}{2AE}$$

$$\text{or} \quad P^2 - 2WP - \frac{2AEWh}{l} = 0$$

Therefore
$$P = \frac{2W + \sqrt{4W^2 + \frac{8AEWh}{l}}}{2}$$

(neglecting negative value)

Therefore
$$P = W\left[1 + \sqrt{\left(1 + \frac{2AEh}{Wl}\right)}\right] \qquad \ldots(4)$$

If, however the value of δl is very small as compared to h, then

$$Wh = \frac{P\delta l}{2} = \frac{P}{2}\frac{Pl}{AE} = \frac{P^2 l}{2AE}$$

Therefore

$$P^2 = \frac{2WAEh}{l}$$

$$or$$

$$\frac{P^2}{A^2} = \frac{2WEh}{Al}$$

$$or$$

$$\frac{P}{A} = \sigma = \sqrt{\frac{2WEh}{Al}} \qquad \ldots(5)$$