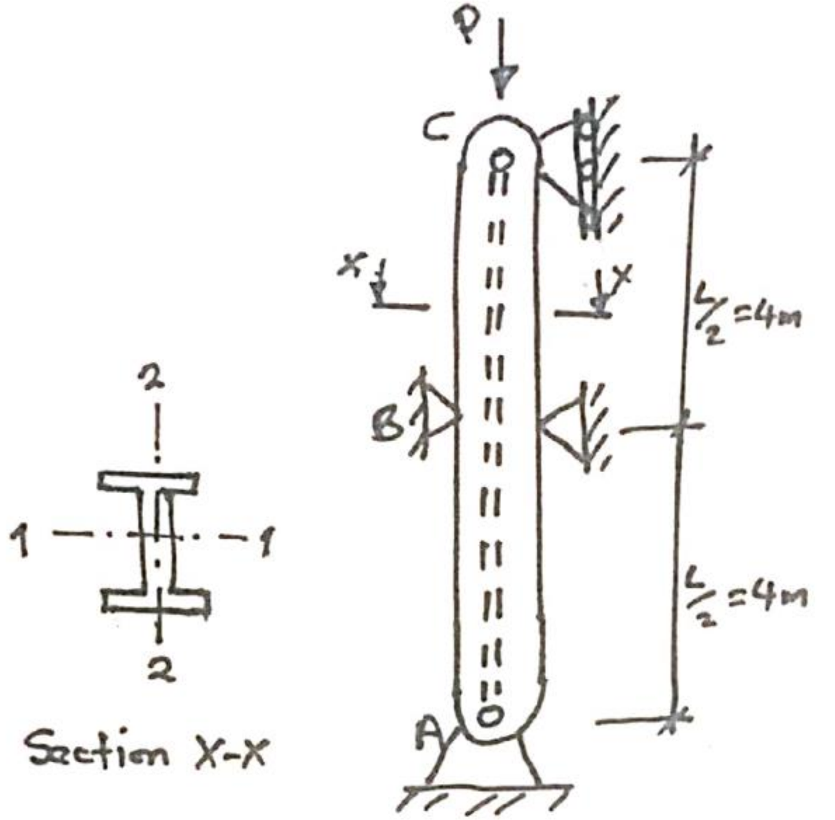
Problem 1



$L = 8\,m$

$E = 200\,GPa$

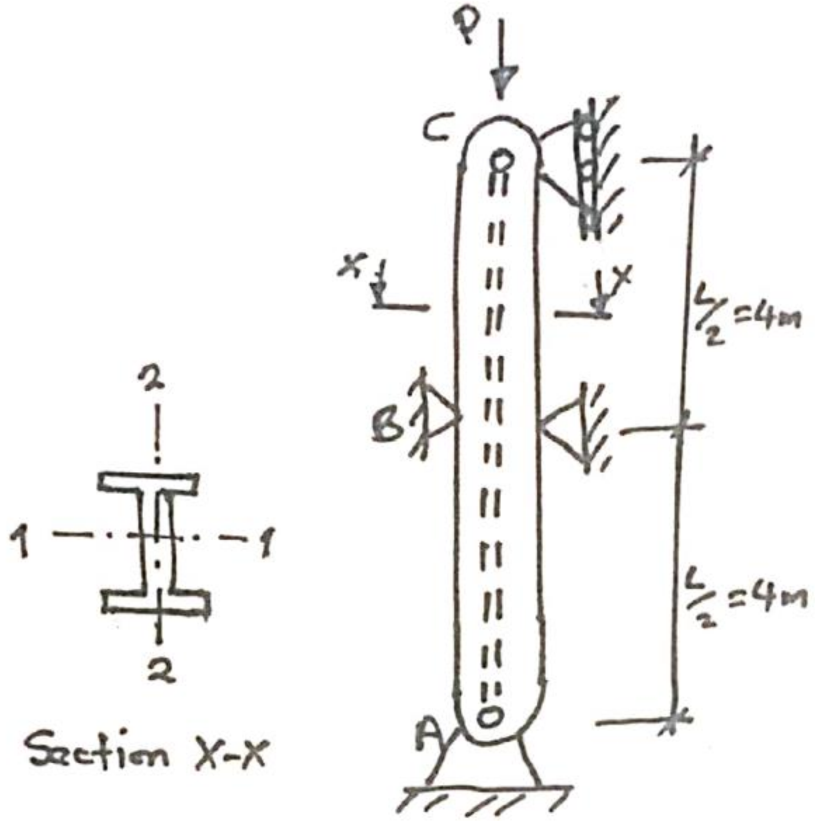
$\sigma_y = 300\,GPa$

$I_1 = 3060\,cm^4$

$I_2 = 162\,cm^4$

$A = 39.5\,cm^2$

Determine the allowable load P_{allow} using a factor of Safety $n = 2.5$ with respect to Euler buckling of the column.



Critical loads:

If the column buckles in the plane of the figure

$$P_{cr} = \frac{\pi^2 E I_2}{(L/2)^2} = \frac{4\pi^2 E I_2}{L^2}$$

$$= \frac{4\pi^2 (200\ GPa)\cdot(162\ cm^4)}{(8m)^2} = 200\ kN$$

If the column buckles perpendicular to the plane of the figure

$$P_{cr} = \frac{\pi^2 E I_1}{L^2} = \frac{\pi^2 \cdot (200\ GPa)\cdot(3060\ cm^4)}{(8m)^2} = 943.8\ kN$$

Therefore the critical load for the column (the Smaller of the two values is

$P_{cr} = 200\,kN$ and buckling will occur in that plane of the figure.

Critical Stresses:

$$\sigma_{cr} = \frac{P_{cr}}{A} = \frac{943.8\,kN}{39.5\,cm^2} = 238.9\,MPa < \sigma_y = 300\,MPa$$

Since this stress is less the proportional limit, both critical-load calculations are satisfactory.

Allowable load (axial):

$$P_{all} = \frac{P_{cr}}{n} = \frac{200\,kN}{2.5} = 79.9\,kN$$

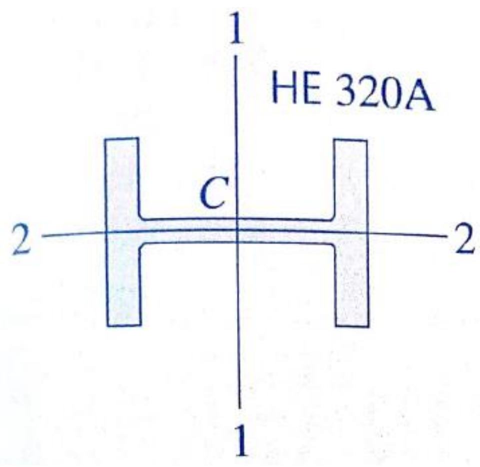
n - desired factor of Safety

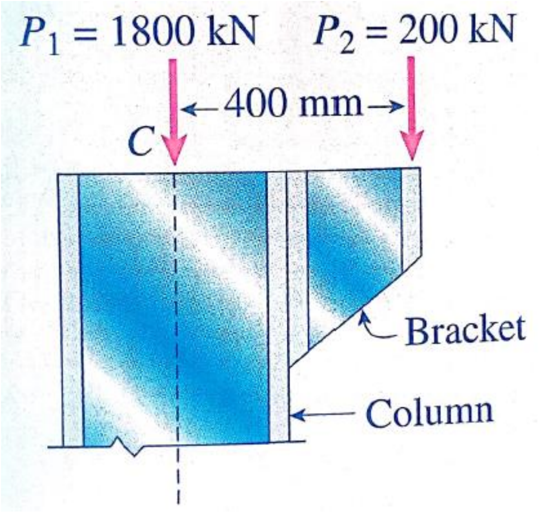
Problem 2

A steel wide flange column HE 320 A is pin supported at ends and has a length of 7.5m. The column supports a centrally applied load $P_1 = 1800$ kN and an eccentrically applied load $P_2 = 200$ kN. Bending takes place about axis 1-1 of the cross section, and the eccentric load acts on axis 2-2 at a distance of 400 mm from the centroid C.

(a) Using the secant formula, assuming E = 210 GPa, calculate the maximum compressive stress in the column.

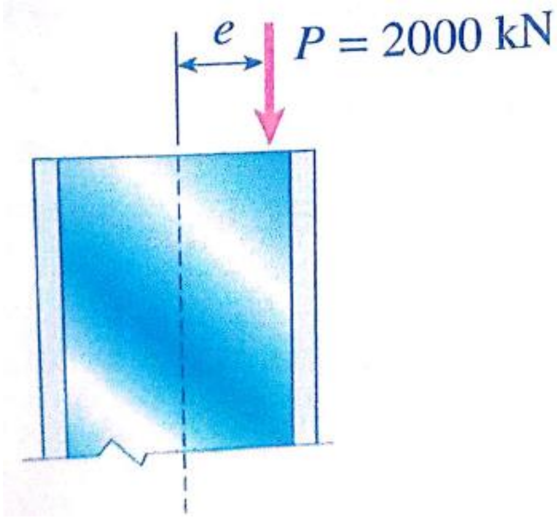
(b) If the yield stress is $\sigma_y = 300$ MPa, what is the factor of safety with respect to yielding?





Soln

a) P_1 and P_2 are statically equivalent to a single load acting with an eccentricity of 40 mm.



Apply the secant formula to find σ_{max}.

For Section HE 32A : $A = 124.4\,cm^2$ $r = 13.58\,cm$ $c = \frac{310\,mm}{2}$

$= 155\,mm$

$(= y_c)$

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{L}{2r}\sqrt{\frac{P}{AE}}\right)\right]$$

$$\frac{P}{A} = \frac{2000\,kN}{124.4\,cm} = 160.77\,MPa \qquad \frac{ec}{r^2} = \frac{(40\,mm)(155\,mm)}{(13.58\,cm)^2} = 0.336$$

$$\frac{L}{r} = \frac{7.5\,\mathrm{m}}{13.58\,\mathrm{cm}} = 55.23 \qquad \frac{P}{EA} = \frac{2000\,\mathrm{kN}}{(210\,\mathrm{GPa})(124.4\,\mathrm{cm}^2)} = 765.6 \times 10^{-6}$$

$$\sigma_{max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{L}{2r}\sqrt{\frac{P}{AE}}\right)\right]$$

$$= (160.77\,\mathrm{MPa})(1 + 0.466) = \underline{235.6\,\mathrm{MPa}}$$

(b) Factor of Safety with respect to yielding.

- Find P acting at eccentricity e that will produce $\sigma_{Max} = \sigma_y = 300$ MPa.
- Since P is suffient enough to produce a yielding of material, $\therefore P \Rightarrow P_y$
- Note: You can not determine P_y by multipying load $P(= 2000\,kN)$ by ratio σ_y / σ_{Max}.

  Reason: Dealing with non-linear relationship between load and stress.

$\therefore$ Substitute $\sigma_{max} = \sigma_y = 300$ MPa in secant formula.

- Solve for P which becomes P_y.

$$\sigma_y = \frac{P_y}{A}\left[1 + \frac{e\overset{y_c}{c}}{r^2}\sec\left(\frac{L}{2r}\sqrt{\frac{P_y}{AE}}\right)\right]$$

$$300\ \text{MPa} = \frac{P_y}{124.4\ \text{cm}^2}\left[1 + 0.336\cdot\sec\left(\frac{55.23}{2}\sqrt{\frac{P_y}{(210\ \text{GPa})(124.4\ \text{cm}^2)}}\right)\right]$$

or $$3732\ \text{kN} = P_y\left[1 + 0.336\cdot\sec\left(5.403\times10^{-4}\sqrt{P_y}\right)\right]$$

Solve in term of P_y $\quad P_y = 2473\ \text{kN}$

This Load will yield Material (in Compression) at the cross section of Maximum bending Moment.

Factor of Safety

$$n = \frac{P_y}{P} = \frac{2473 \text{ kN}}{2000 \text{ kN}} = \underline{1.236}$$