UNIT – IV

BEAM DEFLECTION

PART – A

1) Write the equation giving maximum deflection in case of a simply supported beam subjected to a point load at mid span (Apr/May 2018)

12.4. DEFLECTION OF A SIMPLY SUPPORTED BEAM CARRYING A POINT LOAD AT THE CENTRE

A simply supported beam *AB* of length *L* and carrying a point load *W* at the centre is shown in Fig. 12.3.

As the load is symmetrically applied the reactions R_A and R_B will be equal. Also the maximum deflection will be at the centre.

[redacted]

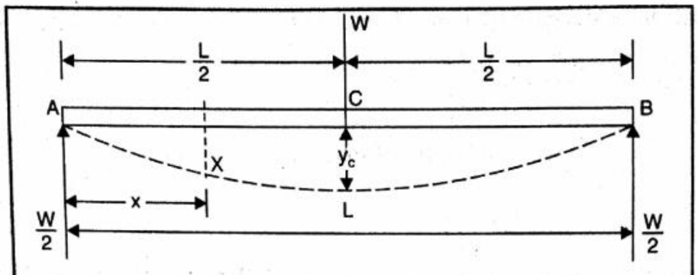


Fig. 12.3

Now $$R_A = R_B = \frac{W}{2}$$

Consider a section X at a distance x from A. The bending moment at this section is given by,

$$M_x = R_A \times x$$
$$= \frac{W}{2} \times x$$

(Plus sign is as B.M. for left portion at X is clockwise)

But B.M. at any section is also given by equation (12.3) as

$$M = EI \frac{d^2y}{dx^2}$$

Equating the two values of B.M., we get

$$EI \frac{d^2y}{dx^2} = \frac{W}{2} \times x \qquad ...(i)$$

On integration, we get

$$EI \frac{dy}{dx} = \frac{W}{2} \times \frac{x^2}{2} + C_1 \qquad ...(ii)$$

where C_1 is the constant of integration. And its value is obtained from boundary conditions. The boundary condition is that at $x = \frac{L}{2}$, slope $\left(\frac{dy}{dx}\right) = 0$ (As the maximum deflection is at the centre, hence slope at the centre will be zero). Substituting this boundary condition in equation (*ii*), we get

$$0 = \frac{W}{4} \times \left(\frac{L}{2}\right)^2 + C_1$$

or $$C_1 = -\frac{WL^2}{16}$$

Substituting the value of C_1 in equation (*ii*), we get

$$EI \frac{dy}{dx} = \frac{Wx^2}{4} - \frac{WL^2}{16} \qquad ...(iii)$$

The above equation is known the *slope equation*. We can find the slope at any point on the beam by substituting the values of x. Slope is maximum at A. At A, $x = 0$ and hence slope at A will be obtained by substituting $x = 0$ in equation (*iii*).

$$\therefore \quad EI\left(\frac{dy}{dx}\right)_{\text{at } A} = \frac{W}{4}\times 0 - \frac{WL^2}{16}$$

$$\left[\left(\frac{dy}{dx}\right)_{\text{at } A} \text{ is the slope at } A \text{ and is represented by } \theta_A\right]$$

or $$EI \times \theta_A = -\frac{WL^2}{16}$$

$$\therefore \quad \theta_A = -\frac{WL^2}{16EI}$$

The slope at point B will be equal to θ_A, since the load is symmetrically applied.

$$\therefore \quad \theta_B = \theta_A = -\frac{WL^2}{16EI} \qquad ...(12.6)$$

Equation (12.6) gives the slope in radians.

Deflection at any point

Deflection at any point is obtained by integrating the slope equation (*iii*). Hence integrating equation (*iii*), we get

$$EI \times y = \frac{W}{4}\cdot\frac{x^3}{3} - \frac{WL^2}{16}x + C_2 \qquad ...(iv)$$

where C_2 is another constant of integration. At A, $x = 0$ and the deflection (y) is zero.

Hence substituting these values in equation (*iv*), we get

$$EI \times 0 = 0 - 0 + C_2$$

or $$C_2 = 0$$

Substituting the value of C_2 in equation (*iv*), we get

$$EI \times y = \frac{Wx^3}{12} - \frac{WL^2 . x}{16} \qquad ...(v)$$

The above equation is known as *the deflection equation.* We can find the deflection at any point on the beam by substituting the values of x. The deflection is maximum at centre point C, where $x = \frac{L}{2}$. Let y_c represents the deflection at C. Then substituting $x = \frac{L}{2}$ and $y = y_c$ in equation (*v*), we get

$$EI \times y_c = \frac{W}{12}\left(\frac{L}{2}\right)^3 - \frac{WL^2}{16}\times\left(\frac{L}{2}\right)$$

$$= \frac{WL^3}{96} - \frac{WL^3}{32} = \frac{WL^3 - 3WL^3}{96}$$

$$= -\frac{2WL^3}{96} = -\frac{WL^3}{48}$$

$$\therefore \quad y_c = -\frac{WL^3}{48EI}$$

(Negative sign shows that deflection is downwards)

$$\therefore \text{ Downward deflection, } y_c = \frac{WL^3}{48EI} \qquad ...(12.7)$$

2) State the two theorems of conjugate beam method (Apr/May 2018)

Conjugate Beam Theorem I :

"The slope at any section of a loaded beam relative to the original axis of the beam, is equal to the shear in the conjugate beam at the corresponding section."

We know that, load $= w = \dfrac{M}{EI}$

$\therefore$ Shear $= S_x = \int_0^x w \cdot dx = \int_0^x \dfrac{M}{EI}\, dx$

But, $\int_0^x \dfrac{M}{EI}\, dx = \int_0^x \dfrac{d^2 y}{dx^2} = \dfrac{dy}{dx} = slope$

Conjugate Beam Theorem II :

"The deflection at any given section of a loaded beam, relative to the original position, is equal to the bending moment at the corresponding section of the conjugate beam."

We know that, shear $S_x = \int_0^x \dfrac{M}{EI}\, dx$

$\therefore$ Bending moment, $M_x = \int_0^x S_x \cdot dx = \int_0^x\int_0^x \dfrac{M}{EI}\, dx$

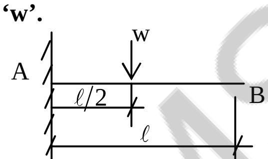
But, $\int_0^x\int_0^x \dfrac{M}{EI}\, dx = \int_0^x\int_0^x \dfrac{d^2 y}{dx^2} = \int_0^x \dfrac{dy}{dx} = y = deflection$ ...*Proved*

The following points are worth noting for the **conjugate beam method:**

(*i*) This method can be *directly used only for simply supported beams.*

(*ii*) In this method for cantilevers and fixed beams, artificial constraints need to be applied to the conjugate beam so that it is supported in a manner consistent with the constraints of the real beam.

3) Write down the equation for the maximum deflection of a cantilever beam carrying a central point load 'w'. (May / June 2017)



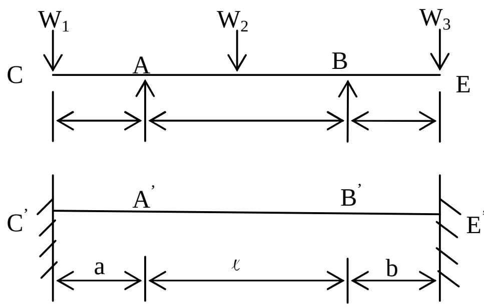
$$y_b = \frac{w_a^{\,3}}{3EI} + \frac{w_a^{\,2}}{2EI}(\ell - a)$$

$$= \frac{w}{3EI}(\ell/2)^3 + \frac{w}{2EI}(\ell/2)^2 \times (\ell/2)$$

$$= \frac{w\ell^3}{24EI} + \frac{w\ell^3}{16EI} = \frac{2w\ell^3 + 3w\ell^3}{48EI}$$

$$= \frac{5w\ell^3}{48EI}$$

4) Draw conjugate beam for a double side over hanging beam **(May / June 2017)**



5) List out the method's available to find the deflection of the beam. **(Nov / Dec 2015, 2016)**

The available methods to find the deflection of beam are

i) Double integration method
ii) Macaulay's method
iii) Moment Area method
iv) Conjugate beam method

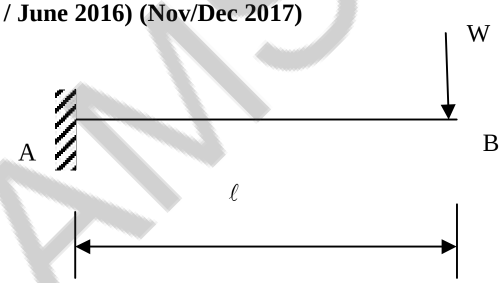
6) State Maxwell's reciprocal theorem (Nov / Dec 2016) (May / June 2016) (Nov / Dec 2017) (Nov/Dec 2018) (Apr/May 2019)

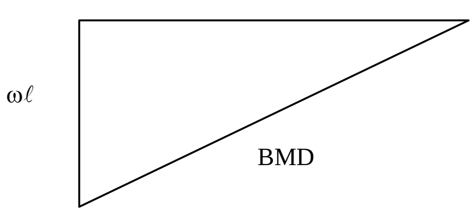
The Maxwell reciprocal theorem states that, "the work done by the first system of load due to displacement caused by a second system of load equal the work done by the second system of load due to displacement caused by the first system of load".

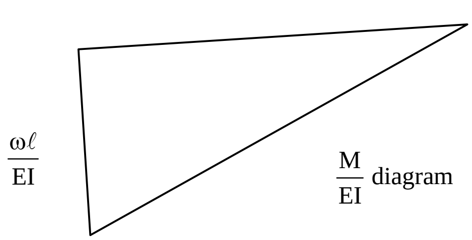
$$\sum_{i=1}^{n}(P_i)_A(\delta_i)_{\mathbf{B}} = \sum_{j=1}^{m}(P_j)_A(\delta_j)_{\mathbf{B}}$$

7) How the deflection & slope is calculated for the Cantilever beam by conjugate beam method?

(May / June 2016) (Nov/Dec 2017) **(Nov/Dec 2018)**







Total load on conjugate beam = Area of load diagram

$$P = A = \frac{1}{2} \times \ell \times \frac{\omega\ell}{EI} = \frac{-\omega\ell^2}{2EI}$$

We know that,

Slope at B = shear force at B for the conjugate beam

$$\boxed{\theta_B = -P = \frac{\omega\ell^2}{2EI}}$$

Deflection at B = B.M at B for the conjugate beam

$$= -p \times \frac{2}{3} \times \ell$$

$$= \left[\frac{\omega\ell^2}{2EI} \times \frac{2}{3} \times \ell\right]$$

$$y_B = \frac{\omega\ell^3}{3EI}$$

8) What is the equation used in the case of double integration method? (Nov / Dec 2015)

The B.M at any point is given by the differential equation

$$M = EI\frac{d^2y}{dx^2}$$

Integration the above equation, we get,

$$\int M = \int EI\frac{d^2y}{dx^2} = EI\frac{dy}{dx}$$

Integration above equation twice, we get , $\rightarrow$ slope equation

$$\iint M = \iint EI\frac{d^2y}{dx^2} = EIy$$

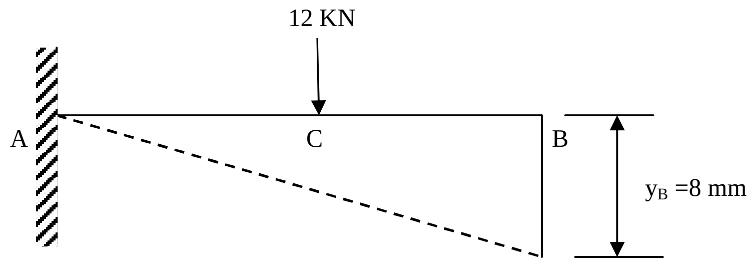
$$\rightarrow \text{Deflection equation}$$

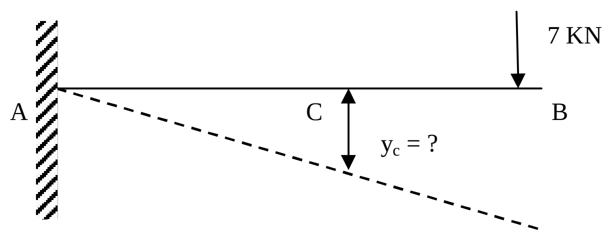
9) What are the advantages of Macaulay's over other method for the calculation of slope & deflection? (Apr / May 2015)

The procedure of finding slope and deflection for a SSB with an eccentric point load is very Laborious. There is a convenient method, that method was devised by Mr. M.H.Macaulay and is known as Macaulay's method.

In this method, B.M at any section is expressed and the integration is carried out.

10) In a cantilever beam, the measured deflection at, free end was 8 mm when a concentrated load of 12 KN was applied at it's mid span. What will be the deflection at mid – span when the same beam carries a concentrated load of 7KN at the free end? (Apr / May 2015)





Maxwell Reciprocal theorem,

$$\sum \frac{1}{2} P_i \delta_i = \sum \frac{1}{2} P_j \delta_j$$
$$12 \times 8 = 7 \times y_c$$
$$\frac{12 \times 8}{7} = y_c$$
$$y_c = 13.71 \text{ mm}$$

11) What is the limitation of double integration method? (Nov / Dec 2014)

* This method is used only for single load

* This method for finding slope & deflection is very laborious

12) Define strain energy? (Nov / Dec 2014)

When an elastic material is deformed due to application of external force, internal resistance is developed in the material of the body, Due to deformation, some work is done by the internal resistance developed in the body, which is stored in the form of energy. This energy is known as strain energy. It is expressed in Nm.

13) What is the relation between slope, deflection and radius of curvature of a beam?

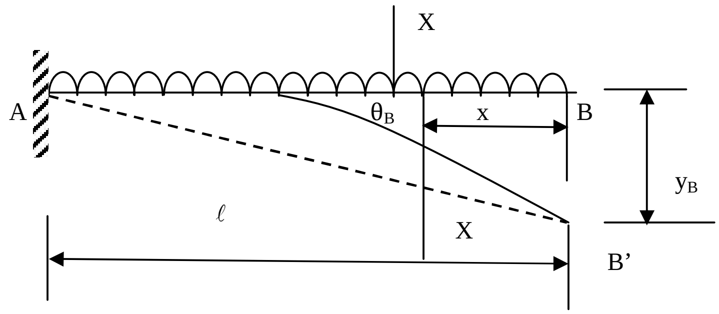
$$\frac{1}{R} = \frac{d^2y}{dx^2}$$

Where, R = radius of curvature

$\theta = dy/dx =$ slope

y = Deflection

14) State the expression for slope and deflection at the free and of a Cantilever beam of length 'l' subjected to a uniformly distributed load of 'w' per unit length.



Consider a section X at a distance x from the free end B,

$$\text{B.M at section XX} = M_{XX} = \frac{-\omega x^2}{2}$$

$$M = EI\frac{d^2y}{dx^2} = \frac{-\omega x^2}{2}$$

Integrate the above equation

$$EI\frac{dy}{dx} = -\frac{\omega x^3}{6} + C_1 \qquad ...1$$

Integration again,

$$EIy = \frac{-\omega x^4}{24} + C_1 x + C_2 \qquad ...2$$

C_1 & C_2 values are obtined from boundary condition.

i) when $x = \ell$; slope $\frac{dy}{dx} = 0$

ii) when $x = \ell$; deflection $y = 0$

Applying BC (i) to equation 1

$$0 = \frac{-\omega \ell^3}{6} + C_1$$

$$\boxed{C_1 = \frac{\omega \ell}{6}}$$

Substitute the C_1 values in equation 1

$$EI\frac{dy}{dx} = \frac{-\omega x^3}{6} + \frac{\omega \ell^3}{6} \qquad \rightarrow 3(\text{slope equ})$$

Max slope →substituting x=0 in equation 3

$$EI = \frac{dy}{dx} = \frac{\omega \ell^3}{6}$$

$$\text{max slope}, \theta_B = \frac{dy}{dx} = \frac{\omega \ell^3}{6EI}$$

Applying B.C(iii) to equation 2

$$0 = \frac{-\omega \ell^4}{6} - \frac{\omega \ell^4}{24} = \frac{-3\omega \ell^4}{24} = \frac{-\omega \ell^4}{8}$$

Substitute C_1 & C_2 values in equation 2

$$EIy = \frac{-\omega\ell^4}{24} + \frac{\omega\ell^4}{6}x - \frac{-\omega\ell^4}{8} \qquad \rightarrow 4$$

Max deflection occurs at the end, → substituting x=0 in equation 4

$$EIy_B = 0 - 0 - \frac{\omega\ell^4}{8}$$

$$y_B = \frac{-\omega\ell^4}{8EI}$$

Max deflection, $\boxed{y_B = \frac{-\omega\ell^4}{8EI}}$

15) In a support beam of 3m span carrying uniformly distribution load throughout the length the slope at the support is 1°. What is the max deflection in the beam? (Apr/May 2019)

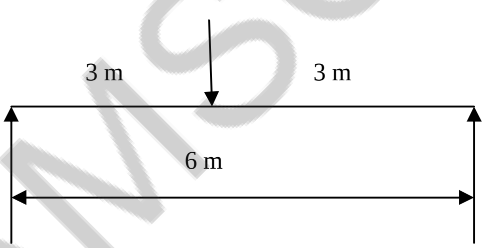
$$\theta_A = \frac{\omega\ell^3}{24EI} = 1° = \frac{\pi}{180°}$$

Max deflection (y_{max}) = $\frac{5}{384}\frac{\omega\ell^4}{EI}$

$$= \frac{\omega\ell^3}{24EI} \times \frac{5\ell}{16} = \frac{\pi}{180°} \times \frac{5\times 3}{16}$$

$$\boxed{y_{max} = 0.0164}$$

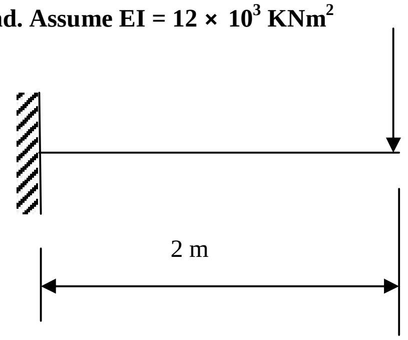
16) Calculate the maximum deflection of a simply support beam carrying a point load of 100 KN at mid span. Span = 6m; EI = 20,000 KN/m²



$$y_{max} = \frac{\omega\ell^3}{48EI} = \frac{100\times 6^3}{48\times 20000} = 0.0225\,m$$

$$\boxed{y_{max} = 22.5 \text{ mm}}$$

17) A cantilever beam of span 2m is carrying a point load of 20 KN in the free end. Calculate the slope at the free end. Assume EI = 12 × 10³ KNm²



$$\theta_B = \frac{\omega\ell^2}{2EI}$$

$$= \frac{20\times 2^2}{2\times 12\times 10^3}$$

$$\boxed{\theta_B = 0.0033 \text{ rad}}$$

18) State the two theorems in the moment area method.

Mohr's theorem 1:

The change of slope between any two point is equal to the net area of the BM diagram between these points divided by EI.

Mohr's theorem 2:

The total deflection between any two point is equal to the moment of the area of the BM diagram between these two point about the last point divided by EI.

19) Define Resilience and proof resilience?

Resilience is ability of a material to absorb energy under elastic deformation and to recover this energy upon removal of load. Resilience is indicated by the area under the stress strain curving to the point of elastic limit. In a technical sense, resilience is the property of a material that allow it return to its original shape after being de formed.

Proof resilience is defined as the maximum energy that can be absorbed within the elastic limit without creating a permanent distortion.

20) Define the term modulus of resilience.

It is the ratio of the proof resilience to the volume of the body.

21) Why moment area method is more useful when compared with double integration?

Moment area method is more useful, as compared to double integration method because many problem which do not have a simple mathematical solution can be simplified by the ending moment area method.

22) Explain the theorem for conjugate beam method?

Theorem I: The slope at any section of a loaded beam, relative to the original axis of the beam is equal to the shear in the conjugated beam at the corresponding section.

Theorem II: the deflection at any given section of a loaded beam, relative to the original position is equal to the bending moment at the corresponding section of the conjugated beam.

23) Define method of singularity function?

In Macaulay's method a single equation is formed for all loading on a beam, the equation is constructed in such a way that the constant of integration apply to all portion of the beam. This method is also called of singularity function.

24) What are the point to be worth for conjugate beam method.

1) This method can be directly used for simply support beam

2) In this method for cantilever and fixed beam, artificial constraints need to be supplied to the conjugate beam so that it is support in a manner consistent with the constraints of the real beam.

PART – B

1) A beam of length 5m and of uniform rectangular section is simply supported at its end. It carries a uniformly distributed load of 9kN/m run over the entire length. Calculate the width and depth of the beam if permissible bending stress is 7 N/mm^2 and central deflection is not to exceed 1cm. Take E=1x10^4 N/mm^2 (Apr/May 2019) (Nov/Dec 2018)

$$L = 5m = 5000mm$$
$$w = 9kN/m$$
$$W = wL = 9x5 = 45kN = 45000N$$
$$\pm_b = 7N/mm^2$$
$$y_C = 1cm = 10mm$$
$$E = 1x10^4 N/mm^2$$
$$I = \frac{bd^3}{12}$$
$$y_C = \frac{5}{384}\frac{WL^3}{EI}$$
$$10 = \frac{5}{384}\frac{45000x5000^3x12}{1x10^4 xbd^3}$$
$$bd^3 = 878.906x10^7 mm^4 \rightarrow 1$$
$$M = \frac{wl^2}{8} = \frac{Wl}{8} = \frac{45000x5000}{8} = 28125000Nmm$$

Bending equation

$$\frac{M}{I} = \frac{\pm_b}{y}$$
$$\frac{28125000}{\frac{bd^3}{12}} = \frac{7}{\frac{d}{2}} \Rightarrow bd^2 = 24107142.85mm^3 \rightarrow 2$$

divide equation 1 by equation 2, we get,

d=364.58mm subs. in equation 2, we get

b=181.36mm

2) A simply supported beam of length 5m carries a point load of 5kN at a distance of 3m from the left end. If E=2x10^5 N/mm^2 and I=10^8 mm^4 determine the slope at the left support and deflection under the point load using conjugate beam method. (Apr/May 2019) (Nov/Dec 2017)

Sol. Given :

Length, $L = 5$ m

Point load, $W = 5$ kN

Distance AC, $a = 3$ m

Distance BC, $b = 5 - 3 = 2$ m

Value of $E = 2 \times 10^5$ N/mm^2 $= 2 \times 10^5 \times 10^6$ N/m^2

$= 2 \times 10^5 \times 10^3$ kN/m^2 $= 2 \times 10^8$ kN/m^2

Value of $I = 1 \times 10^8$ mm^4 $= 10^{-4}$ m^4

Let R_A = Reaction at A

and R_B = Reaction at B.

Taking moments about A, we get

$$R_B \times 5 = 5 \times 3$$

$$\therefore \quad R_B = \frac{5 \times 3}{5} = 3 \text{ kN}$$

and R_A = Total load $- R_B = 5 - 3 = 2$ kN

The B.M. at $A = 0$

B.M. at $B = 0$

B.M. at $C = R_A \times 3 = 2 \times 3 = 6$ kNm.

Now B.M. diagram is drawn as shown in Fig. 14.3 (*b*).

Now construct the conjugate beam as shown in Fig. 14.3 (*c*). The vertical load at C^* on conjugate beam

$$= \frac{\text{B.M. at } C}{EI} = \frac{6 \text{ kNm}}{EI}$$

Now calculate the reaction at A^* and B^* for conjugate beam

Let $R_A{}^*$ = Reaction at A^* for conjugate beam

$R_B{}^*$ = Reaction at B^* for conjugate beam.

Taking moments about A^*, we get

$R_B{}^* \times 5$ = Load on $A^*C^*D^*$ × distance of C.G. of $A^*C^*D^*$ from A^* + Load on $B^*C^*D^*$ × Distance of C.G. of $B^*C^*D^*$ from A^*

$$= \left(\frac{1}{2} \times 3 \times \frac{6}{EI}\right) \times \left(\frac{2}{3} \times 3\right) + \left(\frac{1}{2} \times 2 \times \frac{6}{EI}\right) \times \left(3 + \frac{1}{3} \times 2\right)$$

$$= \frac{18}{EI} + \frac{6}{EI} \times \frac{11}{3} = \frac{8}{EI} + \frac{22}{EI} = \frac{40}{EI}$$

$$\therefore \quad R_B{}^* = \frac{40}{EI} \times \frac{1}{5} = \frac{8}{EI}$$

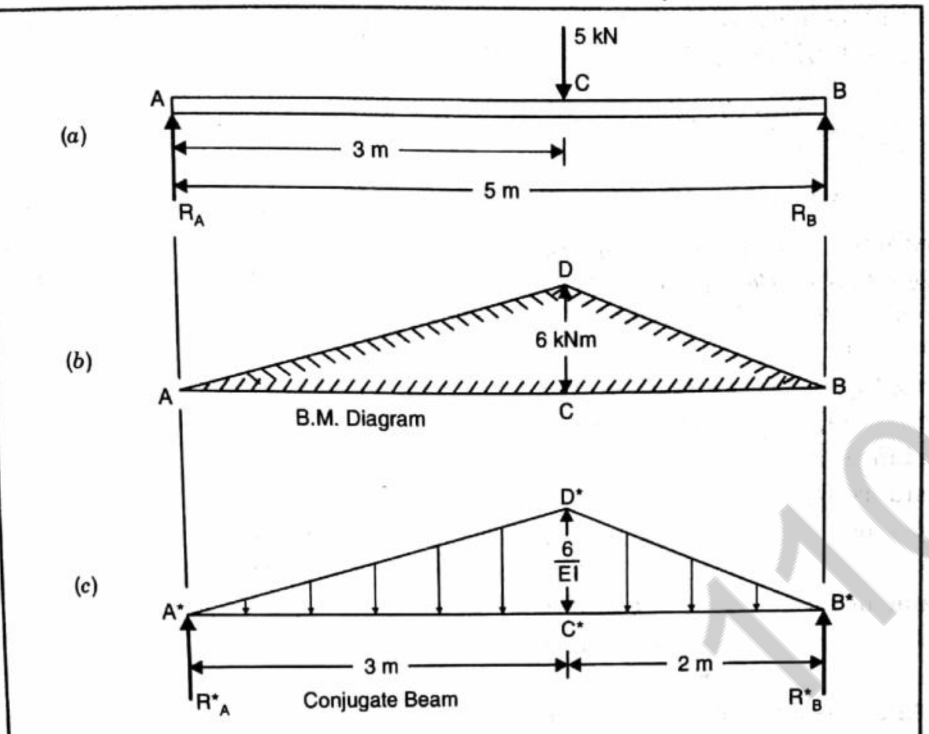


Fig. 14.3

$\therefore$

$$R_A^* = \text{Total load } (i.e., \text{ load } A^*B^*D^*) - R_B^*$$

$$= \left(\frac{1}{2} \times 5 \times \frac{6}{EI}\right) - \frac{8}{EI}$$

$$= \frac{15}{EI} - \frac{8}{EI} = \frac{7}{EI}$$

Let θ_A = Slope at A for the given beam *i.e.,* $\left(\frac{dy}{dx}\right)$ at A

y_C = Deflection at C for the given beam

Then according to conjugate beam method,

$$\theta_A = \text{Shear force at } A^* \text{ for conjugate beam} = R_A^*$$

$$= \frac{7}{EI} = \frac{7}{2 \times 10^8 \times 10^{-4}} \quad (\because E = 2 \times 10^8 \text{ kN/m}^2 \text{ and } I = 10^{-4} \text{ m}^4)$$

= **0.00035 radians. Ans.**

$$y_C = \text{B.M. at } C^* \text{ for conjugate beam}$$

$$= R_A^* \times 3 - \text{Load } A^*C^*D^* \times \text{Distance of C.G. of } A^*C^*D^* \text{ from } C^*$$

$$= \frac{7}{EI} \times 3 - \left(\frac{1}{2} \times 3 \times \frac{6}{EI}\right) \times \left(\frac{1}{3} \times 3\right)$$

$$= \frac{21}{EI} - \frac{9}{EI} - \frac{12}{EI}$$

$$= \frac{12}{2 \times 10^8 \times 10^{-4}} = \frac{6}{10^4} \text{ m} = \frac{6 \times 1000}{10000} \text{ mm} = \mathbf{0.6 \text{ mm.}} \quad \textbf{Ans.}$$

3) Derive the equation for slope and deflection of a simply supported beam of length 'L' carrying point load 'W' at the centre by Mohr's theorem. (Nov/Dec 2018)

Fig. 12.20 (*a*) shows a simply supported beam *AB* of length *L* and carrying a uniformly distributed load of *w*/unit length over the entire span. The B.M. diagram is shown in Fig. 12.20 (*b*). This is a case of symmetrical loading, hence slope is zero at the centre *i.e.*, at point *C*.

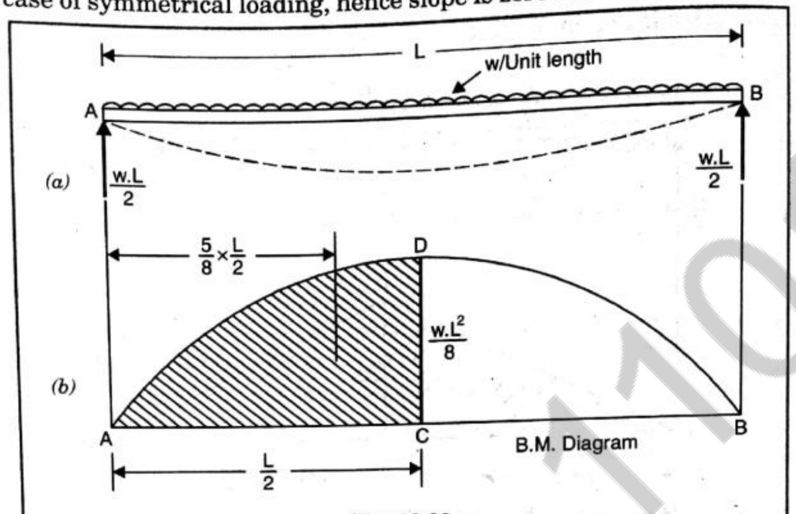


Fig. 12.20

(*i*) Now using Mohr's theorem for slope, we get

Slope at $A = \dfrac{\text{Area of B.M. diagram between } A \text{ and } C}{EI}$

But area of B.M. diagram between *A* and *C*

$$= \text{Area of parabola } ACD$$

$$= \frac{2}{3} \times AC \times CD$$

$$= \frac{2}{3} \times \frac{L}{2} \times \frac{wL^2}{8} = \frac{w \cdot L^3}{24}$$

$\therefore$ Slope at $A = \dfrac{w \cdot L^3}{24EI}$

(*ii*) Now using Mohr's theorem for deflection, we get from equation (12.17) as

$$y = \frac{A\bar{x}}{EI}$$

where A = Area of B.M. diagram between A and C

$$= \frac{w \cdot L^3}{24}$$

and $\bar{x}$ = Distance of C.G. of area A from A

$$= \frac{5}{8} \times AC = \frac{5}{8} \times \frac{L}{2} = \frac{5L}{16}$$

$$\therefore \quad y = \frac{\dfrac{w.L^3}{24} \times \dfrac{5L}{16}}{EI} = \frac{5}{384} \frac{w.L^4}{EI}.$$

4. A cantilever of length 2m carries a uniformly distributed load of 2.5kN/m run for a length of 1.25m from the fixed end and a point load of 1kN at the free end. Find the deflection at the free end, if the section is rectangular 12cm wide and 24cm deep and E=1x10^4 N/mm^2 (Apr/May 2018)

Length, $L = 2 \text{ m} = 2000 \text{ mm}$

U.d.l., $w = 2.5 \text{ kN/m} = 2.5 \times 1000 \text{ N/m}$

$$= \frac{2.5 \times 1000}{1000} \text{ N/mm} = 2.5 \text{ N/mm}$$

Point load at free end, $W = 1 \text{ kN} = 1000 \text{ N}$

Distance AC, $a = 1.25 \text{ m} = 1250 \text{ mm}$

Width, $b = 12 \text{ cm}$

Depth, $d = 24 \text{ cm}$

Value of $I = \frac{bd^3}{12} = \frac{12 \times 24^3}{12}$

$= 13824 \text{ cm}^4 = 13824 \times 10^4 \text{ mm}^4 = 1.3824 \times 10^8 \text{ mm}^4$

Value of $E = 1 \times 10^4 \text{ N/mm}^2$

Let y_1 = Deflection at the free end due to point load 1 kN alone

y_2 = Deflection at the free end due to u.d.l. on length AC.

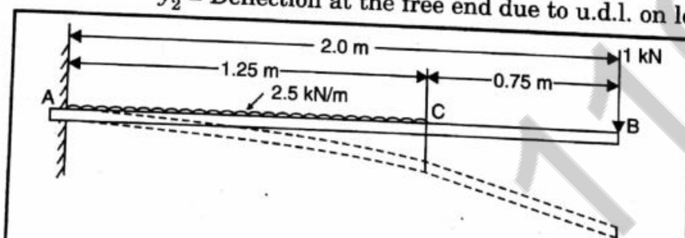


Fig. 13.7

(*i*) Now the downward deflection at the free end due to point load of 1 kN (or 1000 N) at the free end is given by equation (13.2 A) as

$$y_1 = \frac{WL^3}{3EI} = \frac{1000 \times 2000^3}{3 \times 10^4 \times 1.3824 \times 10^8} = 1.929 \text{ mm.}$$

(*ii*) The downward deflection at the free end due to uniformly distributed load of 2.5 N/mm on a length of 1.25 m (or 1250 mm) is given by equation (13.8) as

[redacted]

$$y_2 = \frac{wa^4}{8EI} + \frac{w \cdot a^3}{6EI}(L - a)$$

$$= \frac{2.5 \times 1250^4}{8 \times 10^4 \times 1.3824 \times 10^8} + \frac{2.5 \times 1250^3}{6 \times 10^4 \times 1.3824 \times 10^8}(2000 - 1250)$$

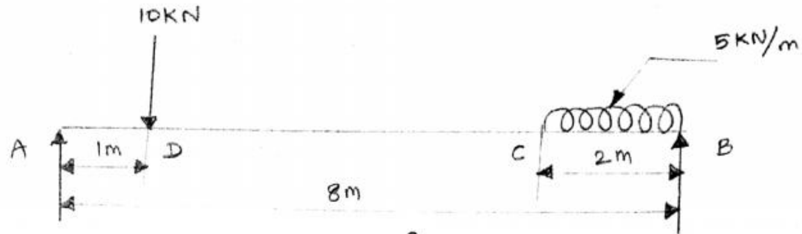
$$= 0.5519 + 0.4415 = 0.9934$$

∴ Total deflection at the free end due to point load and u.d.l.

$= y_1 + y_2 = 1.929 + 0.9934 =$ **2.9224 mm. Ans.**

5) A beam AB of 8m span is simply supported at the ends. It carries a point load of 10kN at a distance of 1m from the end A and a uniformly distributed load of 5kN/m for a length of 2m from the end B. If I= 10 x 10^{-6}m^4, determine : i) Deflection at the mid span ii) Maximum deflection iii) slope at the end A (Apr/May 2018)

14) b) Apr - 2018



$I = 10 \times 10^{-6}\ m^4 \qquad E = 200 \times 10^9\ N/m^2$

To find, R_A & R_B, $\quad R_A + R_B = 10 + 5 \times 2 = 20KN \rightarrow (1)$

$$\Sigma M_A = 0 \Rightarrow \quad -10 \times 1 - (5 \times 2) \times \left(6 + \tfrac{2}{2}\right) + 8R_B = 0$$

$$-10 - 70 + 8R_B = 0$$

$$8R_B = 80$$

$$\boxed{R_B = 10KN} \text{ subs. in (1),}$$

we get $\boxed{R_A = 10KN}$

$$M_x = EI\frac{d^2y}{dx^2} = 10x \,\Big|\, -10(x-1) \,\Big|\, -5(x-6) \times \frac{(x-6)}{2} \rightarrow (2)$$

Integrating, we get,

$$EI\frac{dy}{dx} = \frac{10x^2}{2} + C_1 \,\Big|\, -\frac{10(x-1)^2}{2} \,\Big|\, -\frac{5}{2}\frac{(x-6)^3}{3}$$

$$= 5x^2 + C_1 \,\Big|\, -5(x-1)^2 \,\Big|\, -\frac{5}{6}(x-6)^3 \rightarrow (3)$$

Integrating again, we get,

$$EIy = \frac{5x^3}{3} + C_1x + C_2 \,\Big|\, -\frac{5(x-1)^3}{3} \,\Big|\, -\frac{5}{6}\frac{(x-6)^4}{4}$$

$$= \frac{5x^3}{3} + C_1x + C_2 \,\Big|\, -\frac{5}{3}(x-1)^3 \,\Big|\, -\frac{5}{24}(x-6)^4 \rightarrow (4)$$

when $x = 0;\ y = 0$ subs. in (4) $\Rightarrow \boxed{C_2 = 0}$

$x = 8;\ y = 0$ subs. in (4)

$$0 = \frac{5(8)^3}{3} + 8C_1 - \frac{5}{3}(7)^3 - \frac{5}{24}(2)^4$$

$$0 = 853.33 + 8C_1 - 571.67 - 3.33$$

$$-278.33 = 8C_1$$

$$\boxed{C_1 = -34.79}$$

Hence slope & Deflection eqns. are,

$$EI\frac{dy}{dx} = 5x^2 - 34.79 - 5(x-1)^2 - \frac{5}{6}(x-6)^3$$

→ slope eqn.

$$EIy = \frac{5x^3}{3} - 34.79x - 5/3\,(x-1)^3 - 5/24\,(x-6)^4$$

→ Deflection eqn.

(i) Deflection at mid span:

$$EIy_{mid} \underset{(x=4m)}{=} 5/3\,(4)^3 - 34.79(4) - 5/3\,(3)^3 = -77.49$$

$$y_{mid} = \frac{-77.49}{EI} = \frac{-77.49}{200\times10^9 \times 10\times10^{-6}} = 3.87\times10^{-5}\ m$$

$$\boxed{y_{mid} = 0.0387\ mm}$$

(ii) Max. Deflection:

For max. defle., equate slope at the section to zero,

we get, $EI\frac{dy}{dx} = 5x^2 - 34.79 - 5(x-1)^2 = 0$

$$5x^2 - 34.79 - 5(x^2 - 2x + 1) = 0$$

$$5x^2 - 34.79 - 5x^2 + 10x - 5 = 0$$

$$10x = +39.79$$

$$\boxed{x = +3.979\ m}$$

$$EIy_{max} = 5/3\,(3.979)^3 - 34.79(3.979) - 5/3\,(2.979)^3$$

$$EIy_{max} = -77.49$$

$$\boxed{y_{max} = 0.0387\ mm}$$

(iii) slope at end A,

putting $x=0$ in slope eqn., we get

$$EI\frac{dy}{dx} = -34.79$$

$$\theta_A = \frac{-34.79}{EI} = -1.74\times10^{-5}$$

$$\boxed{\theta_A = -0.00099^\circ}$$

6) A cantilever of length 3m is carrying a point load of 50kN at a distance of 2m from the fixed end. If E=2x10^5 N/mm^2 and I=10^8 mm^4 find i) slope at the free end and ii) deflection at the free end. (Nov/Dec 2017)

$$L = 3m = 3000mm$$
$$W = 50kN = 50000N$$
$$a = 2m = 2000mm$$
$$I = 10^8 mm^4$$
$$E = 2x10^5 N/mm^2$$
$$i) slope$$
$$-_B = \frac{Wa^2}{2EI} = \frac{50000x2000^2}{2x2x10^5x10^8} = 0.005rad$$
$$ii) Deflection$$
$$y_B = \frac{Wa^3}{3EI} + \frac{Wa^2}{2EI}(L-a)$$
$$y_B = \frac{50000x2000^3}{3x2x10^5x10^8} + \frac{50000x2000^2}{2x2x10^5x10^8}(3000-2000)$$
$$y_B = 11.67mm$$

7) Determine the slope at the two supports and deflection under the loads. Use conjugate beam method E = 200 GN/m^2, I for right half is 2 x 10^{8}mm^4, I for left half is 1 x 10^8 mm^4 the beam is given in fig. Q.14 (b). (May / June 2017)

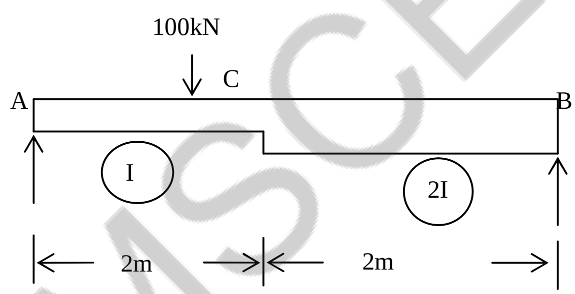


Fig. Q .14 (b)

Solution.

Given:

Length, L = 4m

Length AC = Length BC = 2m

Point load, W = 100kN

Moment of inertia for AC

$$I = 1 \times 10^8 \text{ mm}^4 = \frac{10^8}{10^{12}} \text{m}^4 = 10^{-4} \text{m}^4$$

Moment of inertia for BC

$= 2 \times 10^8 mm^4$

$= 2 \times 10^{-4} m^4 = 2I$

Value of $E = 200\ GN/m^2 = 200 \times 10^9\ N/m^2$

$= 200 \times 10^6\ kN/m^2$.

The reactions at A and B will be equal, as point load is acting at the centre,

$$\therefore \quad R_A = R_B = \frac{100}{2} = 50kN$$

Now B.M. at A and B are zero.

B.M. at C = R_A x 2 = 50 x 2 = 100 kNm

Now B.M. can be drawn as shown in Fig.14 (b)

Now we can construct the conjugate beam by dividing B.M. at any section by the product of E and M.O.I.

The conjugate beam is shown in Fig.14 (c). The loading are shown on the conjugate beam. The loading on the length A^*C^* will be $A^*C^*D^*$ whereas the loading on length B^*C^* will be $B^*C^*E^*$.

$$\text{The ordinate } C^*D^* = \frac{\text{B.M. at C}}{E \times \text{M.O.I for AC}} = \frac{100}{EI}$$

$$\text{The ordinate } C^*E^* = \frac{\text{B.M. at C}}{\text{product of E and M.O.I for BC}} = \frac{100}{E \times 2I} = \frac{50}{EI}$$

Let R_A^* = Reaction at A^* for conjugate beam

R_B^* = Reaction at B^* for conjugate beam

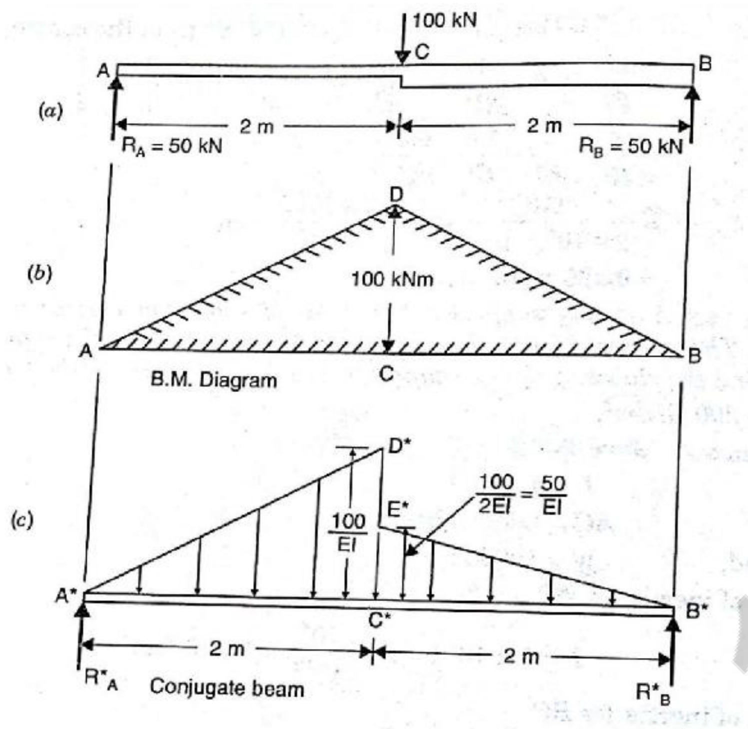


Fig.14

First calculate R_A^* and R_B^*

Taking moments of all forces about A^*, we get

R_B^* x 4 = Load $A^*C^*D^*$ x Distance of C.G. of $A^*C^*D^*$ from A +

Load $B^*C^*E^*$ x Distance of C.G. of $B^*C^*E^*$ from A^*

$$= \left(\frac{1}{2}\times 2\times\frac{100}{EI}\right)\times\left(\frac{2}{3}\times 2\right)+\left(\frac{1}{2}\times 2\times\frac{50}{EI}\right)\times\left(2\times\frac{1}{3}\times 2\right)$$

$$= \frac{400}{3EI}+\frac{400}{3EI}=\frac{800}{3EI}$$

$$R_B^* = \frac{200}{3EI}$$

R_A^* = Total load on conjugate beam - R_B^*

$$= \left(\frac{1}{2}\times 2\times\frac{100}{EI}+\frac{1}{2}\times 2\times\frac{50}{EI}\right)-\frac{200}{3EI}$$

$$= \frac{150}{EI}-\frac{200}{3EI}=\frac{250}{3EI}$$

i) Slopes at the supports

Let θ_A = Slope at A i.e., $\left(\frac{dy}{dx}\right)$ at A for the given beam

$$\Theta_B = \text{Slope at B i.e., } \left(\frac{dy}{dx}\right) \text{ at B for the given beam}$$

Then according to the conjugate beam method,

Θ_A = shear force at A^* for conjugate beam = R_A^*

$$= \frac{250}{3EI}$$

$$= \frac{250}{3 \times 200 \times 10^6 \times 10^4} = 0.004166\text{rad.} \quad \text{Ans.}$$

Θ_B = shear force at B^* for conjugate beam = R_B^*

$$= \frac{200}{3EI}$$

$$= \frac{200}{3 \times 200 \times 10^6 \times 10^{-4}} = 0.003333\text{rad.} \quad \text{Ans.}$$

(iii) Deflection under the load

Let y_c = Deflection at C for the given beam.

Then according to the conjugate beam method,

Y_c =B.M. at point C^* of the conjugate beam

= R_A^* x 2 –(Load $A^*C^*D^*$) x Distance of C.G. of $A^*C^*D^*$ from C^*

$$\frac{250}{3EI} \times 2 - \left(\frac{1}{2} \times 2 \times \frac{100}{EI}\right) \times \left(\frac{1}{3} \times 2\right)$$

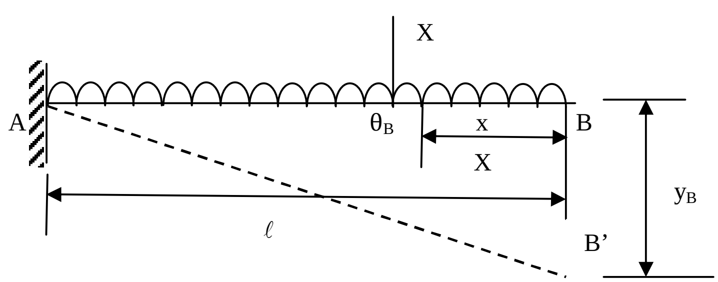
$$= \; = \frac{500}{3EI} - \frac{200}{3EI} = \frac{100}{EI}$$

$$= \frac{100}{200 \times 10^6 \times 10^{-4}} \text{m}$$

$$= \frac{1}{200} \text{m} = \frac{1}{200} \times 1000 = 5\text{mm.} \quad \text{Ans..}$$

8) Cantilever of length (l) carrying uniformly distributed load w KN per unit run over whole length. Derive the formula to find the slope and deflection at the free end by double integration method. Calculate the deflection if w = 20 KN/m, l = 2.30 m and EI = 12000 KNm^2 (13)

(Nov / Dec 2016)



Cantilever AB of length (l) fixed at and free at end B carrying a UDL of w per unit length over the whole span,

Consider section XX at a distance x from the free end B

B.M at section XX = $\omega x\ x/z = \frac{-\omega x^2}{2}$

$$M = EI\frac{d^2y}{dx^2} = \frac{-\omega x^2}{2}$$

Integration the above equation, $EI\frac{dy}{dx} = \frac{-\omega x^3}{6} + C_1$ ...1

Integration again, $EIy = \frac{-\omega x^4}{24} + C_1x + C_2$ ...2

C_1 & C_2 → values are obtained from the boundary condition,

i) When x = ℓ, slope $\frac{dy}{dx} = 0$

ii) When x = ℓ, slope $y = 0$

Applying Boundary condition i) in equation 1 we get,

$$0 = \frac{-\omega \ell^3}{6} + C_1$$

$C_1 = \frac{\omega \ell^3}{6}$ sub in equa 1 we get

slop equation $EI\frac{dy}{dx} = \frac{-\omega x^3}{6} + \frac{\omega \ell^3}{6}$ ...3

Max slop can be determine by substituting x = 0 in equ 3

$$EI\left(\frac{dy}{dx}\right)_{B\ \text{at}\ (x=0)} = \frac{\omega \ell^3}{6}$$

$$EI\ \theta_{B\ \text{at}\ (x=0)} = \frac{\omega \ell^3}{6}$$

$$\boxed{\theta_B = \frac{\omega \ell^3}{6EI}}$$

Apply ii) Boundary condition to equation 2,

$$0 = \frac{-\omega \ell^4}{24} + \frac{\omega \ell^3}{6}\ell + C_2$$

$$\boxed{C_2 = \frac{\omega \ell^4}{6} - \frac{\omega \ell^4}{24} = \frac{-3\omega \ell^4}{24} = \frac{-\omega \ell^4}{8}}$$ ...4

Sub, C_1 & C_2 value in equation 2 we get, Deflection equation $\boxed{EI\,y = \frac{-\omega z^4}{24} + \frac{\omega \ell^3}{6} x - \frac{\omega \ell^4}{8}}$ ...5

Max deflection occur at z = 0 in equation 5

$$EI\, y_{B\ (x=0)} = \frac{-\omega \ell^4}{8}$$

$$\boxed{y_B = \frac{-\omega \ell^4}{8EI}} \quad [\text{sign indicate downward deflection}]$$

$\omega = 20\ KN/m \quad \ell = 2.30\,m \quad EI = 12000\,KNm^2$

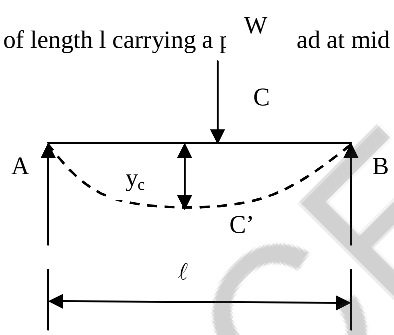
$$y_B = \frac{20 \times 10^3 \times 2.3^4}{8 \times 12000 \times 10^3} = 5.83 \times 10^{-3}\ m$$

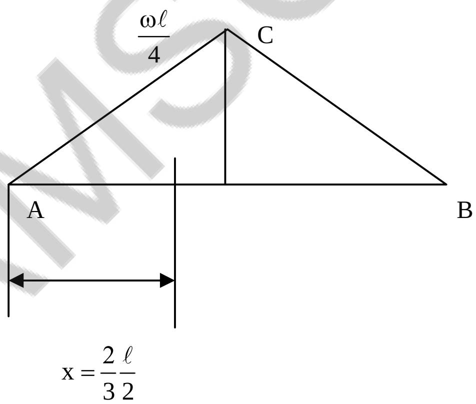
$$\boxed{y_B = 5.38\ mm}$$

9) Derive the formula to find the deflection of a simply supported beam with point load w at the centre by moment area method (8 mark)

(Nov / Dec 2016)

A SSB of length l carrying a p ad at mid – span,





Loading is symmetric the maximum deflection occurs at mid span C. The slope at C is zero. Slope at A & B is maximum,

$$\text{Slope at } A = \theta_a = \frac{\text{Area of BMD between A \& C}}{EI} = \frac{A}{EI}$$

$$A = \frac{1}{2} \times \frac{\ell}{2} \times \frac{\omega\ell}{4} = \frac{\omega\ell^2}{16}$$

$$\boxed{\theta_a = \frac{\omega\ell^2}{16EI}}$$

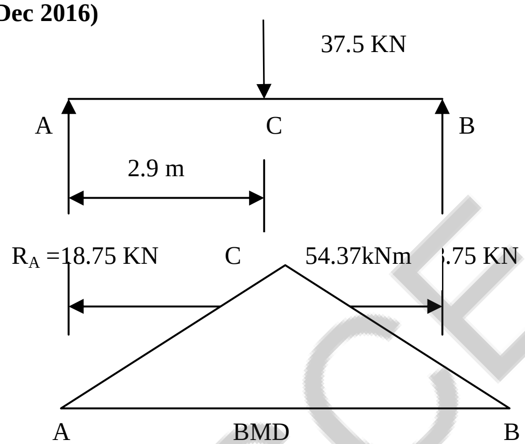
$$\bar{x} = \frac{2}{3}\frac{\ell}{2} = \frac{\ell}{3}$$

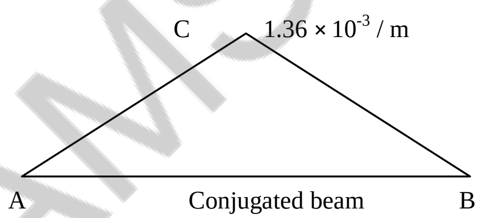
$$y_c = \frac{A\bar{x}}{EI} = \frac{\frac{\omega\ell^2}{16} \times \frac{\ell}{3}}{EI}$$

$$\boxed{y_c = \frac{\omega\ell^3}{48EI}}$$

10) A simply supplied beam of span 5.80 m carries a central point load of 37.5 KN, Find the max. slope and deflection, Let EI = 40000 KNm^2. Use conjugate beam method, (5)

(Nov /Dec 2016)





BMD:

$R_A \,\&\, R_B$ $\qquad R_A + R_B = 37.5 \text{ KN} \qquad ...1$

$\sum M_A = 0 \qquad 5.8R_B = 37.5 \times 2.9$

$\boxed{R_B = 18.75 \text{ KN}}$ substitute in 1, we get

$\boxed{R_A = 18.75 \text{ KN}}$

$$M_A = M_B = 0$$
$$M_c = 18.75 \times 2.9$$
$$= 54.375 \text{ KNm}$$
$$\frac{M}{EI} = \frac{54.375}{40000} = 1.36 \times 10^{-3} / m$$

Total load on conjugated beam =Area of M/EI diagram

$$P = \frac{1}{2} \times 5.8 \times 1.36 \times 10^{-3}$$
$$P = 3.94 \times 10^{-3}$$

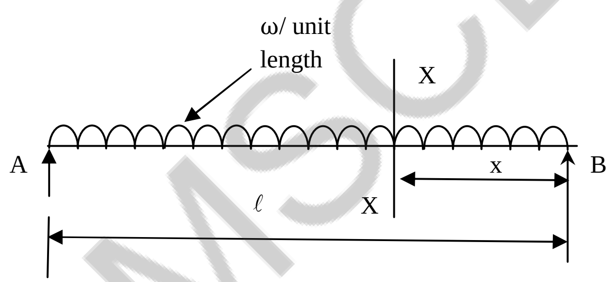
Reaction at each support for conjugate beam,

$$R_A = R_B = \frac{1}{2}P = 1.972 \times 10^{-3} \text{ radians}$$

Deflection at c = B.M at C for the conjugate beam,

$$= 1.972 \times 10^{-3} \times 2.9 - \frac{1}{2} \times 2.9 \times 1.36 \times 10^{-3} \times \frac{1}{3} \times 2.9$$
$$= 5.7188 \times 10^{-3} - 1.9062 \times 10^{-3}$$
$$y_c = 3.8125 \times 10^{-3} \text{m}$$
$$\boxed{y_c = 3.8125 \text{ mm}}$$

11) A SSB subjected to UDL of w KN/m for the entire span. Calculate the maximum deflection by double integration method (16 mark) (Apr / May 2016)



SSI $R_A = \frac{\omega\ell}{2}$ d carrying a UDL of ω per m length $R_B = \frac{\omega\ell}{2}$ e span

The reaction at A & B are, $R_A = R_B = \frac{\omega\ell}{2}$

Consider a section XX at a distance x from B

B.M at XX = $\frac{\omega\ell}{2}x - \omega x\frac{x}{2}$

$$M_x = \frac{\omega\ell}{2}x - \frac{\omega x^2}{2}$$
$$M_x = EI\frac{d^2y}{dx^2} = \frac{\omega\ell}{2}x - \frac{\omega x^2}{2} \qquad ...1$$

Integrating the above equation

$$EI\frac{dy}{dx} = \frac{\omega\ell}{4}x^2 - \frac{\omega x^3}{6} + C_1 \qquad ...2$$

Integration again,

$$EI\,y = \frac{\omega\ell x^3}{12} - \frac{\omega x^4}{24} + C_1 x + C_2 \qquad ...3$$

Varies of C_1 & C_2 → obtained by applying Boundary condtion,

i) when $x = \frac{\ell}{2} \Rightarrow \text{slop}\ \frac{dy}{dx} = 0$

ii) when $x = 0 \Rightarrow \text{deflection}\ y = 0$

Apply B.C i) to equation 2

$$0 = \frac{\omega\ell}{4}\left(\frac{\ell}{2}\right)^2 - \frac{\omega}{6}\left(\frac{\ell}{2}\right)^3 + C_1$$

$$0 = \frac{\omega\ell^3}{16} - \frac{\omega\ell^3}{48} + C_1$$

$$\boxed{C_1 = \frac{\omega\ell^3}{48} - \frac{\omega\ell^3}{16} = -\frac{\omega\ell^3}{24}} \quad \text{sub in Equ 2}$$

Slop equation $\boxed{EI\frac{dy}{dx} = \frac{\omega\ell}{4}x^2 - \frac{\omega x^3}{6} - \frac{\omega\ell^3}{24}}$ ...4

Max slop occur between A & B

Max slop substitute x=0 ; in equa 4

$$EI\frac{dy}{dx} = EI\theta_B = \frac{-\omega\ell^3}{24}$$

$$\boxed{\theta_B = \frac{-\omega\ell^3}{24EI}} \quad -\text{ve sign slop in neg direction}$$

$$\boxed{\theta_A = \theta_B = \frac{\omega\ell^3}{24EI}}$$

Applying boundary condition ii) ,in equation 3

$C_2 = 0$

Substitute C_1 & C_2 values ion equation 3, we get

$$EIy = \frac{\omega\ell x^3}{12} - \frac{\omega x^4}{24} - \frac{\omega\ell^3 x}{24} \qquad ...5$$

the deflection is minimum at mid point C.

To find max deflection x= $\frac{\ell}{2}$ sub in equa 5

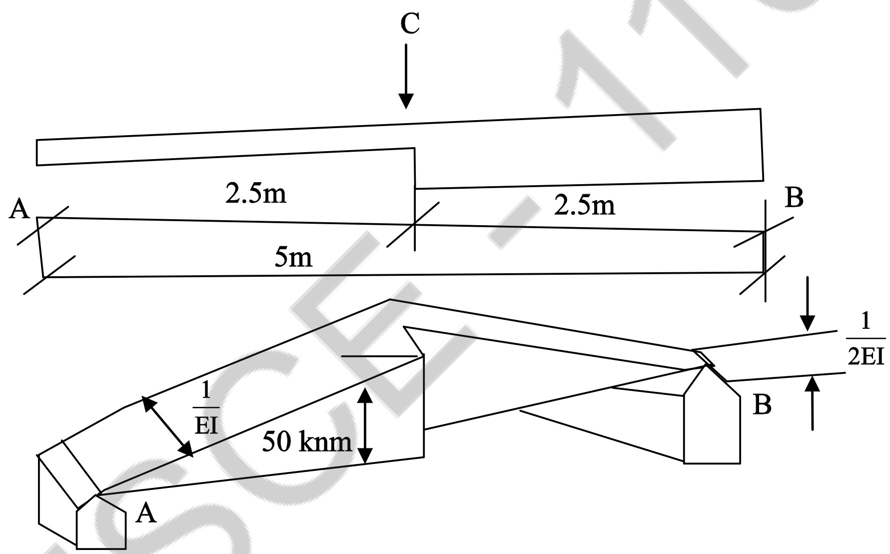
$$EIy_c = \frac{w\ell\left(\frac{\ell}{2}\right)^3}{12} - \frac{\omega}{24}\left(\frac{\ell}{2}\right)^4 - \frac{\omega\ell^3}{24}\left(\frac{\ell}{2}\right)$$

$$= \frac{\omega\ell^4}{96} - \frac{\omega\ell^4}{384} - \frac{\omega\ell^4}{384} = \frac{5\omega\ell^4}{384}$$

$$y_c = \frac{5w\ell^4}{384\ EI}$$

12) A SSB AB of span 5m carries a point of 40 KN at its centre. The values of moments of inertia for the left half is 2 × 10 ^{8} mm^4 and for the right half of portion is 4 × 10^8 mm^4. Find the slope at the two support and deflection under the load. Take E = 200 GN/m^2 (16 mark)

(Apr / May 2016)



Slope at two supports, $(BM)_{max} = \frac{\omega\ell}{4} = \frac{40\times 5}{4} = 50\,KNm$

Draw conjugate beam, Take M_A

$$R_B \times 5 = \frac{1}{EI}\left[\frac{1}{2}\times 50\times 2.5\times\frac{5}{3}\right] + \frac{1}{2EI}\left[\frac{1}{2}\times 50\times 2.5\right]\left[2.5+\frac{2.5}{3}\right]$$

$$= \frac{1}{3EI}[312.5] + \frac{1}{2EI}\left[\frac{1}{2}\times 50\times 2.5\times\frac{10}{3}\right]$$

$$5R_B = \frac{312.5}{3EI} + \frac{312.5}{3EI} = \frac{625}{3EI}$$

$$\boxed{R_B = \frac{125}{3EI}\ KN}$$

$$R_A + R_B = \frac{1}{EI}\left[\frac{1}{2}\times 50\times 2.5\right]+\frac{1}{2EI}\left[\frac{1}{2}\times 50\times 2.5\right]$$

$$= \frac{62.5}{EI}+\frac{62.5}{2EI}=\frac{187.5}{2EI}$$

$$R_A = \frac{187.5}{2EI}-\frac{125}{3EI}=\frac{562.5-250}{6EI}=\frac{312.5}{6EI}\ KN$$

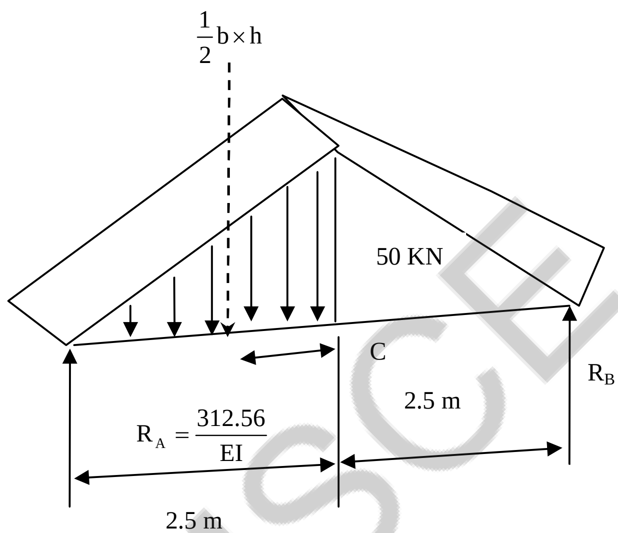
$$\text{shear force at A}, \theta_A = F_A = \frac{312.5}{6EI}=\frac{312.5\times 10^3}{6\times 200\times 10^9\times 2\times 10^{-4}}$$

$$\theta_A = 0.0013 \text{ rad}$$

$$\text{shear force at A}, \theta_B = F_B = \frac{125}{3EI}=\frac{125\times 10^3}{3\times 200\times 10^9\times 2\times 10^{-4}}$$

$$\theta_B = 0.00104 \text{ rad}$$

Deflection under load(y_c)



$$M_c = R_A\times 2.5 - \frac{1}{EI}\left[\frac{1}{2}\times 50\times 2.5\times\frac{2.5}{3}\right]$$

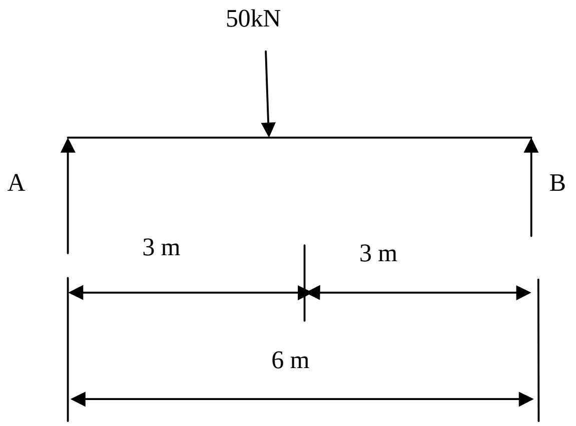
$$M_c = \frac{312.5}{6EI}\times 2.5-\frac{312.5}{6EI}=\frac{468.75}{6EI}$$

$$y_c = M_c = \frac{468.75}{6EI}=1.95\times 10^{-3}\,m$$

$$\boxed{y_c = 1.95 \text{ mm}}$$

13) A beam 6m long, simply supported at its end, is carrying a point load of 50 KN at its centre. The moments of inertia of the beam is given as equal to 78 × 10^6mm^4. If E for the material of the beam = 2.1 × 10 ^{5} N/mm^2, calculate i) deflection at the centre of the beam & ii) slope at the supports (16 mark)

(Nov / Dec 2015)



$\ell = 6\text{m} \quad \omega = 50\,\text{KN} \qquad I = 78\times10^{6}\,\text{mm}^{4} \quad E = 2.1\times10^{5}\ \text{N/mm}^{2}$

Double Integration method

ii) $\theta_A = \theta_B = \dfrac{\omega\ell^2}{16\,EI} = \dfrac{50\times10^3\times6000^2}{16\times2.1\times10^5\times78\times10^6} = 6.87\times10^{-3}$ radians

i) $y_c = \dfrac{\omega\ell^3}{48\,EI} = \dfrac{50\times10^3\times6000^3}{48\times2.1\times10^5\times78\times10^6}$

$y_c = 13.74$ mm

14) A beam of length 6m is simply supported at ends and carries two point loads of 48 kN and 40 kN at distance of 1m and 3m respectively from the left support as shown in fig.

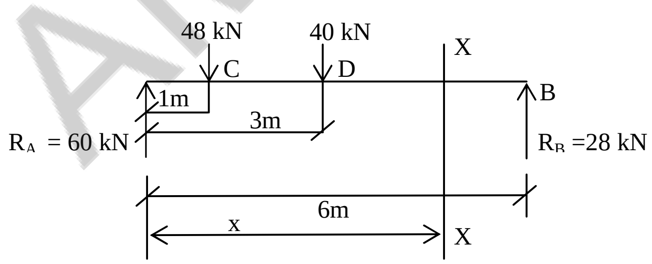
Using Macauley's method find

(i) deflection under each load

(ii) maximum deflection &

(iii) the point at which maximum deflection occurs,

Given, E = 2 x 10^5 N/mm^2 & I = 85 x 10^6 mm^4 (Nov/Dec 2015) (16)



R_A & R_B

$R_A + R_B = 88$ kN $\rightarrow(1)$

$\Sigma M_A = 0 \Longrightarrow -48\text{x}1 - 40\text{x}3 + 6R_B = 0$

$6R_B = 168$

$$\boxed{R_B = 28\,kN}$$

Substitute in eqn (1)

$$\boxed{R_A = 60\,kN}$$

Consider the selection X in the last part of the beam at a distance x from the left support A. The BM at this selection is given by,

$$EI\frac{d^2y}{dx^2} = R_A x \left| -48(x-1) \right| -40(x-3)$$
$$= 60x \left| -48(x-1) \right| -40(x-3)$$

Integrating the above equation, we get,

$$EI\frac{dy}{dx} = 60\frac{x^2}{2} + c_1 \left| -48\frac{(x-1)^2}{2} \right| -40\frac{(x-3)^2}{2}$$
$$= 30x^2 + c_1 \left| -24(x-1)^2 \right| -20(x-3)^2 \rightarrow (1)$$

Integrate the above equation, again,

$$EIy = 30\frac{x^3}{3} + c_1 x + c_2 \left| \frac{-24(x-1)^3}{3} \right| - \frac{20(x-3)^3}{3}$$
$$= 10x^3 + c_1 x + c_2 \left| -8(x-1)^3 \right| \frac{-20}{3}(x-3)^3 \rightarrow (2)$$

To find values of c_1 & c_2 use, two boundary condition,

(i) At x = 0; y = 0

(ii) At x = 6m ; y = 0

Substitute boundary condition (i) in equation (2) we get

x = 0 ; y = 0 $\Rightarrow \boxed{c_2 = 0}$

$\Downarrow$

lies first part of the beam so consider equation, upto first line

substitute boundary condition (ii) in equation (2) , we get,

x = 6m ; y = 0

$0 = 10 \times 6^3 + c_1 \times 6 + 0 - 8(6-1)^3 - 20/3\ (6-3)^3$

$0 = 2160 + 6c_1 - 8 \times 5^3 - 20/3 \times 3^3$

$0 = 2160 + 6c_1 - 1000 - 180 = 980 + 6c_1$

$$\boxed{c_1 = \frac{-980}{6} = -163.33}$$

Substitute c_1 & c_2 value in equation (2),

$$EIy = 10x^3 - 163.33x \left| -8(x-1)^3 \right| - \frac{20}{3}(x-3)^3 \qquad \rightarrow (3)$$

(1) Deflection under each load:

At point c,

Substitute x =1 in equation (3) upto first part of vertical line,

$EIy_c = 10 \times 1^3 - 163.33 \times 1$

$= -153.33 \text{ kNm}^3$

$EIy_c = -153.33 \times 10^{12} \text{ Nmm}^3$

$$y_c = \frac{-153.33 \times 10^{12}}{EI} = \frac{-153.33 \times 10^{12}}{2 \times 10^5 \times 85 \times 10^6}$$

$$\boxed{y_c = -9.016 \text{ mm}}$$

At point D,

Substitute x = 3 in eqn (3) upto second part of vertical line,

$EIy_D = 10 \times 3^3 - 163.33 \times 3 - 8(3-1)^3$

$= 270 - 489.99 - 64 = -283.99 \text{ kNm}^3$

$= -283.99 \times 10^{12} \text{ Nmm}^3$

$$y_D = \frac{-283.99 \times 10^{12}}{EI} = \frac{-283.33 \times 10^{12}}{2 \times 10^5 \times 85 \times 10^6}$$

$$\boxed{y_D = -16.7 \text{ mm}}$$

(2) Maximum Deflection;

Deflection is max between section C & D

For maximum deflection, dy/dx = 0 substituting in eqn (1)

Consider the eqn (1) upto second vertical line,

$30x^2 + c_1 - 24(x-1)^2 = 0$

$6x^2 + 48x - 187.33 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-48 \pm \sqrt{48^2 - 4 \times 6 \times 187.33}}{2 \times 6} = 2.87\text{m}$$

Substitute , x = 2.87m in eqn(3) ,upto second vertical line, we get,

$EIy_{max} = 10 \times 2.87^3 - 163.33 \times 2.87 - 8(2.87-1)^3$

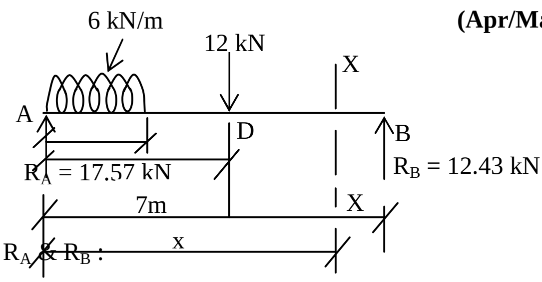
$= 284.67 \text{ kNm}^3 = -284.67 \times 10^{12} \text{ Nmm}^3$

$$y_{max} = \frac{-284.67\times10^{12}}{2\times10^5\times85\times10^6} = -16.745\text{mm}$$

$$\boxed{y_{max} = 16.745\text{mm}}$$

15) A horizontal beam of uniform section and 7m long is simply supported at it ends. The beam is subjected to a UDL of 6 KN/m over a length of 3m from the left end and a concentrated load of 12 KN at 5m from the left end. Find the maximum deflection in the beam using Macauley's method.

(Apr/May 2015) 16 Marks



To find R_A & R_B :

$R_A + R_B = 6 \times 3 + 12 = 30$ KN $\rightarrow (1)$

$\Sigma M_A = 0 \Rightarrow 7R_B = 12 \times 5 + 3 \times 3 \times 3/2 = 87$

$$\boxed{R_B = 12.43\,\text{KN}}$$

$$\boxed{R_A = 17.57\,\text{KN}}$$

$M_{XX} = R_A\,x - 6X\,3X\,(x\text{-}1.5) - 12X\,(x\text{-}5)$

$M_{XX} = EI\,{}^{d^2y}\!/\!_{dx^2} = 17.57x\ |\text{-}18(x\text{-}1.5)|\ \text{-}\ 12(x\text{-}5)$

$$EI\frac{d^2y}{dx^2} = 17.57x\,|-18(x-1.5)\,|-12(x-5)$$

Integrate

$$EI\frac{dy}{dx} = 17.57\,{}^{x^2}\!/_2 + c_1\left|\frac{-18(x-1.5)^2}{2}\right| - \frac{12(x-5)^2}{2}$$

$$EI\frac{dy}{dx} = 8.785x^2 + c_1\left|-9(x-1.5)^2\right| - 6(x-5)^2 \qquad \rightarrow (1)$$

Integrate

$$EIy = 8.785\,{}^{x^3}\!/_3 + c_1x + c_2\left|\frac{-9(x-1.5)^3}{3}\right| - \frac{6(x-5)^3}{3}$$

$$= 2.93x^3 + c_1x + c_2\left|-3(x-1.5)^3\right| - 2(x-5)^3 \qquad \rightarrow (2)$$

To find values of c_1 & c_2 use boundary

(i) At x = 0 $\Longrightarrow$ y = 0 $\longrightarrow$ (i)

(ii) At x = 7m $\Longrightarrow$ y = 0 $\longrightarrow$ (ii)

Substitute B.C in equation (2), we get, consider the term upto first vertical line

$$\boxed{0 = c_2}$$

Substitute B.C (ii) in equation (2) , we get

X = 7m ; y = 0

$$0 = 2.93(7)^3 + 7c_1 - 3(7\text{-}1.5)^3 \text{-}2(7\text{-}5)^3$$

$7c_1 = -2.93(7)^3 + 3(7\text{-}1.5)^3 +2(7\text{-}5)^3$

$7c_1 = -489.865$

$$\boxed{c_1 = -69.98}$$

Substitute c_1 & c_2 values in equation (2) , we get,

$$EIy = 2.93x^3 \text{-}69.98\ x\ |\text{-}3(x\text{-}1.5)^3\ |\ \text{-}2(x\text{-}5)^3 \rightarrow (3)$$

Assume deflection maximum between c & D_1 we get,

For maximum deflection dy/dx = 0

Substitute in equation (1),

Consider upto second vertical line

$0 = 8.785x^2 \text{-}69.98 \text{-}9(x\text{-}1.5)^2$

$= 8.785x^2 - 69.98 - 9(x^2\text{-}3x+2.25)$

$= 8.785x^2 - 69.98 - 9x^2 + 27x - 20.25$

$0 = -0.215x^2 + 27x - 90.23$

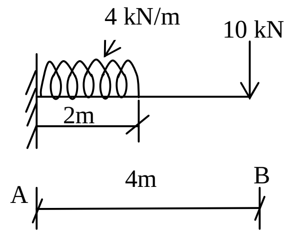
$0 = 0.215x^2 -27x + 90.23$

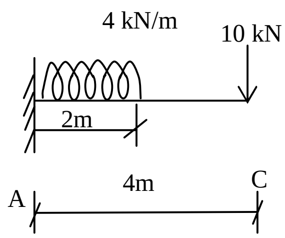
$\boxed{x = 3.435m}$ substituting in equation (3) upto second vertical line,

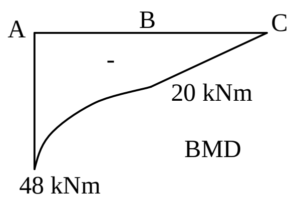
$$EIy_{max} = 2.93(3.435)^3 - 69.98(3.435) -3(3.435\text{-}1.5)^3$$

$$\boxed{y_{max} = \frac{-143.36}{EI}}$$

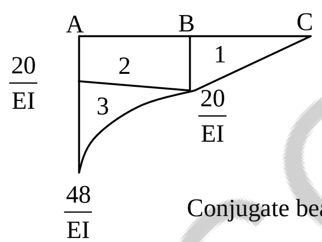
16) A cantilever of span 4m carries a UDL of 4 KN/m over a length of 2m from the fixed end and a concentrated load of 10 KN at the free end. Determine the slope and deflection of the cantilever at the free and using conjugate beam method. Assume EI uniform throughout.







BMD



Conjugate beam

B.M at C = 0

B.M at B = -10 x 2 = -20 KNm

B.M at A = -10 x 4 – 4 x 2 x $\left(\frac{\cancel{2}^{1}}{\cancel{2}}\right)$

= -10 x 4 = 4 x 2 x 1 = -48 KNm

Total load on beam = Area of $\frac{M}{EI}$ diagram

$$P = -\frac{1}{\cancel{2}} \times \cancel{2} \times \frac{20}{EI} - 2 \times \frac{20}{EI} - \frac{1}{3} \times 2 \times \left(\frac{48}{EI} - \frac{20}{EI}\right)$$

$$= \frac{-20}{EI} - \frac{40}{EI} - \frac{1}{3} \times 2 \times \frac{28}{EI}$$

$$P = \frac{-60 - 120 - 56}{3EI} = \frac{-236}{3EI}$$

Slope at C,

$$\theta_c = SF\,at\,C = -P = \frac{236}{3EI}$$

For finding BM at C for conjugate beam the total load can be considered as UVL and which is divided into one triangle & one rectangle and one parabolic curve on conjugate beam

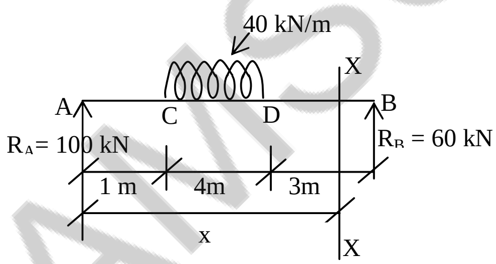
$$B.M\ at\ c = \left[\frac{1}{\cancel{2}} \times \cancel{2} \times \frac{20}{EI} \times \frac{2}{3} \times 2\right] + \left[2 \times \frac{20}{EI} \times \left(2 + \frac{\overset{1}{\cancel{2}}}{\cancel{2}}\right)\right] + \left[\frac{1}{2} \times 2 \times \left[\frac{48}{EI} - \frac{20}{EI}\right] \times \left(\frac{\cancel{3}}{\underset{2}{\cancel{4}}} \times 2 + 2\right)\right]$$

$$= \frac{80}{3EI} + \frac{120}{EI} + \left[\frac{1}{3} \times \cancel{2} \times \frac{28}{EI} \times \frac{7}{\cancel{2}}\right]$$

$$\boxed{\left.\begin{matrix}\text{Deflection}\\ \text{at c}\end{matrix}\right\} = \left.\begin{matrix}\text{BM at}\\ \text{c}\end{matrix}\right\} = \frac{80}{3EI} + \frac{360}{3EI} + \frac{196}{3EI} = \frac{636}{3EI}}$$

17) Determine the deflection of the beam at its midspan and also the position of maximum deflection & max. Deflection Take $E = 2 \times 10^5\ N/mm^2$ and $I = 4.3 \times 10^8\ mm^4$. Use Macaulay's method. The beam is given in fig

(Nov/Dec 2014) (May / June 2017) (16)



R_A & R_B :

$R_A + R_B = 40 \text{ X } 4 = 160 \quad \rightarrow (1)$

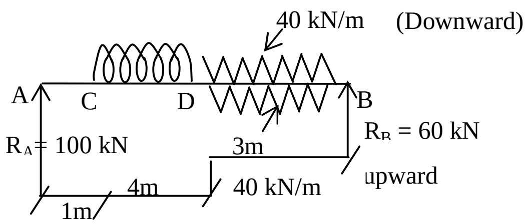
$\Sigma M_A = 0 \Rightarrow 8R_B = 40 \text{ X } 4 \text{ X } 3$

$8R_B = 480$

$\boxed{R_B = 60\,KN}$ substitute in (1)

$$\boxed{R_A = 100\,KN}$$

To obtain general expressions for the B.M at a distance x from the left end A, which will apply for all values of x, it is necessary to extend the UDL upto the support B, compensating with an equal upward load of 40 KN/m over the span DB as shown in figure, now Macauley's method can be applied.



B.M at any section at a distance x from end A is given by,

$$EI\frac{d^2y}{dx^2} = R_A x \left| -40(x-1)\frac{(x-1)}{2} \right| + 40(x-5)\frac{(x-5)}{2}$$

$$EI\frac{d^2y}{dx^2} = 100x \left| -20(x-1)^2 \right| + 20(x-5)^2 \qquad \rightarrow (1)$$

Integrate the above equation, we get,

$$EI\frac{dy}{dx} = \frac{100x^2}{2} + c_1 \left| -20\frac{(x-1)^3}{3} \right| + 20\frac{(x-5)^4}{3} \qquad \rightarrow (2)$$

Integrate again, we get,

$$EIy = 50\,{x^3}/{3} + c_1x + c_2 \left| \frac{-20}{3}\frac{(x-1)^4}{4} \right| + \frac{20}{3}\frac{(x-5)^4}{4}$$

$$= 50\,{x^3}/{3} + c_1x + c_2 \left| \frac{-5}{3}(x-1)^4 \right| + \frac{5}{3}\frac{(x-5)^4}{4} \qquad \rightarrow (3)$$

The value of c_1 & c_2 are obtained from boundary condition (i) x = 0 ; y = 0 9ii) x = 8m y = 0

Substituting x = 0; y = 0 in equation (3) upto first dotted line, we get $\boxed{c_2 = 0}$

Substituting (ii) B.C x = 8; y = 0 in equation(3),

$$0 = \frac{50}{3}\times 8^3 + c_1\times 8 + 0 - {5}/{3}(8-1)^4 + {5}/{3}(8-5)^4$$

$$0 = 8533.33 + 8c_1 - 4001.66 + 135$$

$$8c_1 = -4666.67$$

$$\boxed{c_1 = \frac{-4666.67}{8} = -583.33}$$

Substituting the values of c_1 & c_2 in equation (3) we get,

$$EIy = \frac{50}{3}x^3 - 583.33x \Big| -\frac{5}{3}(x-1)^4 \Big| + \frac{5}{3}(x-5)^4 \qquad \rightarrow (4)$$

a) Deflection at centre

substitute x = 4 in equation (4) , upto second vertical line,

$$EIy_{(x=4)} = \frac{50}{3}4^3 - 583.33 \times 4 - \frac{5}{3}(4-1)^4$$
$$= -1401.66 \times 10^3 \times 10^9 \text{ Nmm}^3$$
$$= -1401.66 \times 10^{12} \text{ Nmm}^3$$

$$y = \frac{-1401.66 \times 10^{12}}{2 \times 10^5 \times 4.5 \times 10^8} = -16.29 \text{ mm}$$

(– sign indicates downward)

b) Position of maximum deflection

For maximum deflection dy/dx = 0; equating the slope given by eqn (2) upto second vertical line;

$$0 = 50x^2 + c_1 - \frac{20}{3}(x-1)^3$$
$$\boxed{0 = 50x^2 - 583.33 - 6.667(x-1)^3} \qquad \rightarrow (5)$$

The above equation is solved by trial & error method

Let,

x =1 ; R.H.S of equation of eqn (5),

$= 50(1)^2 - 583.33 – 6.667(1-1)^3$

= -533.33

x =2 ; then R.H.S

$= 50 \times 4 - 583.33 – 6.667 (1)^3$

= -390.00

x = 3; then R.H.S

$= 50 \times 9 – 583.33 – 6.667(2)^3$

=-136.69

x = 4; then R.H.S

$= 50 \times 16 - 583.33 – 6.667(3)^3$

= + 36.58

x value lies between x = 3 & x = 4

Let x = 3.82 then R.H.S

$= 50 \times 3.82 - 583.33 - 6.667(3.82-1)^3$

$= -3.22$

X = 3.83 then R.H.S

$= 50 \times 3.83 - 583.33 - 6.667(3.83-1)^3$

$= -0.99$

Maximum deflection will be at a distance of 3.83 m from support A.

c) Maximum deflection

substitute x = 3.83 m in eqn (4) upto second vertical line, we get maximum deflection,

$$EIy_{max} = \frac{50}{3}(3.83)^3 - 583.33 \times 3.83 - \frac{5}{3}(3.83-1)^4$$
$$= -1404.69\,KNm^3 = -1404.69 \times 10^{12}\,Nmm^3$$

$$\boxed{y_{max} = \frac{-1404.69 \times 10^{12}}{2 \times 10^5 \times 4.3 \times 10^8} = -16.33mm}$$