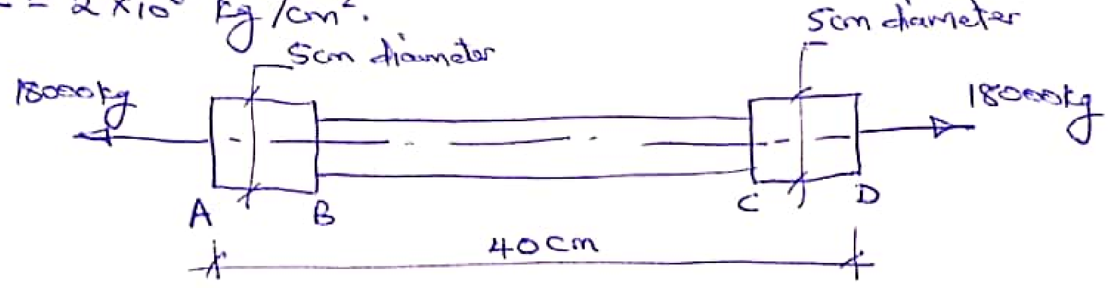
Example # 1

A bar ABCD shown below is subjected to a tensile load of 18000 kg. If stress in the material is limited to 1400 kg/cm², find the diameter of the portion BC and its length if the total elongation of the bar ABCD is to be 0,024 cm. Take $E = 2\times10^6$ kg/cm².



Solution

Required X-section area of BC

$$= \frac{P}{[\sigma]} = \frac{18000}{1400} = 12{,}857\ cm^2$$

If diameter of BC is d then

$$\frac{\pi d^2}{4} = 12.857 \quad \therefore\ d = \sqrt{\frac{12.857}{\pi}} = 4.046\ cm$$

X-section area of AB or CD $= \frac{\pi 5^2}{4} = 19.63\ cm^2$

If length of BC is x cm then the combined lengths of AB and CD is $(40-x)$ cm

Elongation of part (AB + CD) $= \delta l_1 = \frac{Pl}{AE} = \frac{18000(40-x)}{19.63\cdot 2\times10^6}$

Elongation of part BC $= \delta l_2 = \frac{Pl}{AE} = \frac{18000\cdot x}{12.857\cdot 2\times10^6}$

$\therefore$ Total elongation of ABCD $= \delta l_1 + \delta l_2 = 0{,}024$

$$\frac{18000(40-x)}{19.63\cdot 2\times10^6} + \frac{18000\,x}{12.857\cdot 2\times10^6} = 0{,}024 \quad \text{OR} \quad 0{,}000458(40-x) + 0{,}0007x = 0{,}024$$

OR $0{,}000242\,x = 0{,}00568$

$x = 23.47$ cm

Example 9.14. *An eccentrically loaded lap riveted joint is to be designed for a steel bracket as shown in Fig. 9.24.*

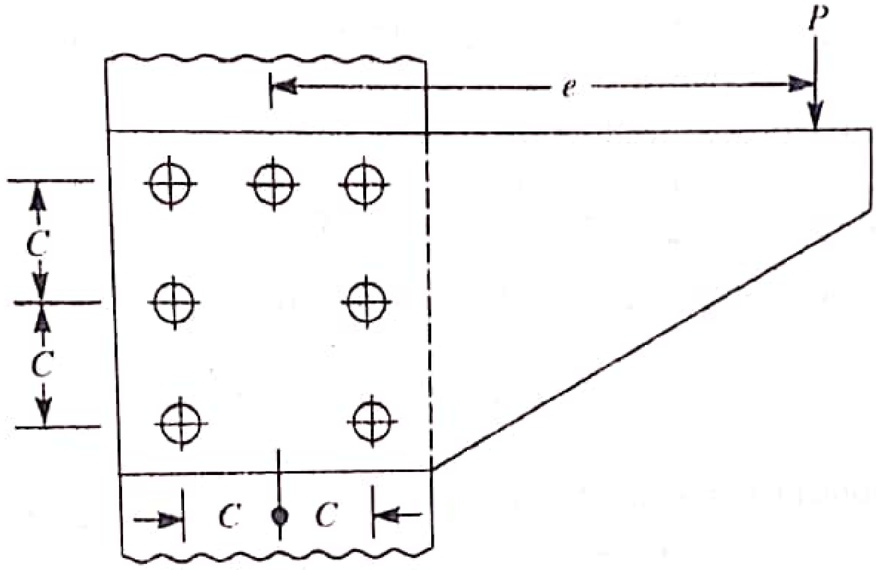


Fig. 9.24

The bracket plate is 25 mm thick. All rivets are to be of the same size. Load on the bracket, P = 50 kN ; rivet spacing, C = 100 mm ; load arm, e = 400 mm.

Permissible shear stress is 65 MPa and crushing stress is 120 MPa. Determine the size of the rivets to be used for the joint.

Solution. Given : t = 25 mm ; P = 50 kN = 50 × 10^3 N ; e = 400 mm ; n = 7 ; τ = 65 MPa = 65 N/mm^2 ; σ_c = 120 MPa = 120 N/mm^2

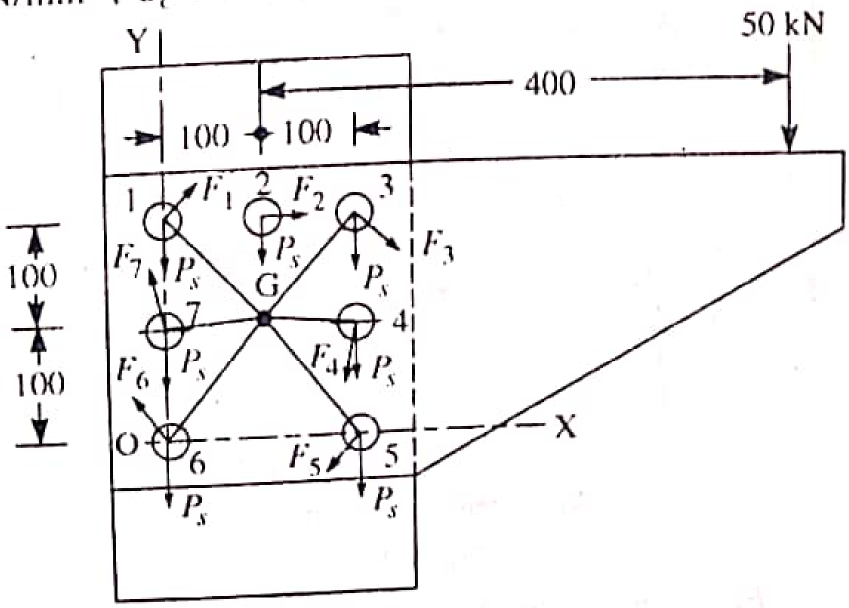


Fig. 9.25

First of all, let us find the centre of gravity (G) of the rivet system.

Let $\bar{x}$ = Distance of centre of gravity from OY,

$\bar{y}$ = Distance of centre of gravity from OX,

$x_1, x_2, x_3... =$ Distances of centre of gravity of each rivet from OY, and
$y_1, y_2, y_3... =$ Distances of centre of gravity of each rivet from OX.

We know that
$$\bar{x} = \frac{x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7}{n}$$
$$= \frac{100 + 200 + 200 + 200}{7} = 100 \text{ mm} \quad ...(\because x_1 = x_6 = x_7 = 0)$$

and
$$\bar{y} = \frac{y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7}{n}$$
$$= \frac{200 + 200 + 200 + 100 + 100}{7} = 114.3 \text{ mm} \quad ...(\because y_5 = y_6 = 0)$$

∴ The centre of gravity (G) of the rivet system lies at a distance of 100 mm from OY and 114.3 mm from OX, as shown in Fig. 9.25.

We know that direct shear load on each rivet,
$$P_s = \frac{P}{n} = \frac{50 \times 10^3}{7} = 7143 \text{ N}$$

The direct shear load acts parallel to the direction of load P *i.e.*, vertically downward as shown in Fig. 9.25.

Turning moment produced by the load P due to eccentricity (e)
$$= P \times e = 50 \times 10^3 \times 400 = 20 \times 10^6 \text{ N-mm}$$

This turning moment is resisted by seven rivets as shown in Fig. 9.25.

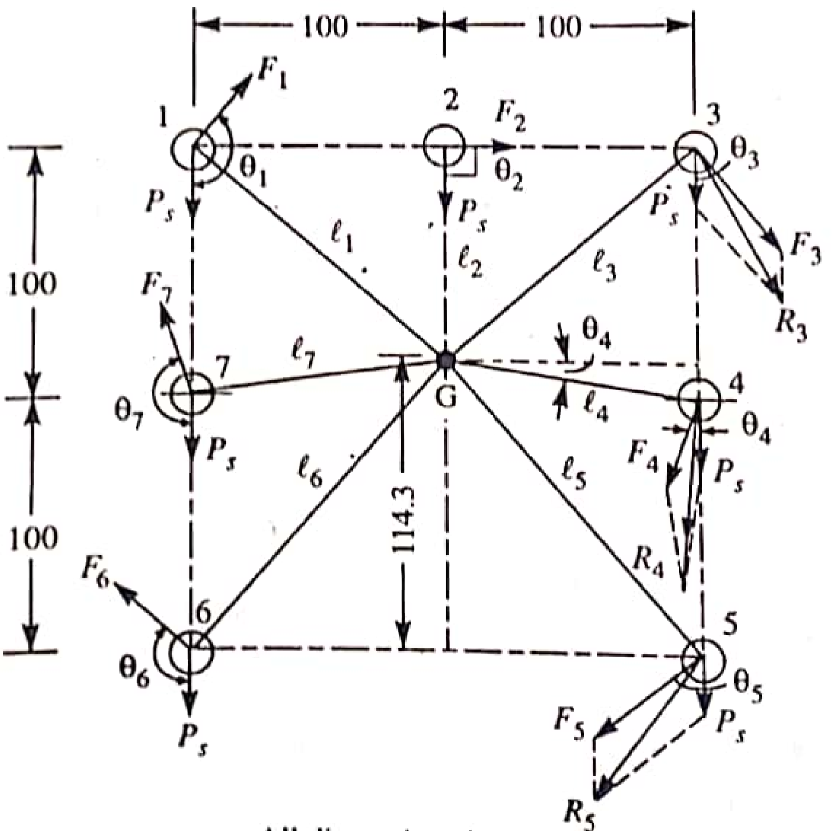


Fig. 9.26

Let F_1, F_2, F_3, F_4, F_5, F_6 and F_7 be the secondary shear load on the rivets 1, 2, 3, 4, 5, 6 and 7 placed at distances l_1, l_2, l_3, l_4, l_5, l_6 and l_7 respectively from the centre of gravity of the rivet system as shown in Fig. 9.26.

From the geometry of the figure, we find that
$$l_1 = l_3 = \sqrt{(100)^2 + (200 - 114.3)^2} = 131.7 \text{ mm}$$

$$l_2 = 200 - 114.3 = 85.7 \text{ mm}$$

$$l_4 = l_7 = \sqrt{(100)^2 + (114.3 - 100)^2} = 101 \text{ mm}$$

and

$$l_5 = l_6 = \sqrt{(100)^2 + (114.3)^2} = 152 \text{ mm}$$

Now equating the turning moment due to eccentricity of the load to the resisting moment of the rivets, we have

$$P \times e = \frac{F_1}{l_1}\left[(l_1)^2 + (l_2)^2 + (l_3)^2 + (l_4)^2 + (l_5)^2 + (l_6)^2 + (l_7)^2\right]$$

$$= \frac{F_1}{l_1}\left[2(l_1)^2 + (l_2)^2 + 2(l_4)^2 + 2(l_5)^2\right] \quad ...(\because l_1 = l_3 \,;\, l_4 = l_7 \text{ and } l_5 = l_6)$$

$$50 \times 10^3 \times 400 = \frac{F_1}{131.7}\left[2\,(131.7)^2 + (85.7)^2 + 2(101)^2 + 2(152)^2\right]$$

$$20 \times 10^6 \times 131.7 = F_1(34\ 690 + 7345 + 20\ 402 + 46\ 208) = 108\ 645\ F_1$$

$$\therefore \quad F_1 = 20 \times 10^6 \times 131.7 / 108\ 645 = 24\ 244 \text{ N}$$

Since the secondary shear loads are proportional to their radial distances from the centre of gravity, therefore

$$F_2 = F_1 \times \frac{l_2}{l_1} = 24\ 244 \times \frac{85.7}{131.7} = 15\ 776 \text{ N}$$

$$F_3 = F_1 \times \frac{l_3}{l_1} = F_1 = 2\ 244 \text{ N} \quad ...(\because l_1 = l_3)$$

$$F_4 = F_1 \times \frac{l_4}{l_1} = 24\ 244 \times \frac{101}{131.7} = 18\ 593 \text{ N}$$

$$F_5 = F_1 \times \frac{l_5}{l_1} = 24\ 244 \times \frac{152}{131.7} = 27\ 981 \text{ N}$$

$$F_6 = F_1 \times \frac{l_6}{l_1} = F_5 = 27\ 981 \text{ N} \quad ...(\because l_6 = l_5)$$

$$F_7 = F_1 \times \frac{l_7}{l_1} = F_4 = 18\ 593 \text{ N} \quad ...(\because l_7 = l_4)$$

By drawing the direct and secondary shear loads on each rivet, we see that the rivets 3, 4 and 5 are heavily loaded. Let us now find the angles between the direct and secondary shear load for these three rivets. From the geometry of Fig. 9.26, we find that

$$\cos\theta_3 = \frac{100}{l_3} = \frac{100}{131.7} = 0.76$$

$$\cos\theta_4 = \frac{100}{l_4} = \frac{100}{101} = 0.99$$

and

$$\cos\theta_5 = \frac{100}{l_5} = \frac{100}{152} = 0.658$$

Now resultant shear load on rivet 3,

$$R_3 = \sqrt{(P_s)^2 + (F_3)^2 + 2\,P_s \times F_3 \times \cos\theta_3}$$

$$= \sqrt{(7143)^2 + (24\ 244)^2 + 2 \times 7143 \times 24\ 244 \times 0.76} = 30\ 033 \text{ N}$$

Resultant shear load on rivet 4,

$$R_4 = \sqrt{(P_s)^2 + (F_4)^2 + 2\,P_s \times F_4 \times \cos\theta_4}$$

$$= \sqrt{(7143)^2 + (18\ 593)^2 + 2 \times 7143 \times 18\ 593 \times 0.99} = 25\ 684 \text{ N}$$

and resultant shear load on rivet 5,

$$R_5 = \sqrt{(P_5)^2 + (F_5)^2 + 2\,P_5 \times F_5 \times \cos\theta_5}$$

$$= \sqrt{(7143)^2 + (27\,981)^2 + 2 \times 7143 \times 27\,981 \times 0.658} = 33\,121 \text{ N}$$

The resultant shear load may be determined graphically, as shown in Fig. 9.26.

From above we see that the maximum resultant shear load is on rivet 5. If d is the diameter of rivet hole, then maximum resultant shear load (R_5),

$$33\,121 = \frac{\pi}{4} \times d^2 \times \tau = \frac{\pi}{4} \times d^2 \times 65 = 51\ d^2$$

$$\therefore \qquad d^2 = 33\,121 / 51 = 649.4 \quad \text{or} \quad d = 25.5 \text{ mm}$$

From Table 9.7, we see that according to IS : 1929–1982 (Reaffirmed 1996), the standard diameter of the rivet hole (d) is 25.5 mm and the corresponding diameter of rivet is 24 mm.

Let us now check the joint for crushing stress. We know that

$$\text{Crushing stress} = \frac{\text{Max. load}}{\text{Crushing area}} = \frac{R_5}{d \times t} = \frac{33\,121}{25.5 \times 25} = 51.95 \text{ N/mm}^2 = 51.95 \text{ MPa}$$

Since this stress is well below the given crushing stress of 120 MPa, therefore the design is satisfactory.