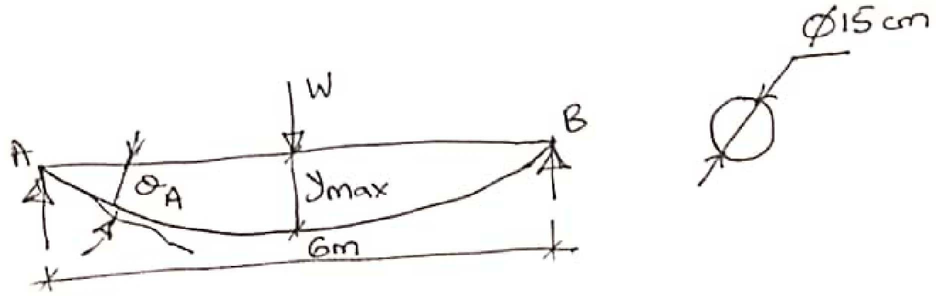
Example 1

A simply supported steel beam 6 metres long is circular in cross-section and is of 15 cm diameter. What heaviest central point load can be placed on it so that the maximum deflection of the beam does not exceed 1.035 cm? Calculate the slope at the supports. For steel take $E = 2.1 \times 10^6 \ kg/cm^2$.

Solution

Given: $y_{max} = 1.035 cm$; $l = 600 cm$; $E = 2.1 \times 10^6 \ Kg/cm^2$

$d = 15 cm.$



Moment of inertia for the beam

$$I = \frac{\pi \cdot 15^4}{64} = 2485.047 \ cm^4$$

But $$y_{max} = \frac{Wl^3}{48EI} \Rightarrow 1.035 = \frac{W \cdot 600^3}{48 \cdot (2.1 \times 10^6) \cdot 2485.047}$$

$$W = \frac{1.035 \cdot 48 \cdot 2{,}1 \times 10^6 \cdot 2485{,}047}{600^3} = \underline{\underline{1200{,}28 \ Kg}}$$

Slope $$\theta_A = \frac{Wl^2}{16EI}$$ But $$y_{max} = \frac{Wl^3}{48EI}$$

$$\therefore \quad \theta_A = \frac{Wl^2}{16EI} = \frac{3 \times y_{max}}{l}$$

$$\theta_A = \frac{3 \times 1{,}035}{600} = \underline{\underline{0{,}00518 \ radian}}$$

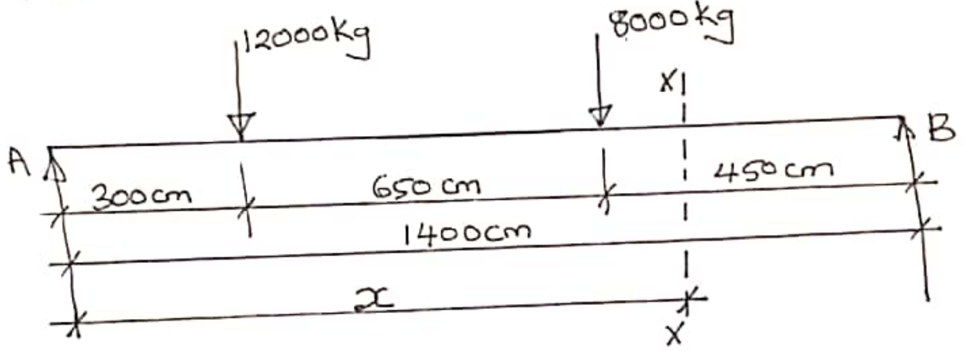
By symmetry $$\theta_B = -\theta_A = \underline{\underline{-0{,}00518 \ radian}}$$

Example #2

A horizontal girder of steel having uniform section is 14 metre long and simply supported at its ends. It carries concentrated loads of 12 tonne and 8 tonne at two points 3 metres and 4.5 metre from the two ends respectively. I for the section of the girders is $16 \times 10^4\ cm^4$ and $E_s = 2.1 \times 10^6\ kg/cm^2$. Calculate the deflection of the girder at points under the two loads.

Solution. Let R_A and R_B be the support reactions at A and B.

Taking moments about B we have:



$$R_A \cdot 1400 = 12000 \cdot (1400 - 300) + 8000 \cdot 450$$

$$\therefore \quad R_A = 12000\ kg$$

$$R_B = (12000 + 8000) - 12000 = 8000\ kg$$

Consider a section X – X at a distance x from support A.

$$M_x = 12000x - 12000(x - 300) - 8000[x - (300 + 650)]$$

$$= 12000x - 12000(x - 300) - 8000(x - 950)$$

$$\therefore \quad EI\frac{d^2y}{dx^2} = -M_x = -12000x + [12000(x-300)] + [8000(x-950)]$$

Successive integrating this eqn we have:

$$EI\frac{dy}{dx} = -\frac{12000x^2}{2} + \left[\frac{12000(x-300)^2}{2}\right] + \left[\frac{8000(x-950)^2}{2}\right] + C_1 \quad \ldots (i)$$

$$\& \quad EIy = -\frac{12000x^3}{6} + \left[\frac{12000(x-300)^3}{6}\right] + \left[\frac{8000(x-950)^3}{6}\right] + C_1x + C_2 \quad \ldots (ii)$$

Where C_1 and C_2 are Constants of integration and are determined by applying Conditions of zero deflection at A and B. (i.e at $x = 0$ and $x = 1400$; $y = 0$).

Applying these Conditions to eqn (ii) we get:

$$C_2 = 0 \text{ and } C_1 = 193{,}179 \times 10^7$$

$$\therefore \quad EIy = -\frac{12000x^3}{6} + \left[\frac{12000(x-300)^3}{6}\right] + \left[\frac{8000(x-950)^3}{6}\right] + 193{,}179 \times 10^7 x \quad \ldots (iii)$$

For deflection under 12 tonnes load put $x = 300$ in the eqn (iii) and neglect terms within square brackets that become negative

$$\therefore \quad y_c = \frac{1}{EI}\left[\frac{-12000 \times 300^3}{6} + 193{,}179 \times 10^7 \times 300\right]$$

$$= \frac{1}{2{,}1 \times 10^6 \times 16 \times 10^4} \times 525{,}537 \times 10^9 = \underline{\underline{1{,}564 \text{ cm}}}$$

For deflection under 8 tonnes load put $x = (1400 - 450) = 950$ in equation (ii)

$$y_D = \frac{1}{EI}\left[\frac{-12000 \times 950^3}{6} + \frac{12000 \times 650^3}{6} + 193.179 \times 10^7 \times 950\right]$$

$$= \frac{1}{2.1 \times 10^6 \times 16 \times 10^4} \times 6697.005 \times 10^8 = \underline{\underline{1.9931 \text{ cm}}}$$