Example

Calculate the strain energy in ~~it~~ a bar 2.5m long and 5 cm in diameter when it is subjected to a tensile load of 10 tonnes. What will then be the modulus of resilience of the material of the bar? Take $E = 2.0 \times 10^6$ kg/cm^2

Solution

X-sectional area of bar

$$A = \frac{\pi 5^2}{4} = 19,635 \text{ cm}^2$$

Volume of bar $V = A \times L = 19,635 \times 2.5 \times 100 \text{ cm}^3 = 4908.75 \text{ cm}^3$

Load applied $P = 10$ tonnes $= 10000$ kg

strain energy $U = \frac{\sigma^2}{2E} \times \text{Volume} = \frac{\left(\frac{P}{A}\right)^2}{2E} \times \text{Volume}$

$$= \frac{\left(\frac{10000}{19,645}\right)^2}{2 \times 2 \times 10^6} \times 4908,75 = \underline{318,31} \text{ kg.cm}$$

$$\text{Modulus of resilience} = \frac{\text{Strain energy}}{\text{Volume}} = \frac{318,31}{4908,75} = \underline{0,065} \text{ kg.cm/cm}^3$$

Example 2

A rod 1.2 cm in diameter stretched 0.32 cm under a steady load of 1000 kg. What stress would be produced in the bar by a weight of 70 kg which falls through 7.5 cm before commencing to stretch the rod, the rod being initially unstressed. The value of E may be taken as 21×10^5 kg/cm^2.

Solution

Let the length of the rod be l

Cross sectional area of the rod $A = \frac{\pi \cdot 1.25^2}{4} = 1.23 \text{ cm}^2$

Stress in the rod $\sigma = \frac{P}{A} = \frac{1000}{1.23} = 813$ kg/cm^2

Corresponding strain in the rod

$$e = \frac{\sigma}{E} = \frac{813}{21 \times 10^5} = 0.0003871$$

But $e = \frac{\delta l}{L}$ or $l = \frac{\delta l}{e} = \frac{0.32}{0.0003871} = 826.56$ cm

If P be the gradually applied load equivalent to 70 kg of impact load then:

$$P = W\left[1 + \sqrt{\left(1 + \frac{2AEh}{Wl}\right)}\right]$$

$$= 70\left[1 + \sqrt{\left(1 + \frac{2 \times 1.23 \times 21 \times 10^5 \times 7.5}{70 \times 826.56}\right)}\right] = 1882.77 \text{ kg}$$

$\therefore$ Stress produced by impact load

$$= \frac{1882.77}{A} = \frac{1882.77}{1.23} = 1530.71 \text{ kg/cm}^2$$