

\*\* A circular cast iron specimen of 6cm diameter and 180 mm height is subjected to axial compression load equal 14 ton until the failure. If the friction angle is 12°.

-Determine the respective stresses in the specimen.

### Solve

$$\phi = 12^\circ \quad \theta = 45 + \phi / 2 = 45 + 12/2 = 51^\circ$$

$$A = \frac{\pi \times D^2}{4} = \frac{\pi \times 6^2}{4} = 28.27 \text{ cm}^2$$

$$B = \frac{A}{\cos \theta} = \frac{28.27}{\cos 51} = 44.928 \text{ cm}^2$$

$$N = P \cdot \cos \theta = 14000 \times \cos 51 = 8810.485 \text{ kg}$$

$$Q = P \cdot \sin \theta = 14000 \times \sin 51 = 10880.043 \text{ kg}$$

$$F = N \cdot \tan \phi = 8810.485 \times \tan 12 = 1872.726 \text{ kg}$$

: \*\* لحساب الاجهادات المتولده :

$$* \sigma_p = \frac{P}{A} = \frac{14000}{28.27} = 495.22 \text{ kg/cm}^2$$

$$* \sigma_N = \frac{N}{B} = \frac{8810.485}{44.928} = 196.13 \text{ kg/cm}^2$$

$$* \sigma_Q = \frac{Q}{B} = \frac{10880.043}{44.928} = 242.2 \text{ kg/cm}^2$$

$$* \sigma_F = \frac{F}{B} = \frac{1872.726}{44.928} = 41.7 \text{ kg/cm}^2$$

$$* \sigma_{Failure} = \sigma_r = \sigma_Q - \sigma_F = 242.2 - 41.7 = 200.5 \text{ kg/cm}^2$$

\*\* A reinforced concrete column 50 cm x 50 cm in section is reinforced with 8 steel bars of 19 mm diameter. The column is carrying a load of 200 tones. Find the stresses in the concrete and steel bars.

Take E for steel =  $2 \times 10^6$  kg/cm<sup>2</sup> and E for concrete =  $180 \times 10^3$  kg/cm<sup>2</sup>

## Solve

$$A_{\text{total}} = 50 \times 50 = 2500 \text{ cm}^2$$

$$A_{\text{steel}} = \frac{\pi \times D^2}{4} \times n = \frac{\pi \times 1.9^2}{4} \times 8 = 22.68 \text{ cm}^2$$

$$A_{\text{total}} = A_{\text{concrete}} + A_{\text{steel}}$$

$$A_{\text{concrete}} = A_{\text{total}} - A_{\text{steel}} = 2500 - 22.68 = 2477.32 \text{ cm}^2$$

بما ان العمود محمل بحمل ضغط منتظم فان انفعال الخرسانه مساوى انفعال الحديد

$$\varepsilon_{\text{Steel}} = \varepsilon_{\text{concrete}}$$

$$L_{\text{concrete}} = L_{\text{Steel}}$$

$$\Delta_{\text{Concrete}} = \Delta_{\text{Steel}} \quad \dots \dots \dots \text{Equ 1}$$

$$\frac{P_c \times L_c}{E_c \times A_c} = \frac{P_s \times L_s}{E_s \times A_s}$$

$$\frac{P_c}{180 \times 10^3 \times 2477.32} = \frac{P_s}{2 \times 10^6 \times 22.68}$$

$$P_c = 9.831 P_s$$

$$P_{\text{total}} = P_{\text{concrete}} + P_{\text{steel}} \quad \dots \dots \dots \text{Equ 2}$$

$$200 \times 10^3 = P_{\text{concrete}} + P_{\text{steel}}$$

From 1, 2

$$P_{\text{steel}} = 18466.14 \text{ kg} = 18.47 \text{ ton}$$

$$P_{\text{concrete}} = 181533.8 \text{ kg} = 181.53 \text{ ton}$$

$$\sigma_c = \frac{P_c}{A_c} = \frac{181533.8}{2477.32} = 73.28 \text{ kg/cm}^2$$

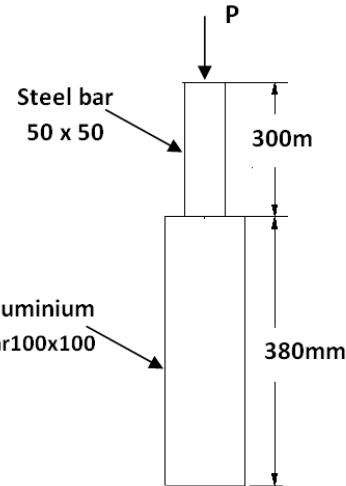
$$\sigma_s = \frac{P_s}{A_s} = \frac{18466.14}{22.68} = 814.2 \text{ kg/cm}^2$$

\*\* A member formed by connecting a steel bar to an aluminum bar is shown in figure. Assuming that the bars are prevented from buckling sidewise, calculate the magnitude of force  $P$ , which will cause the total length of member to decrease 0.25mm. The values of elastic modulus for steel and aluminum are 210 kN/mm<sup>2</sup> and 70 kN/mm<sup>2</sup> respectively.

$$\Delta L_{\text{total}} = 0.25 \text{ mm}$$

$$E_s = 210 \text{ KN/mm}^2$$

$$E_{Al} = 70 \text{ KN/mm}^2$$



## Solve

مقدار الانضغاط الكلى يساوى مجموع الانضغاط الحادث فى الحديد والالومنيوم

$$\Delta L_{\text{total}} = \Delta L_s + \Delta L_{\text{Al}} \quad \dots \text{Equ 1}$$

$P \equiv P_s \equiv P_{Al}$  ..... Eqn 2

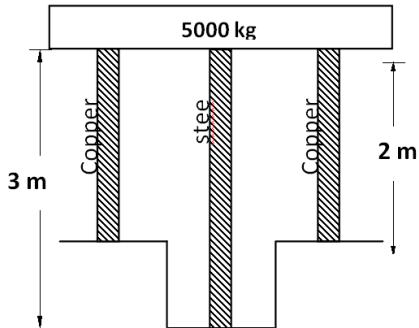
$$\Delta L_{\text{total}} = \frac{P_s \times L_s}{E_s \times A_s} + \frac{P_{Al} \times L_{Al}}{E_{Al} \times A_{Al}}$$

$$0.25 = \frac{P \times 300}{210 \times (50 \times 50)} + \frac{P \times 380}{70 \times (100 \times 100)}$$

$$P = 224.42 \text{ KN}$$

\* A steel rod of cross-sectional area  $16 \text{ cm}^2$  and two copper rods each of cross-sectional area of  $10 \text{ cm}^2$  together support a load of 5000 kg as shown in figure. Find the stress in the rods.

-Take: E for steel =  $2.0 \times 10^6 \text{ kg/cm}^2$  and E for copper =  $1.0 \times 10^6 \text{ kg/cm}^2$



### Solve

$$P_{\text{total}} = 2 P_{\text{copper}} + P_{\text{steel}} \quad \dots \quad \text{Equ 1}$$

$$5000 = 2 P_{\text{copper}} + P_{\text{steel}}$$

$$\Delta_{\text{copper}} = \Delta_{\text{steel}} \quad \dots \quad \text{Equ 2} \quad \text{لأن الطول مختلف}$$

$$\frac{P_c \times L_c}{E_c \times A_c} = \frac{P_s \times L_s}{E_s \times A_s}$$

$$\frac{P_c \times 200}{1 \times 10^6 \times 10} = \frac{P_s \times 300}{2 \times 10^6 \times 16} \quad P_c = 0.468 P_s$$

$$5000 = 2 \times 0.468 P_s + P_s$$

$$P_s = 2582.6 \text{ Kg}$$

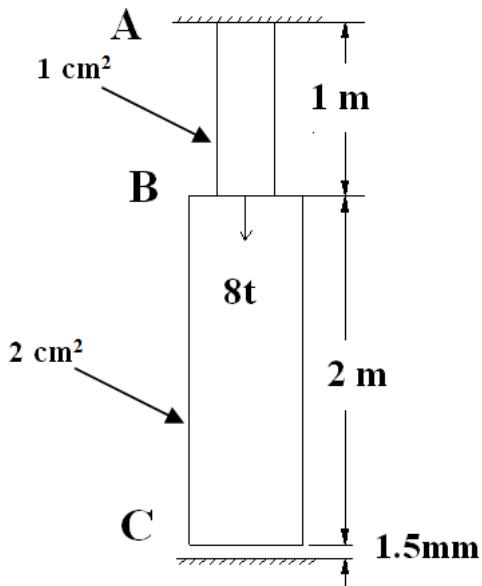
$$P_c = 0.468 \times 2582.6 = 1208.67 \text{ Kg}$$

$$\sigma_c = \frac{P_c}{A_c} = \frac{1208.67}{10} = 120.867 \text{ kg/cm}^2$$

$$\sigma_s = \frac{P_s}{A_s} = \frac{2582.6}{16} = 161.4125 \text{ kg/cm}^2$$

\* A composite bar ABC, rigidly fixed at A and 1.5 mm above the lower support, it loaded as shown in figure, if the cross-sectional area of the section AB is  $1\text{cm}^2$ , and that of the section BC is  $2\text{ cm}^2$ .

-Determine the reaction at the ends and the stresses in the two sections. - Take  $E = 2.0 \times 10^6 \text{ kg/cm}^2$



### Solve

عند اول التحميل فان 8 طن تؤثر على الجزء AB فقط ويوجد حالتان :

If  $\Delta_{AB} \leq 1.5 \text{ mm}$        $\therefore R_A = 8 \text{ ton}, R_C = 0.0 \text{ ton}$

If  $\Delta_{AB} \geq 1.5 \text{ mm}$        $\therefore R_A = \text{قيمة}, R_C = \text{قيمة}$

$$\Delta_{AB} = \frac{P \times L}{E \times A} = \frac{8000 \times 100}{2 \times 10^6 \times 1} = 0.4 \text{ cm} = 4 \text{ mm} \geq 1.5 \text{ mm} \quad \therefore \text{يوجد رد فعل عند C}$$

$$R_A + R_c = 8000 \text{ kg} \quad , \quad R_A = P_1 + R_{A2}$$

أولاً يتم إيجاد الحمل الذي يحدث استطالة مقدارها 1.5 mm في الجزء AB (  $P_1$  )

Take  $\Delta_{AB} = 1.5 \text{ mm}$

$$\Delta_{AB} = \frac{P \times L}{E \times A} = \frac{P_1 \times 100}{2 \times 10^6 \times 1} = 0.15 \text{ cm} \quad \therefore P_1 = 3000 \text{ Kg} \quad \text{القوه المسببه لجعل الشكل Fixed-Fixed}$$

$$R_{A2} + R_c = 8000 - 3000 = 5000 \text{ Kg} \quad \text{الحمل المتبقى من 8000kg}$$

ثانياً: عند وصول الجزء BC عند النقطه C فان مقدار الاستطالة للجزء AB هيساوى مقدار الانضغاط للجزء BC

$$\therefore \Delta_{AB} = \Delta_{BC}$$

$$\frac{R_{A2} \times L_{AB}}{E \times A_{AB}} = \frac{R_c \times L_{BC}}{E \times A_{BC}}$$

$$\frac{R_{A2} \times 100}{1} = \frac{R_c \times 200}{2}$$

$$\therefore R_{A2} = R_c$$

$$R_{A2} = R_c = 2500 \text{ Kg}$$

$$R_A = P_1 + R_{A2}$$

$$\therefore R_A = 3000 + 2500 = 5500 \text{ Kg}$$

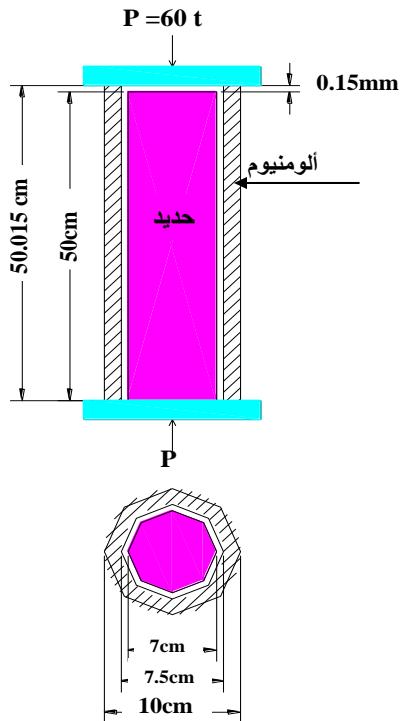
$$\sigma_{AB} = \frac{R_{AB}}{A_{AB}} = \frac{5500}{1} = 5500 \text{ Kg/cm}^2$$

$$\sigma_{BC} = \frac{R_c}{A_{BC}} = \frac{2500}{2} = 1250 \text{ Kg/cm}^2$$

\* A solid steel bar, 50 cm long and 7 cm diameter, is placed inside an aluminum tube having 7.5 cm inside diameter and 10 cm outside. The aluminum cylinder is 0.015 cm longer than the steel bar. An axial load of 60000 kg is applied to the bar and cylinder through rigid cover plates as shown in figure.

-Find the stresses developed in the steel bar and aluminum tube.

-Assume: E for steel =  $2.2 \times 10^6 \text{ kg/cm}^2$  and, E for aluminum =  $0.7 \times 10^6 \text{ kg/cm}^2$



## Solve

$$A_s = \frac{\pi \times D^2}{4} = \frac{\pi \times 7^2}{4} = 38.49 \text{ cm}^2$$

$$A_{Al} = \frac{\pi \times D^2}{4} = \frac{\pi \times (10^2 - 7.5)^2}{4} = 34.36 \text{ cm}^2$$

مسورة الألومنيوم أطول من قضيب الحديد 0.015 سم وبالتالي لا يوجد قوه تأثر على الحديد حتى الان  
اولاً: تعين الحمل الذي يحدث انضغاط قيمته 0.015 سم في مسورة الألومنيوم لينطبقوا على بعض ( $P_1$ )

Take  $\Delta_{Al} = 0.015 \text{ cm}$

$$\Delta_{Al} = \frac{P \times L}{E \times A}$$

$$0.015 = \frac{P_1 \times 50.015}{0.7 \times 10^6 \times 34.36} \quad \therefore P_1 = 7210 \text{ Kg} \quad \text{القوه المسببه لجعل الشكل Fixed-Fixed}$$

$P_R = 60000 - 7210 = 52790 \text{ Kg}$  على مسورة الألومنيوم و قضيب الحديد

$$P_R = P_s + P_{Al} = 52790 \text{ Kg}$$

$$52790 = \sigma_s \times A_s + \sigma_{Al} \times A_{Al}$$

**ثانياً:** عند وصول ماسورة الالمنيوم الى قضيب الحديد فان مقدار الانضغاط سيكون متساويا

$$\epsilon_s = \epsilon_{Al}$$

$$\frac{\sigma_s}{E_s} = \frac{\sigma_{Al}}{E_{Al}} \quad \frac{\sigma_s}{2.2 \times 10^6} = \frac{\sigma_{Al}}{0.7 \times 10^6} \quad \sigma_s = 3.143 \sigma_{Al} \quad \dots \text{Equ 2}$$

$$\sigma_s = 1068 \text{ Kg/cm}^2 , \quad \sigma_{Al\ 1} = 340 \text{ Kg/cm}^2$$

الايجاد المؤثر على ماسورة الالومنيوم عباره عن مجموع اجهادين الاول من تأثير الحمل الذى سبب انضغاط 0.015 سم والثانى من تأثير الحمل المتبقى على ماسورة الالومنيوم و قضيب الحديد معا

$$\sigma_{Al} = \sigma_{Al\ 1} + \sigma_{Al\ 2} = 340 + \frac{P_1}{A_{Al}} = 340 + \frac{7210}{34.36} = 550 \text{ Kg/cm}^2$$

$$\sigma_s = 1068 \text{ Kg/cm}^2$$

\*\* A load of 30t is applied on a short concrete column 25 x25 cm. The column is reinforced by steel bars of total area 56 cm<sup>2</sup>. If the modulus of elasticity for steel is 15 times that of concrete, find the stresses in steel and concrete. If the stresses in concrete should not exceed 40 kg/cm<sup>2</sup>, find the area of steel required so that the column may support a load of 60 t.

### Solve

$$A_{\text{total}} = 25 \times 25 = 625 \text{ cm}^2$$

$$A_{\text{total}} = A_{\text{concrete}} + A_{\text{steel}}$$

$$A_{\text{concrete}} = A_{\text{total}} - A_{\text{steel}} = 625 - 56 = 569 \text{ cm}^2$$

### Case i

بما ان العمود محمل بحمل ضغط منتظم فان انفعال الخرسانه مساوى انفعال الحديد

$$\varepsilon_{\text{Steel}} = \varepsilon_{\text{concrete}}$$

$$\frac{\sigma_s}{E_s} = \frac{\sigma_c}{E_c}$$

$$\frac{\sigma_s}{15 \cdot E_c} = \frac{\sigma_c}{E_c} \quad \sigma_s = 15 \cdot \sigma_c \quad \dots \dots \dots \text{Equ 1}$$

$$P_{\text{total}} = P_{\text{concrete}} + P_{\text{steel}}$$

$$30000 = \sigma_s \cdot A_s + \sigma_c \cdot A_c$$

$$30000 = \sigma_s \cdot 56 + \sigma_c \cdot 569 \quad \dots \dots \dots \text{Equ 2}$$

$$30000 = 15 \cdot \sigma_c \cdot 56 + \sigma_c \cdot 569$$

$$\sigma_c = 21.29 \text{ kg/cm}^2, \sigma_s = 319.35 \text{ kg/cm}^2$$

### Case ii

$$\sigma_c = 40 \text{ kg/cm}^2$$

$$\sigma_s = 15 \cdot \sigma_c = 15 \cdot 40 = 600 \text{ kg/cm}^2$$

$$A_{\text{concrete}} = A_{\text{total}} - A_{\text{steel}} = 625 - A_s$$

$$P_{\text{total}} = P_{\text{concrete}} + P_{\text{steel}}$$

$$60000 = \sigma_s \cdot A_s + \sigma_c \cdot A_c$$

$$60000 = 600 \cdot A_s + 40 \cdot (625 - A_s)$$

$$A_s = 62.5 \text{ cm}^2$$