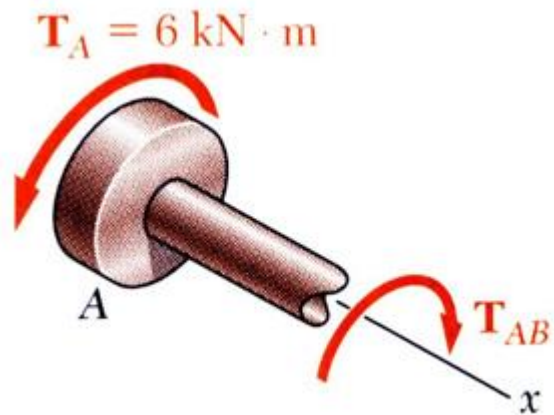


Shaft  $BC$  is hollow with inner and outer diameters of  $90\text{ mm}$  and  $120\text{ mm}$ , respectively. Shafts  $AB$  and  $CD$  are solid of diameter  $d$ . For the loading shown, determine (a) the minimum and maximum shearing stress in shaft  $BC$ , (b) the required diameter  $d$  of shafts  $AB$  and  $CD$  if the allowable shearing stress in these shafts is  $65\text{ MPa}$ .

### SOLUTION:

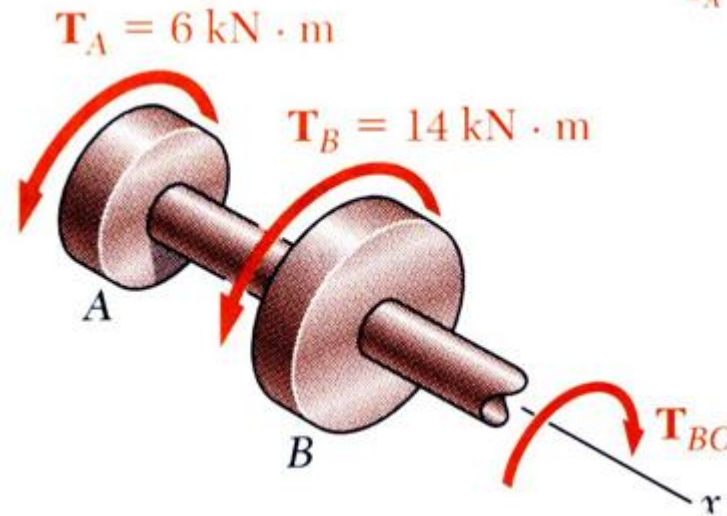
- Cut sections through shafts  $AB$  and  $BC$  and perform static equilibrium analysis to find torque loadings
- Apply elastic torsion formulas to find minimum and maximum stress on shaft  $BC$
- Given allowable shearing stress and applied torque, invert the elastic torsion formula to find the required diameter

- Cut sections through shafts  $AB$  and  $BC$  and perform static equilibrium analysis to find torque loadings



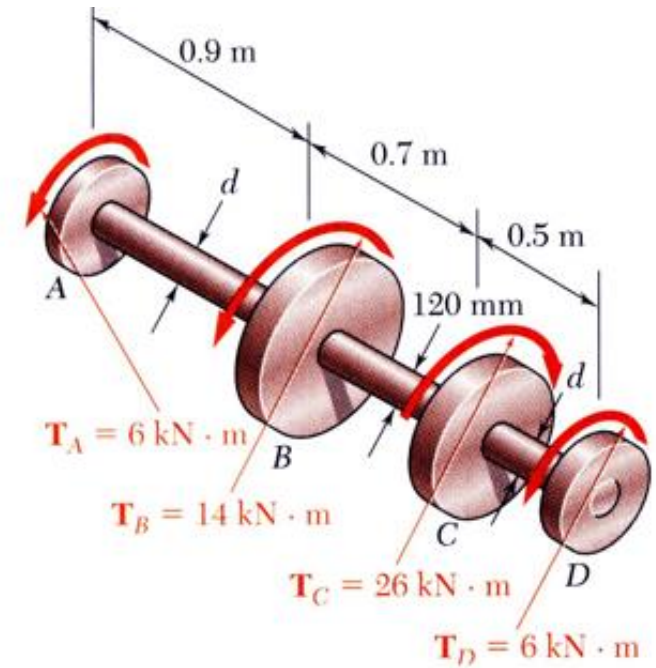
$$\sum M_x = 0 = (6 \text{ kN} \cdot \text{m}) - T_{AB}$$

$$T_{AB} = 6 \text{ kN} \cdot \text{m} = T_{CD}$$

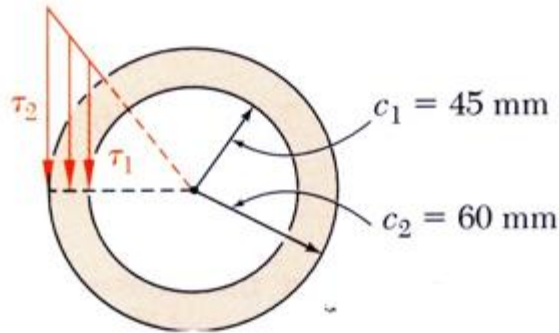


$$\sum M_x = 0 = (6 \text{ kN} \cdot \text{m}) + (14 \text{ kN} \cdot \text{m}) - T_{BC}$$

$$T_{BC} = 20 \text{ kN} \cdot \text{m}$$



- Apply elastic torsion formulas to find minimum and maximum stress on shaft  $BC$



$$J = \frac{\pi}{2} (c_2^4 - c_1^4) = \frac{\pi}{2} [(0.060)^4 - (0.045)^4]$$

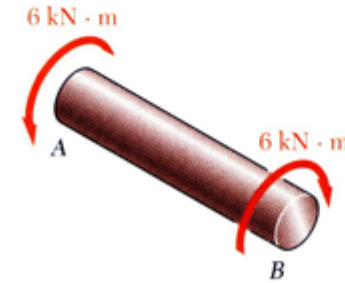
$$= 13.92 \times 10^{-6} \text{ m}^4$$

$$\tau_{\max} = \tau_2 = \frac{T_{BC} c_2}{J} = \frac{(20 \text{ kN} \cdot \text{m})(0.060 \text{ m})}{13.92 \times 10^{-6} \text{ m}^4}$$

$$= 86.2 \text{ MPa}$$

$$\frac{\tau_{\min}}{\tau_{\max}} = \frac{c_1}{c_2} \quad \frac{\tau_{\min}}{86.2 \text{ MPa}} = \frac{45 \text{ mm}}{60 \text{ mm}}$$

- Given allowable shearing stress and applied torque, invert the elastic torsion formula to find the required diameter



$$\tau_{\max} = \frac{Tc}{J} = \frac{Tc}{\frac{\pi}{2} c^4} \quad 65 \text{ MPa} = \frac{6 \text{ kN} \cdot \text{m}}{\frac{\pi}{2} c^3}$$

$$c = 38.9 \times 10^{-3} \text{ m}$$

$$d = 2c = 77.8 \text{ mm}$$