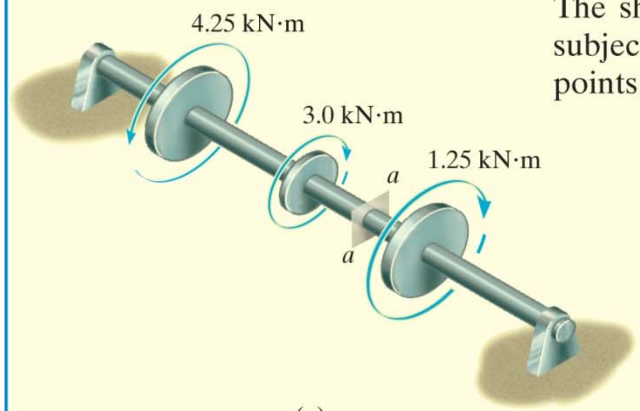
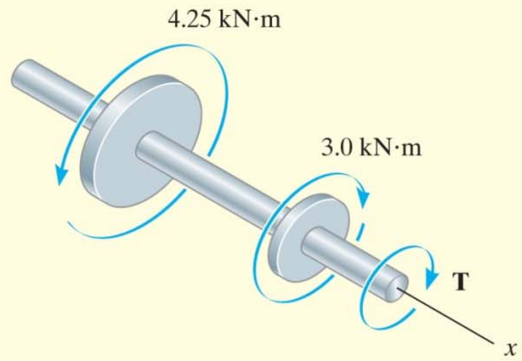
EXAMPLE 5.2

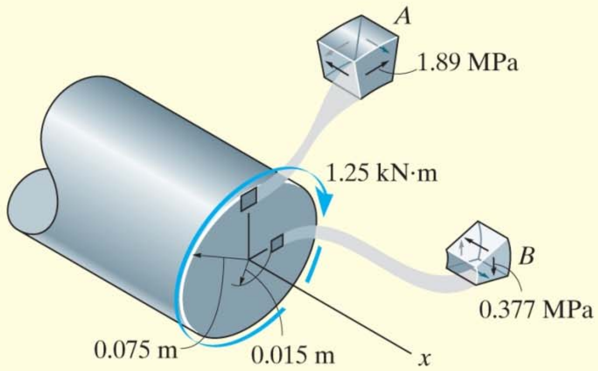


(a)

The shaft shown in Fig. 5–11*a* is supported by two bearings and is subjected to three torques. Determine the shear stress developed at points *A* and *B*, located at section *a–a* of the shaft, Fig. 5–11*c*.



(b)



(c)

Fig. 5–11

SOLUTION

Internal Torque. The bearing reactions on the shaft are zero, provided the shaft's weight is neglected. Furthermore, the applied torques satisfy moment equilibrium about the shaft's axis.

The internal torque at section *a–a* will be determined from the free-body diagram of the left segment, Fig. 5–11*b*. We have

$$\Sigma M_x = 0; \quad 4.25\text{ kN}\cdot\text{m.} - 3.0\text{ kN}\cdot\text{m} - T = 0 \qquad T = 1.25\text{ kN}\cdot\text{m}$$

Section Property. The polar moment of inertia for the shaft is

$$J = \frac{\pi}{2}(0.075\text{ m})^4 = 4.97(10^{-5})\text{ m}^4$$

EXAMPLE 5.2 CONTINUED

Shear Stress. Since point A is at $\rho = c = 0.075$ m,

$$\tau_A = \frac{Tc}{J} = \frac{(1.25\ \text{kN}\cdot\text{m})(0.075\ \text{m})}{4.97(10^{-5})\ \text{m}^4} = 1886\ \text{kPa} = 1.89\ \text{Mpa} \qquad \textit{Ans.}$$

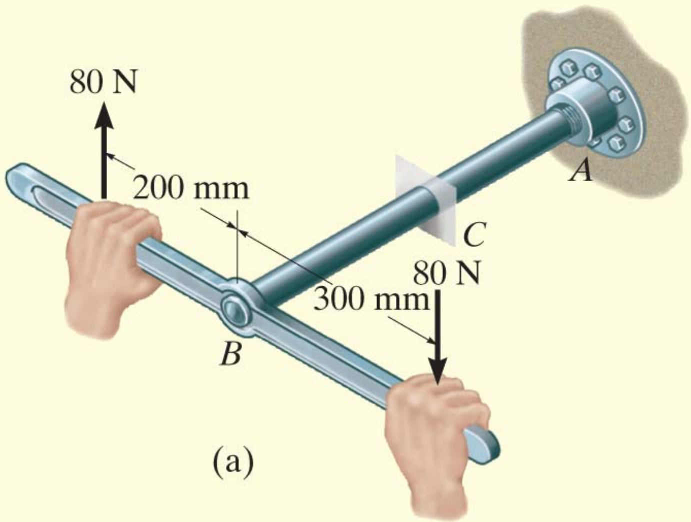
Likewise for point B, at $\rho = 0.015$ m, we have

$$\tau_B = \frac{T\rho}{J} = \frac{(1.25\ \text{kN}\cdot\text{m})(0.015\ \text{m})}{4.97(10^{-5})\text{m}^4} = 377.3\ \text{kPa} = 0.377\ \text{MPa} \qquad \textit{Ans.}$$

NOTE: The directions of these stresses on each element at A and B, Fig. 5–11c, are established from the direction of the resultant internal torque $\mathbf{T}$, shown in Fig. 5–11b. Note carefully how the shear stress acts on the planes of each of these elements.

EXAMPLE 5.3

The pipe shown in Fig. 5–12*a* has an inner diameter of 80 mm and an outer diameter of 100 mm. If its end is tightened against the support at *A* using a torque wrench at *B*, determine the shear stress developed in the material at the inner and outer walls along the central portion of the pipe when the 80-N forces are applied to the wrench.



(a)

EXAMPLE 5.3 (cont)

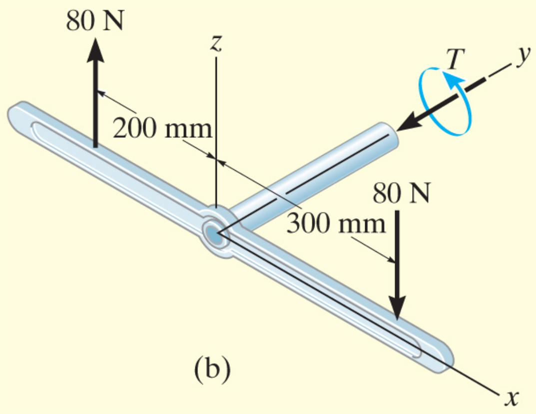
Solutions

- The only unknown at the section is the internal torque T

$$\sum M_y = 0;$$
$$80(0.3) + 80(0.2) - T = 0$$
$$T = 40 \text{ N}\cdot\text{m}$$

- The polar moment of inertia for the pipe's cross-sectional area is

$$J = \frac{\pi}{2}\left[(0.05)^4 - (0.04)^4\right] = 5.796\left(10^{-6}\right)\text{m}^4$$



- For any point lying on the outside surface of the pipe,

$$\rho = c_0 = 0.05\text{ m}$$

$$\tau_0 = \frac{Tc_0}{J} = \frac{40(0.05)}{5.796\left(10^{-6}\right)} = 0.345\text{ MPa} \text{ (Ans)}$$

EXAMPLE 5.3 (cont)

Solutions

- And for any point located on the inside surface, $\rho = c_i = 0.04 \text{ m}$

$$\tau_i = \frac{Tc_i}{J} = \frac{40(0.04)}{5.796(10^{-6})} = 0.276 \text{ MPa} \quad \text{(Ans)}$$

- The resultant internal torque is equal but opposite.

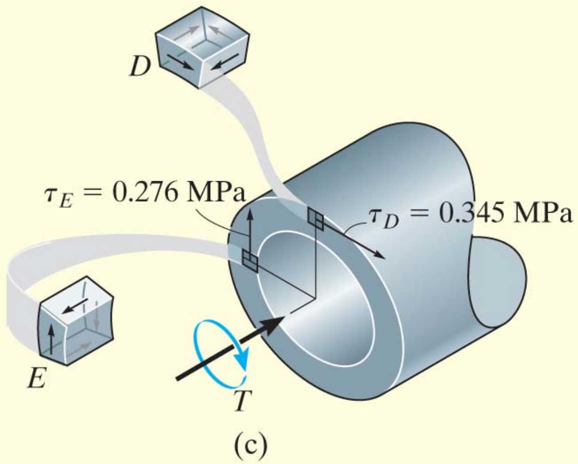


Fig. 5–12

5.3 POWER TRANSMISSION

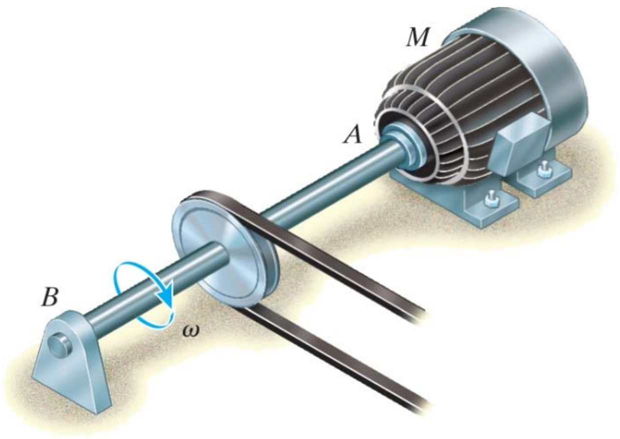


Figure: 05_13_EX04

$$W = FS = T\varphi = M\theta$$

$$P = \frac{dW}{dt} = T\frac{d\varphi}{dt} = T\omega = T\frac{2\pi N}{60} = T2\pi f \quad [Nm/\sec = W]$$

$$T = \frac{J}{c}\tau_{all} = \frac{\pi}{2}c^3\tau_{all}$$

$$\therefore c = \left(\frac{\pi}{2}\frac{T}{\tau_{all}}\right)^{1/3}$$

EXAMPLE 5.4

A solid steel shaft AB, shown in Fig. 5–13, is to be used to transmit 3750 W from the motor M to which it is attached. If the shaft rotates at $\omega = 175$ rpm and the steel has an allowable shear stress of $\tau_{\text{allow}} = 100$ MPa determine the required diameter of the shaft to the nearest mm.

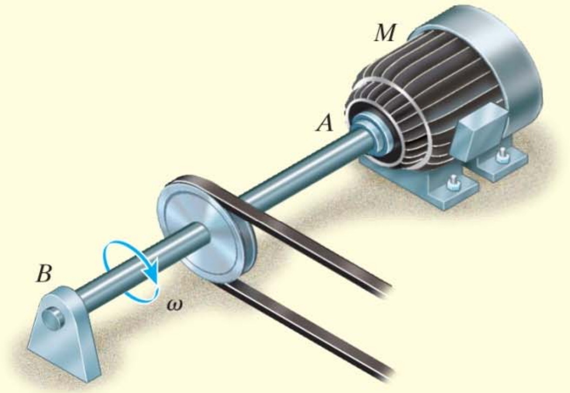


Fig. 5–13

SOLUTION

The torque on the shaft is determined from Eq. 5–10, that is, $P = T\omega$. Expressing P in foot-pounds per second and ω in radians/second, we have

EXAMPLE 5.4 CONTINUED

$$P = 3750\text{ W} = 3750\text{ N}\cdot\text{m/s}$$

$$\omega = \frac{175\text{ rev}}{\text{min}}\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 18.33\text{ rad/s}$$

Thus,

$P = T\omega;$
$$3750\text{ N}\cdot\text{m/s} = T(18.33\text{ rad/s})$$
$$T = 204.6\text{ N}\cdot\text{m}$$

Applying Eq. 5–12 yields

$$\frac{J}{c} = \frac{\pi}{2}\frac{c^4}{c} = \frac{T}{\tau_{\text{allow}}}$$

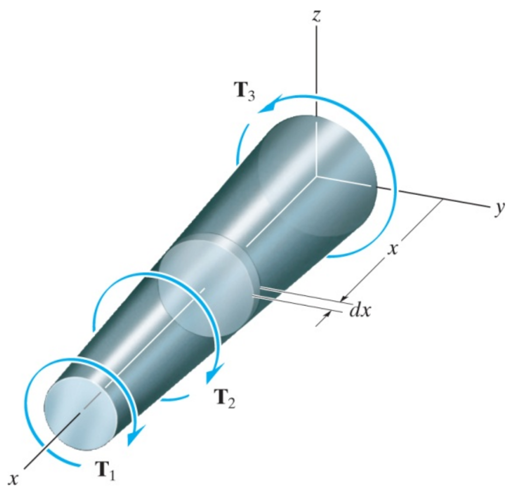
$$c = \left(\frac{2T}{\pi\tau_{\text{allow}}}\right)^{1/3} = \left(\frac{2(204.6\text{ N}\cdot\text{m})(1000\text{ mm/m})}{\pi(100\text{ N/mm}^2)}\right)^{1/3}$$

$$c = 10.92\text{ mm}$$

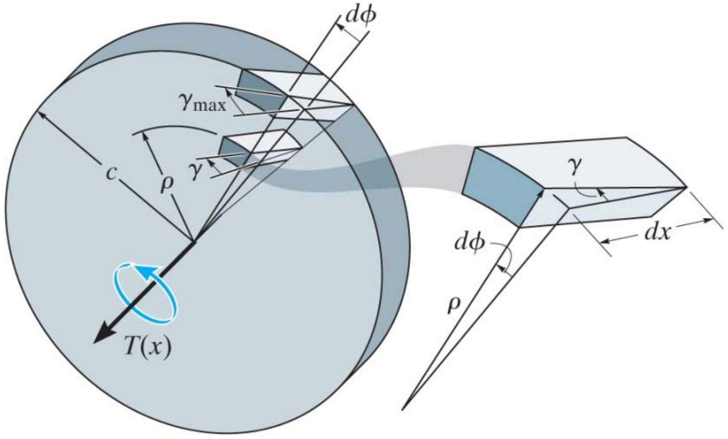
Since $2c = 21.84$ mm, select a shaft having a diameter of

$$d = 22\text{ mm} \qquad\qquad \textit{Ans.}$$

5.4 ANGLE OF TWIST



(a)



(b)

$$d\varphi = \gamma \frac{dx}{\rho}, \quad \gamma = \frac{\tau}{G} = \frac{1}{G}\frac{T\rho}{J} = \frac{T\rho}{GJ}$$

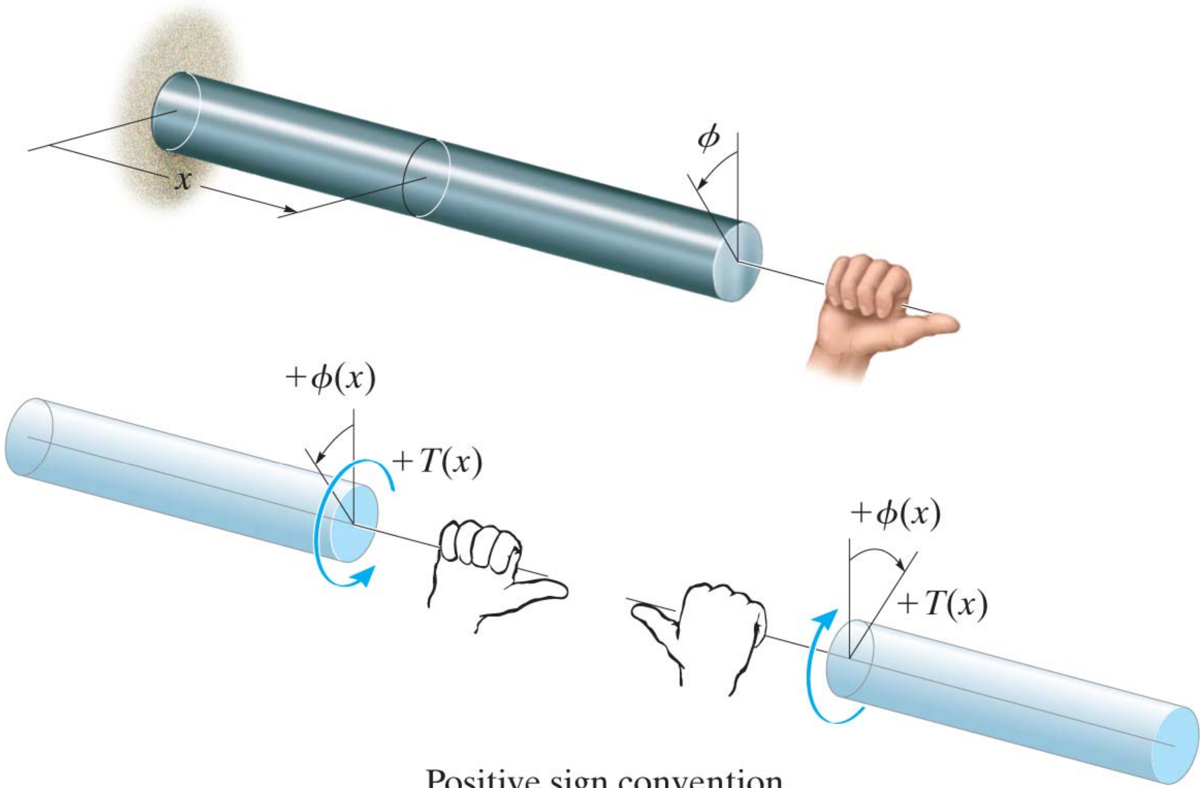
$$d\varphi = \frac{T\rho}{GJ}\frac{dx}{\rho} = \frac{T(x)}{GJ(x)}dx, \quad \therefore \varphi = \int \frac{T(x)}{GJ(x)}dx$$

- For constant torque and cross-sectional area:

$$\varphi = \frac{TL}{GJ}$$

ANGLE OF TWIST (cont)

- Sign convention for both torque and angle of twist
  - positive if (right hand) thumb directs outward from the shaft

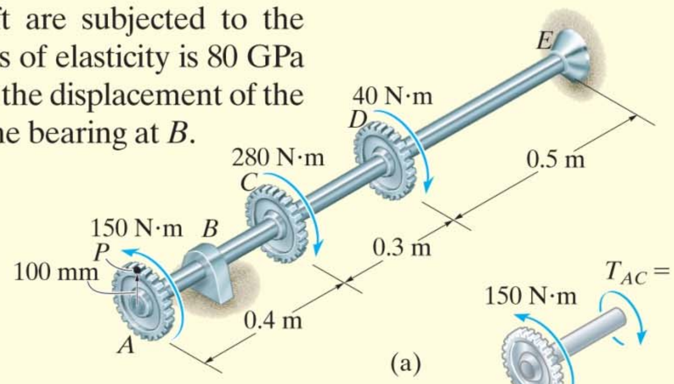


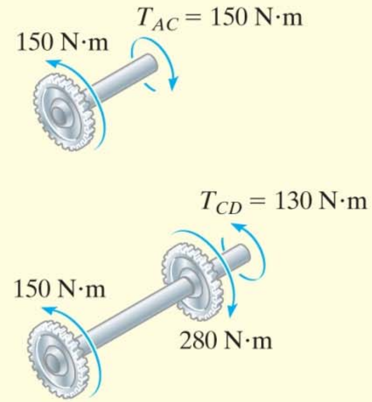
Positive sign convention
for T and ϕ.

Figure: 05_17

EXAMPLE 5.5

The gears attached to the fixed-end steel shaft are subjected to the torques shown in Fig. 5–19*a*. If the shear modulus of elasticity is 80 GPa and the shaft has a diameter of 14 mm, determine the displacement of the tooth *P* on gear *A*. The shaft turns freely within the bearing at *B*.





SOLUTION

Internal Torque. By inspection, the torques in segments *AC*, *CD*, and *DE* are different yet *constant* throughout each segment. Free-body diagrams of appropriate segments of the shaft along with the calculated internal torques are shown in Fig. 5–19*b*. Using the right-hand rule and the established sign convention that positive torque is directed away from the sectioned end of the shaft, we have

$$T_{AC} = +150\ \mathrm{N\cdot m} \qquad T_{CD} = -130\ \mathrm{N\cdot m} \qquad T_{DE} = -170\ \mathrm{N\cdot m}$$

These results are also shown on the torque diagram, Fig. 5–19*c*.

Angle of Twist. The polar moment of inertia for the shaft is

$$J = \frac{\pi}{2}(0.007\text{ m})^4 = 3.771(10^{-9})\text{ m}^4$$

Applying Eq. 5–16 to each segment and adding the results algebraically, we have

$$\phi_A = \sum\frac{TL}{JG} = \frac{(+150\text{ N}\cdot\text{m})(0.4\text{ m})}{3.771(10^{-9})\text{m}^4\,[80(10^9)\text{N/m}^2]} + \frac{(-130\text{ N}\cdot\text{m})(0.3\text{ m})}{3.771(10^{-9})\text{m}^4\,[80(10^9)\text{N/m}^2]} + \frac{(-170\text{ N}\cdot\text{m})(0.5\text{ m})}{3.771(10^{-9})\text{m}^4\,[80(10^9)\text{ N/m}^2]} = -0.2121\text{ rad}$$

Since the answer is negative, by the right-hand rule the thumb is directed *toward* the end *E* of the shaft, and therefore gear *A* will rotate as shown in Fig. 5–19*d*.

The displacement of tooth *P* on gear *A* is

$$s_P = \phi_A r = (0.2121\text{ rad})(100\text{ mm}) = 21.2\text{ mm} \qquad \textit{Ans.}$$

NOTE: Remember that this analysis is valid only if the shear stress does not exceed the proportional limit of the material.

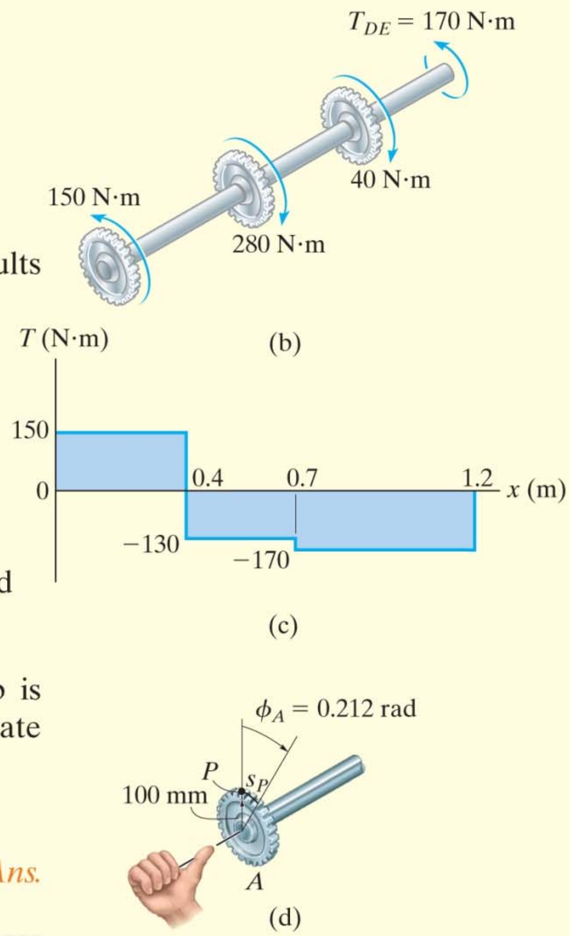


Fig. 5–19

EXAMPLE 5.6

The two solid steel shafts are coupled together using the meshed gears. Determine the angle of twist of end *A* of shaft *AB* when the torque 45 Nm is applied. Take G to be 80 GPa. Shaft *AB* is free to rotate within bearings *E* and *F*, whereas shaft *DC* is fixed at *D*. Each shaft has a diameter of 20 mm.

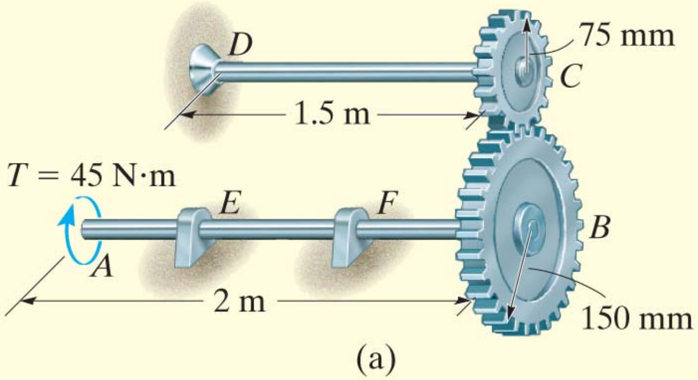


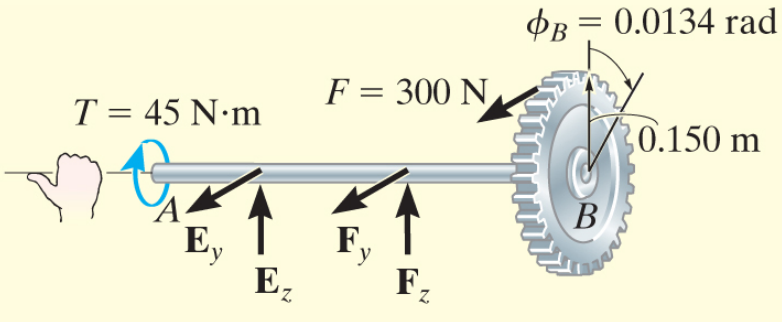
Fig. 5–20

EXAMPLE 5.6 (cont)

Solutions

- From free body diagram,

$$F = 45/0.15 = 300\ \text{N}$$
$$(T_D)_x = 300(0.075) = 22.5\ \text{Nm}$$



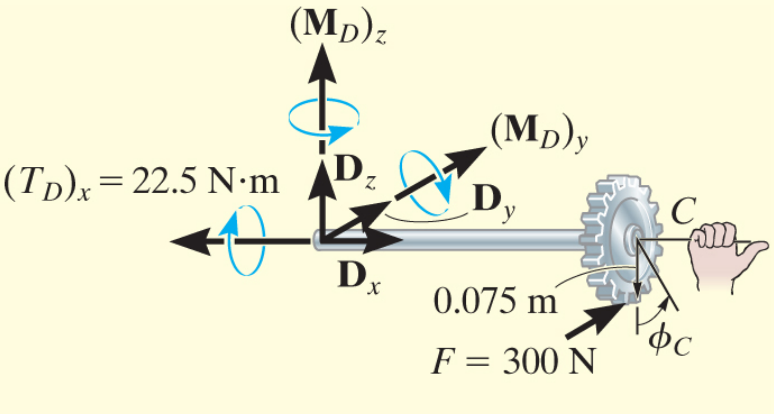
(b)

- Angle of twist at C is

$$\phi_C = \frac{TL_{DC}}{JG} = \frac{(+22.5)(1.5)}{(\pi/2)(0.001)^4\left[80(10)^9\right]} = +0.0269\ \text{rad}$$

- Since the gears at the end of the shaft are in mesh,

$$\phi_B(0.15) = (0.0269)(0.075) \Rightarrow 0.0134\ \text{rad}$$



(c)

EXAMPLE 5.6 (cont)

Solutions

- Since the angle of twist of end *A* with respect to end *B* of shaft *AB* caused by the torque 45 Nm,

$$\phi_{A/B} = \frac{T_{AB}L_{AB}}{JG} = \frac{(+45)(2)}{(\pi/2)(0.010)^4\left[80\left(10^9\right)\right]} = +0.0716\,\text{rad}$$

- The rotation of end *A* is therefore

$$\phi_A = \phi_B + \phi_{A/B} = 0.0134 + 0.0716 = +0.0850\,\text{rad} \quad \text{(Ans)}$$