MEC 3352 – STRENGTH OF MATERIALS II

SOME WORKED EXAMPLES from Ryder

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PRINCIPAL STRESSES AND STRAINS
AND
THEORIES OF FAILURE

Example 7, P. 49: A piece of material is subjected to three perpendicular tensile stresses and the strains in the three directions are in the ratio 3:4:5. If Poisson's ratio is 0.286, find the ratio of the stresses, and their values if the greatest is 60 MPa. (U. L.)

Let the stresses be σ_1, σ_2, and σ_3, and the corresponding strains $3k$, $4k$, and $5k$.

Then
$$3kE = \sigma_1 - 0{\cdot}286(\sigma_2 + \sigma_3) \tag{i}$$
$$4kE = \sigma_2 - 0{\cdot}286(\sigma_3 + \sigma_1) \tag{ii}$$
$$5kE = \sigma_3 - 0{\cdot}286(\sigma_1 + \sigma_2) \tag{iii}$$

Subtract (i) from (iii):
$$\sigma_3 - \sigma_1 - 0{\cdot}286(\sigma_1 - \sigma_3) = 2kE$$
giving
$$\sigma_3 - \sigma_1 = 2kE/1{\cdot}286 \tag{iv}$$
Writing (iii):
$$\sigma_3/0{\cdot}286 - \sigma_1 - \sigma_2 = 5kE/0{\cdot}286 \tag{v}$$
and (ii):
$$\sigma_2 - 0{\cdot}286\sigma_3 - 0{\cdot}286\sigma_1 = 4kE \tag{vi}$$

Add (v) and (vi): $3{\cdot}21\sigma_3 - 1{\cdot}286\sigma_1 = 21{\cdot}5kE$ (vii)

Writing (iv) $1{\cdot}286\sigma_3 - 1{\cdot}286\sigma_1 = 2kE$ (viii)

Subtract (viii) from (vii): $1{\cdot}294\sigma_3 = 19{\cdot}5kE$

or $\sigma_3 = 10{\cdot}14kE$

From (iv): $\sigma_1 = 8{\cdot}58kE$

From (ii): $\sigma_2 = 9{\cdot}34kE$

Ratio of stresses: $\sigma_1; \sigma_2: \sigma_3 = 0{\cdot}847:0{\cdot}921:1$

If the greatest $\sigma_3 = 60\ \text{N/mm}^2$

$$\sigma_1 = 50{\cdot}8\ \text{N/mm}^2 \text{ and } \sigma_2 = 55{\cdot}3\ \text{N/mm}^2$$

Example 9, P. 55: The principal stresses at a point in an elastic material are 60 MPa tensile, 20 MPa tensile and 50 MPa compressive. Calculate the volumetric strain and the resilience. E = 100 GPa; ν = 0.35.

$\sigma_1 = 60,\ \sigma_2 = 20,\ \sigma_3 = -50.$

$$\text{Volumetric strain} = (\sigma_1 + \sigma_2 + \sigma_3)\frac{1-2\nu}{E} \quad \text{(Para. 3.18)}$$

$$= (60 + 20 - 50)\frac{1 - 0{\cdot}7}{100{,}000}$$

$$= 9 \times 10^{-5}$$

Resilience = Strain Energy = U = work done per unit volume.

total work done $= \Sigma \frac{1}{2}\sigma\varepsilon$,

$$U = \tfrac{1}{2}\sigma_1\varepsilon_1 + \tfrac{1}{2}\sigma_2\varepsilon_2 + \tfrac{1}{2}\sigma_3\varepsilon_3$$

$= (1/2E)[\sigma_1(\sigma_1 - \nu\sigma_2 - \nu\sigma_3) + \sigma_2(\sigma_2 - \nu\sigma_3 - \nu\sigma_1) + \sigma_3(\sigma_3 - \nu\sigma_1 - \nu\sigma_2)]$ by Para. 3.14

$= (1/2E)[\sigma_1^2 + \sigma_2^2 + \sigma_3^2 - 2\nu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1])$ per unit volume.

$$\text{Resilience} = [1/(2 \times 100{,}000)]\,[60^2 + 20^2 + (-50)^2 - 2 \times 0{\cdot}35\,(60 \times 20 - 20 \times 50 - 50 \times 60)]$$

$$= 8460/200{,}000$$

$$= 0{\cdot}0423 \text{ N mm/mm}^3$$

Example 10, P. 60: If the principal stresses at a point in an elastic material are $2f$ tensile, f tensile and $\frac{1}{2}f$ compressive, calculate the value of f at failure according to the five different theories.

The elastic limit in simple tension is 200 MPa and Poisson's ratio = 0.3.

(1) *Maximum principal stress theory*

In the complex system, maximum stress $= 2f$

In simple tension, maximum stress $= 200\ \text{N/mm}^2$

Equating gives $f = 100\ \text{N/mm}^2$

(2) *Maximum shear stress theory*

$$\text{Maximum shear stress} = \text{Half difference between principal stresses}$$
$$= \tfrac{1}{2}[2f - (-\tfrac{1}{2}f)]$$
$$= \tfrac{5}{4}f$$

In simple tension, principal stresses are 200, 0, 0, and

$$\text{maximum shear stress} = \tfrac{1}{2} \times 200$$
$$= 100\ \text{N/mm}^2$$ (See also Para. 3.2.)

Equating gives $f = 80\ \text{N/mm}^2$

(3) *Strain energy theory*

In the complex system

$$U = (1/2E)[(2f)^2 + f^2 + (-\tfrac{1}{2}f)^2 - 2 \times 0{\cdot}3(2f.f - f.f/2 - f/2.2f)]$$

(Para. 3.19)

$$= 4{\cdot}95 f^2/2E$$

In simple tension: $U = 200^2/2\mathrm{E}$

Equating gives $f = 200/\sqrt{4{\cdot}95}$

$= 89{\cdot}8\ \mathrm{N/mm^2}$

(4) *Shear strain energy theory*

In the complex system

$$U = (1/12G)[(2f - f)^2 + (f + \tfrac{1}{2}f)^2 + (-\tfrac{1}{2}f - 2f)^2] \quad \text{(Para 3.20.)}$$

$$= 9{\cdot}5 f^2/12G$$

In simple tension (principal stress 200, 0, 0)

$$U = 200^2/6G$$

Equating gives $f = 200/\sqrt{4{\cdot}75}$

$= 91{\cdot}7\ \mathrm{N/mm^2}$

(5) *Maximum strain theory*

Equating the maximum strain in the complex and simple tension cases

$$(1/E)(2f - 0{\cdot}3f + 0{\cdot}3f/2) = 200E$$

or

$$f = 200/1{\cdot}85$$

$$108 \text{ N/mm}^2$$

Example 11, P. 61: The load on a bolt consists of an axial pull of 10 kN together with a transverse shear force of 5 kN. Estimate the diameter of the bolt required according to the:

1. Maximum principal stress theory,
2. Maximum shear stress theory,
3. Strain energy theory and
4. Shear strain energy theory.

Elastic limit in tension is 270 MPa, and a factor of safety of 3 is to be applied. Poisson's ratio = 0.3.

The permissible simple tensile stress is 270/(Factor of safety) $=90 \text{N/mm}^2$.

Let required diameter be d mm, then the applied stresses are

$$\sigma = \frac{10{,}000}{\pi d^2/4} = \frac{40{,}000}{\pi d^2} \text{ N/mm}^2$$

and

$$\tau = \frac{5000}{\pi d^2/4} = \frac{20{,}000}{\pi d^2} \text{ N/mm}^2 \text{ shear (Fig. 3·34),}$$

assuming uniform distribution over the cross-section.

(1) Maximum principal stress in bolt

$$= \tfrac{1}{2}\sigma + \tfrac{1}{2}\sqrt{(\sigma^2 + 4\tau^2)} \text{ (Para. 3.8: } \sigma_x = \sigma, \ \sigma_y = 0)$$

$$= \tfrac{1}{2}\cdot 40{,}000/\pi d^2 + \tfrac{1}{2}\sqrt{[(40{,}000/\pi d^2)^2 + 4(20{,}000/\pi d^2)^2]}$$

$$= (20{,}000/\pi d^2)[1 + \sqrt{(1+1)}]$$

$$= 48{,}290/\pi d^2$$

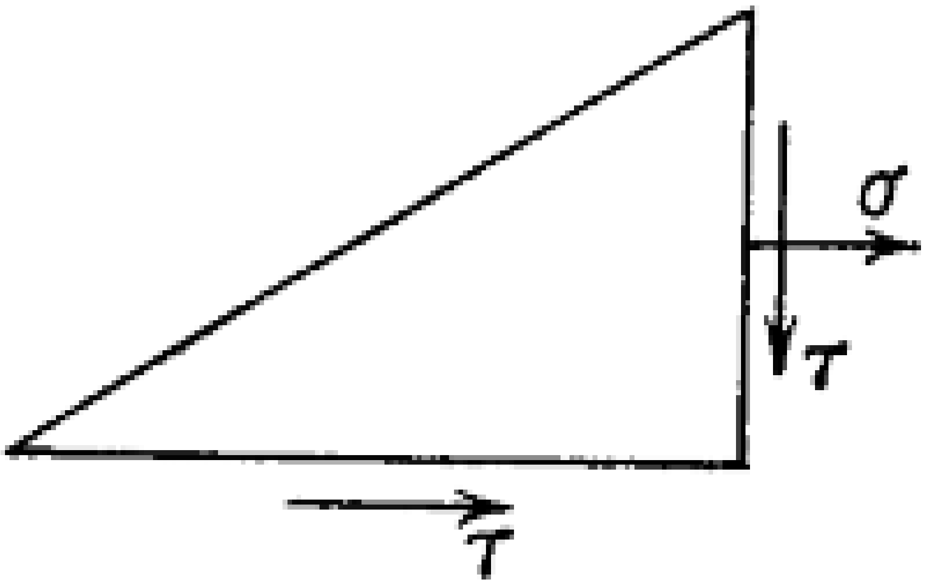

Fig. 3.34

Maximum stress in simple tension = 90
Equating to above gives

$$d = \sqrt{(48{,}290/90\pi)}$$
$$= 13{\cdot}1 \text{ mm.}$$

(2) Maximum shear stress $= \frac{1}{2}\sqrt{(\sigma^2 + 4\tau^2)}$ (Para. 3.10)

$$= 28{,}290/\pi d^2$$

$$= 45 \text{ in simple tension}$$

$$\therefore \quad d = \sqrt{(28{,}290/45\pi)}$$

$$= 14{\cdot}2 \text{ mm.}$$

(3) Principal stresses are $\frac{1}{2}\sigma \pm \frac{1}{2}\sqrt{(\sigma^2 + 4\tau^2)}$, 0, i.e.

$$48{,}290/\pi d^2, \quad 8290/\pi d^2, \quad 0$$

Strain energy $= (1/2E)(48{,}290^2 + 8290^2 + 2 \times 0{\cdot}3 \times 48{,}290 \times 8290)/\pi^2 d^4$

$$= 26{\cdot}4 \times 10^8/(2E\ \pi^2 d^4)$$

$$= 90^2/2E \text{ in simple tension}$$

$$\therefore \quad d = \sqrt[4]{(26{\cdot}4 \times 10^6/81\pi^2)}$$

$$= 13{\cdot}5 \text{ mm.}$$

(4) Shear strain energy

$$= (1/12G)[(48{,}290 + 8290)^2 + 8290^2 + 48{,}290^2]/\pi^2 d^4$$

$$= 90^2/6G \text{ in simple tension}$$

$$\therefore \quad d = \sqrt[4]{[(56\cdot 0 \times 10^6 \times 6)/(81\pi^2 \times 12)]}$$

$$= 13\cdot 7 \text{ mm.}$$

The **<u>largest diameter is 14.2 mm</u>** given by the **<u>Maximum Shear Stress Theory</u>**. This diameter value is chosen in the absence of any other factors to consider because it guards against failure by all the other theories.

ASSIGNMENT PROBLEMS

❖ **Due After One (1) Week.**

From the Problems from the Tutorial Assignment Sheet:

1. ***3D Stresses and Strains:*** Problems 1 and 4

2. ***Theories of Failure:*** Problems 9 and 11

Now

The End

for Real.