MEC 3352 – STRENGTH OF MATERIALS II

SOME WORKED EXAMPLES from Ryder

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THICK CYLINDERS AND SPHERES

Example 7, P. 271: The cylinder of a hydraulic ram is 6 cm internal diameter. Find the thickness required to withstand an internal pressure of 40 MPa, if the maximum tensile stress is limited to 60 MPa and the maximum shear stress to 50 MPa.

If D cm is the external diameter, then the maximum tensile stress is the hoop stress at the inside, i.e.

$$60 = [(D^2 + 36)/(D^2 - 36)]40 \quad \text{from (3)}$$

or $$3D^2 - 108 = 2D^2 + 72$$

$$D = \sqrt{180}$$

$$= 13{\cdot}43 \text{ cm}$$

The maximum shear stress is half the "stress difference" at the inside, i.e.

$$50 = [D^2/(D^2 - 36)]40 \quad \text{from (4)}$$

or $$5D^2 - 180 = 4D^2$$

$$D = \sqrt{180}$$

$$= 13{\cdot}43 \text{ cm} \quad \text{as before}$$

$$\text{Thickness} = \tfrac{1}{2}(13{\cdot}43 - 6)$$

$$= 3{\cdot}72 \text{ cm}$$

Example 8, Pp. 271 & 272: Find the ratio of thickness to internal diameter for a tube subjected to internal pressure when the ratio of pressure to maximum circumferential stress is 0.5.

Find the alteration of thickness of metal in such a tube 8 cm internal diameter when the pressure is 50 MPa. E = 200 GPa; ν = 0.304. (U. L.)

$$\hat{\sigma}_1 = [(d_2^2 + d_1^2)/(d_2^2 - d_1^2)]p_1 \quad \text{from (3)}$$

or $$(d_2^2 - d_1^2) = 0{\cdot}5(d_2^2 + d_1^2)$$

giving $$d_2/d_1 = \sqrt{3}$$

$$\text{Ratio} \quad \frac{\text{Thickness}}{\text{Internal diameter}} = \frac{d_2 - d_1}{2d_1} = \frac{\sqrt{3} - 1}{2}$$

$$= 0{\cdot}366$$

$$d_1 = 80 \text{ mm} \quad d_2 = 80\sqrt{3} = 138{\cdot}6 \text{ mm}$$

At inside

$$p = 50 \text{ N/mm}^2, \quad \sigma_1 = 100 \text{ N/mm}^2$$

$$\sigma_2 = pd_1^2/(d_2^2 - d_1^2) = 25 \text{ N/mm}^2$$

$$\text{Hoop strain} = (100 + 0{\cdot}304 \times 50 - 0{\cdot}304 \times 25)/E$$

$$= 112{\cdot}6/E$$

$$\text{Increase in internal diameter} = (112{\cdot}6/E)80 \text{ mm}$$

At outside

$p = 0, \quad \sigma_1 = 50 \text{ N/mm}^2$ (since $\sigma_1 - p = \text{constant} = 50$)

$\sigma_2 = 25 \text{ N/mm}^2$ as before.

$$\text{Hoop strain} = (50 - 0{\cdot}304 \times 25)/E$$
$$= 47{\cdot}4/E$$

$$\text{Increase in external diameter} = (47{\cdot}4/E)138{\cdot}6 \text{ mm}$$

$$\text{Decrease in thickness} = (112{\cdot}6 \times 80 - 47{\cdot}4 \times 138{\cdot}6)/(2 \times 200{,}000)$$
$$= 0{\cdot}006 \text{ mm}$$

Example 9, Pp. 272 & 273: The maximum stress permitted in a thick cylinder, radii 8 cm and 12 cm, is 20 MPa. The external pressure is 6 MPa. What internal pressure can be applied?

Plot curves showing the variation of hoop and radial stresses through the material.

$$\text{External pressure } 600 = -a + b/576 \quad \text{N/cm}^2$$

$$\text{Maximum stress} = \text{hoop stress at inside}$$

i.e.

$$2000 = a + b/256$$

Adding and solving

$$b = (2600 \times 256 \times 576)/832$$

and

$$a = 2000 - b/256 = 200$$

$$\begin{aligned}\text{Internal pressure} &= -a + b/256 \\ &= -200 + (2600 \times 576)/832 \\ &= 1600 \text{ N/cm}^2\end{aligned}$$

The constant difference between the hoop and radial stresses $= 400$ N/cm^2.

At 10 cm radius

$$\sigma_1 = a + b/400$$
$$= 200 + (2600 \times 256 \times 576)/(832 \times 400)$$
$$= 1350 \text{ N/cm}^2$$

and
$$p = \sigma_1 - 400 = 950 \text{ N/cm}^2$$

See Fig. 15.13 for a graphical representation of the stress variation.

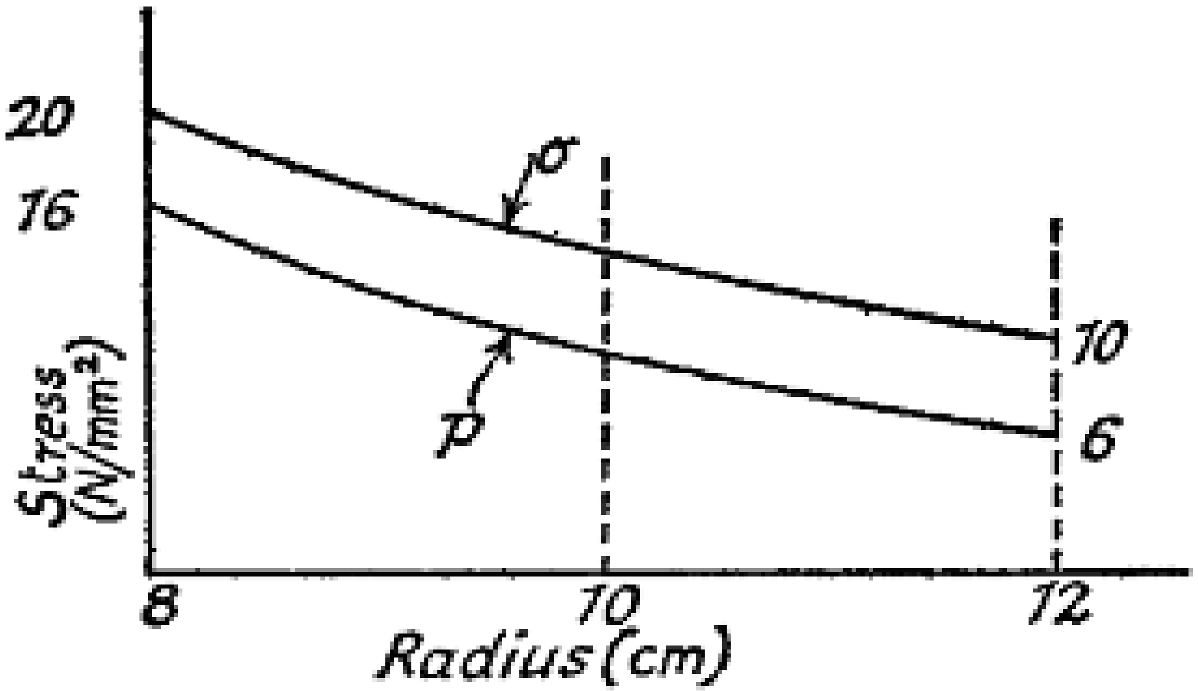


Fig. 15.13

Example 12, Pp. 278 & 279: A tube 4 cm outside by 6 cm outside diameters is to be reinforced by shrinking on a second tube of 8 cm outside diameter. The compound tube is to withstand an internal pressure of 50 MPa and the shrinkage allowance is to be such that the final maximum stress in each tube is to be the same. Calculate this stress and show on a diagram the variation of hoop stress in the two tubes. What is the initial difference of diameters before the shrinking on? E = 207 GPa.

Let p_0 be the common radial pressure due to shrinkage.

For the inner tube:

At the outside $\quad p_0 = -a + b/36$

At the inside $\quad 0 = -a + b/16$

from the general equations (8), Para. 15.8.

Subtract and solve

$$b = -[(36 \times 16)/(36 - 16)]p_0 = -(144/5)p_0$$

and

$$a = b/16 = -(9/5)p_0$$

$$\text{Maximum hoop stress} = a + b/16 = -(18/5)p_0 \qquad \text{(i)}$$

$$\text{Hoop stress at 6 cm diameter} = a + b/36 = -(13/5)p_0 \qquad \text{(ii)}$$

For the outer tube:

At the inside $\qquad p_0 = -a' + b'/36$

At the outside $\qquad 0 = -a' + b'/64$

Subtract and solve

$$b' = [(64 \times 36)/(64 - 36)]p_0 = (576/7)p_0$$

and

$$a' = b'/64 = (9/7)p_0$$

$$\text{Maximum hoop stress} = a' + b'/36 = (25/7)p_0 \qquad \text{(iii)}$$

$$\text{Hoop stress at 8 cm diameter} = a' + b'/64 = (18/7)p_0 \qquad \text{(iv)}$$

The lines marked "shrinkage stresses" on Fig. 15.15 are sketched from results (i) to (iv), the numerical value of p_0 being obtained later.

Stresses due to internal pressure:

At the inside $\quad 50 = -a'' + b''/16$

At the outside $\quad 0 = -a'' + b''/64$

Subtract and solve

$$b'' = [(64 \times 16)/(64 - 16)] \times 50 = (64/3) \times 50$$

and

$$a'' = 50/3$$

Hoop stresses:

4 cm diameter $\quad \sigma_1 = 50/3 + (64 \times 50)/(3 \times 16)$

$$= 83{\cdot}3 \text{ N/mm}^2 \qquad \text{(v)}$$

6 cm diameter $\quad \sigma_1 = 50/3 + (64 \times 50)/(3 \times 36)$

$$= 46{\cdot}4 \text{ N/mm}^2 \qquad \text{(vi)}$$

8 cm diameter $\quad \sigma_1 = 50/3 + (64 \times 50)/(3 \times 64)$

$$= 33{\cdot}3 \text{ N/mm}^2 \qquad \text{(vii)}$$

From results (v), (vi), and (vii) the line of "pressure" stresses is drawn on Fig. 15.15. The final resultant hoop stress in each tube is obtained by taking the algebraic sum of shrinkage and pressure stresses. It was pointed out in Para. 15.8 that the maximum stress occurs at the inside surface. Equating these values for the two tubes gives

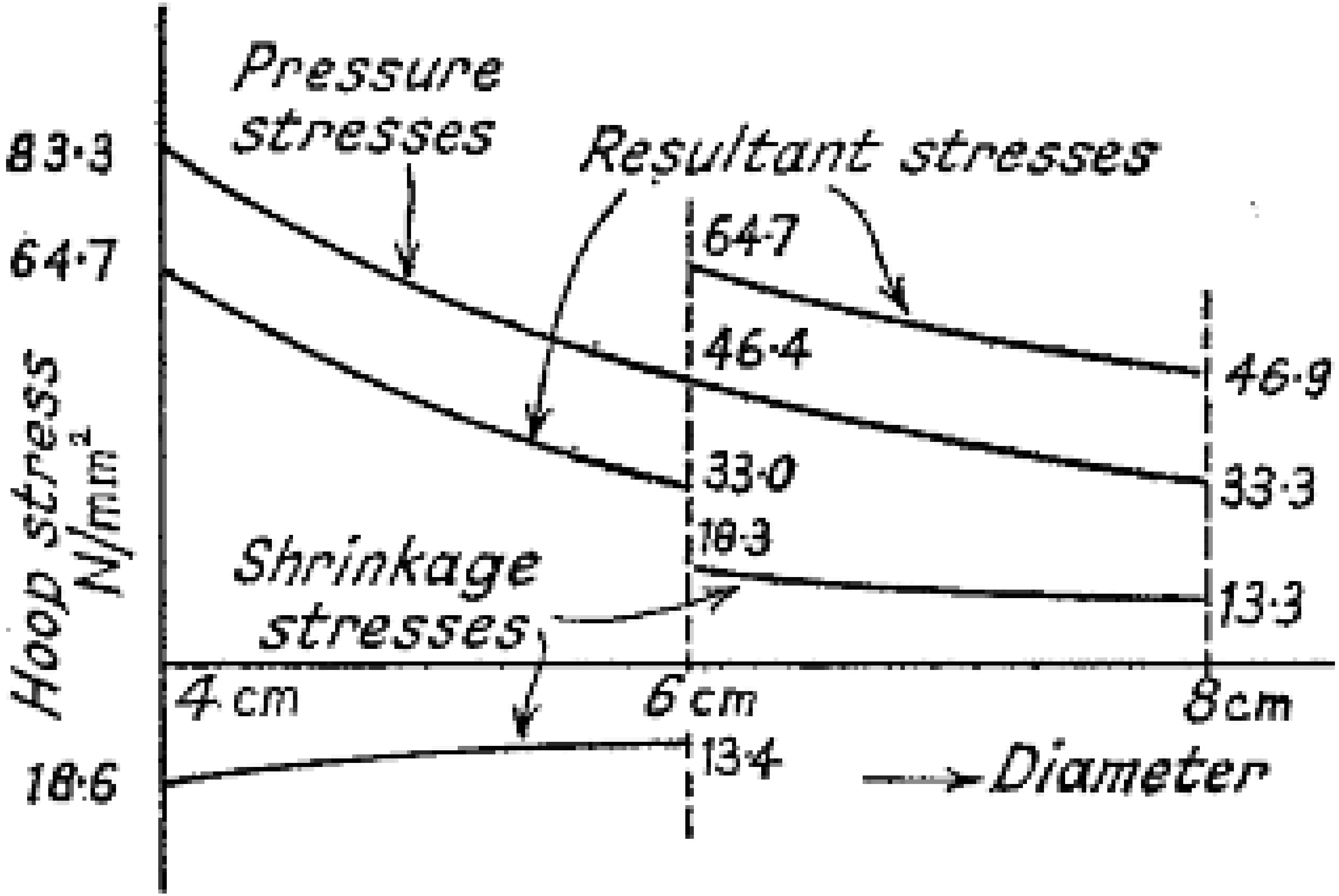
$$\text{(i)} + \text{(v)} = \text{(iii)} + \text{(vi)}$$

i.e.

$$-(18/5)p_0 + 83{\cdot}3 = (25/7)p_0 + 46{\cdot}4$$

or

$$p_0 = 36{\cdot}9 \times 35/251$$
$$= 5{\cdot}15 \text{ N/mm}^2$$



Pressure stresses
Resultant stresses
83·3
64·7
64·7
46·4
46·9
33·0
33·3
18·3
13·3
Shrinkage stresses
Hoop stress N/mm²
4 cm
6 cm
8 cm
18·6
13·4
Diameter

Fig. 15.15

Numerical value of maximum hoop stress

$$= \text{(iii)} + \text{(vi)} = 64{\cdot}7\ \text{N/mm}^2$$

the other values being shown in Fig. 15.15.

Initial difference of diameters at the common surface

$$\begin{aligned}
&= \text{difference of hoop strains} \times \text{diameter} \\
&= (1/E)(\text{difference of hoop shrinkage stresses}) \times \text{diameter} \\
&= [(13{\cdot}4 + 18{\cdot}3)/(207{,}000)] \times 60 \\
&= 0{\cdot}0092\ \text{mm}
\end{aligned}$$

Example 11.14, Rajput, p. 658: A thick spherical shell of 180 mm internal diameter is subjected to an internal fluid pressure of 24 MPa. If the permissible tensile stress is 120 MPa, find the thickness of the shell.

Solution:

Given:

Internal radius of the spherical shell:

$$d_1 = 0.18 \text{ m}$$

Internal fluid pressure, p (or σ_r) = 24 MPa

Permissible tensile stress, σ = 120 MPa

Thickness of the Shell:

We know that:

$$p = -a + \frac{b}{d^3} \qquad \text{(i)}$$

and

$$\sigma_\theta = a + \frac{b}{2d^3} \qquad \text{(ii)}$$

At $d = 0.18$ m, $p = 24$ MPa.

Therefore,

$$24 = -a + \frac{b}{0.18^3} = -a + 171.47b \qquad \text{(iii)}$$

At d = 0.18 m, $\sigma_\theta = 120$ MPa.

Therefore,

$$120 = a + \frac{b}{2 \times 0.18^3} = a + 85.73b \qquad \text{(iv)}$$

Adding (iii) and (iv) and solving,

$$b = 0.56 \quad \text{and} \quad a = 72$$

If d_2 is external radius, at $d = d_2$, $p = 0$

From (i):

$$0 = -72 + \frac{0.56}{d_2^3}$$

$d_2 = 0.198$ m = 198 mm.

Thickness, *t*:

$$t = \frac{d_2 - d_1}{2} = \frac{198 - 180}{2}$$

<u>***t* = 9 mm**</u>

Example 11.15, Rajput, pp. 658-9: A spherical shell of 120 mm internal diameter has to withstand an internal pressure of 30 MPa. If the permissible tensile stress is 80 MPa, find the thickness of the shell.

Solution:

Given:

Internal radius of the spherical shell:

$d_1 = 0.12$ m

Internal fluid pressure, p (or σ_r) = 30 MPa

Permissible tensile stress,σ = 80 MPa

Thickness of the Shell:

We know that:

$$p = -a + \frac{b}{d^3} \qquad \text{(i)}$$

and

$$\sigma_\theta = a + \frac{b}{2d^3} \qquad \text{(ii)}$$

At d = 0.12 m, p = 30 MPa.

Therefore,

$$30 = -a + \frac{b}{0.12^3} = -a + 578.70b \qquad \text{(iii)}$$

At d = 0.12 m, $\sigma_\theta = 80$ MPa.

Therefore,

$$80 = a + \frac{b}{2\times 0.12^3} = a + 289.35b \qquad \text{(iv)}$$

Adding (iii) and (iv) and solving,

$$b = 0.127 \quad \text{and} \quad a = 43.495$$

If d_2 is external radius, at $d = d_2$, $\quad p = 0$

From (i):

$$0 = -43.495 + \frac{0.127}{d_2^3}$$

$d_2 = 0.143$ m = 143 mm.

Thickness, t:

$$t = \frac{d_2 - d_1}{2} = \frac{143 - 120}{2}$$

<u>***t* = 11.5 mm**</u>

The End

But Wait:

Problems, Problems,

Problems…!!!

ASSIGNMENT PROBLEMS

- **Due After One (1) Week.**

From the Problems from the Tutorial Assignment Sheet:

1. ***Thick Cylinders and Spheres:*** Problems 17 and 20

Now
The End
for Real.