THE UNIVERSITY OF ZAMBIA

SCHOOL OF ENGINEERING

DEPARTMENT OF MECHANICAL ENGINEERING

UNIVERSITY EXAMINATIONS

MID-SEMESTER TEST

OCTOBER 2021

ME 3352 – STRENGTH OF MATERIALS II

SOLUTION MANUAL

Candidates are advised to read the following instructions carefully before starting writing:

1. This is a take-home test
2. This test is open book
3. Time allowed is 3 hours
4. Attempt ALL questions
5. All questions carry 25 marks each.
6. This question paper has two (2) printed pages including this cover

Q1.

A steel ring has been shrunk onto the outside of a solid steel disc. The interface radius is 250 mm and the outer radius of the assembly is 356 mm. If the pressure between the ring and the disc is not to fall below 34.5 MPa, and the circumferential stress at the inside of the ring must not exceed 207 MPa,

(a) Determine the maximum speed at which the assembly can be rotated. [15 marks]
(b) What is then the stress at the centre of the disc? [10 marks]
Take $\rho = 7.75\text{Mg/m}^3$ and $\nu = 0.28$.

Solution:

Given:

Compound ring/disc assembly
Interference radius, r_{interf} = 250 mm
Outer radius of assembly, r_{o} = 356 mm
Interference pressure, $p_{\text{interf}} \geq 34.5$ MPa
Ring inside stress, $\sigma_{i_{\text{ring}}} \geq 207$ MPa
$\rho = 7.75\text{Mg/m}^3$
$\nu = 0.28$

(a) Determine the maximum speed at which the assembly can be rotated. [15 marks]

For the ring, when r = 356 mm, $\sigma_r = 0$, and when r = 250 mm, $\sigma_r = -34.5$ MPa; therefore

$$0 = A - \frac{B}{0.126} - \left(\frac{3+0.28}{8} \times 7.75\left(10^3\right)\omega^2 \times 0.126\right) - \left(34.5\left(10^6\right)\right)$$

$$= A - \frac{B}{0.0625} - \left(\frac{3+0.28}{8} \times 7.75\left(10^3\right)\omega^2 \times 0.0625\right)$$

from which

$$B = \left(4280 + 0.025\omega^2\right)10^3 \quad \text{and} \quad A = \left(34{,}000 + 0.6\omega^2\right)10^3$$

Also when r = 250 mm, σ_θ must not exceed 207 MPa; therefore

$$207\left(10^6\right) = A + \frac{B}{0.0625} - \left[\frac{1+(3\times 0.28)}{8} \times 7.75\left(10^3\right)\omega^2 \times 0.0625\right]$$

or

$$207\left(10^3\right) = 34{,}000 + 0.6\omega^2 + 68{,}500 + 0.4\omega^2 - 0.111\omega^2$$

After simplifying,

$$\omega^2 = 117{,}500\,,$$

$$\underline{\omega = 343\,\text{rad/s}}$$

and

$$N = 343 \times \frac{60}{2\pi}$$

N = 3,280 rpm ***Answer.***

(b) What is then the stress at the centre of the disc? [10 marks]

For the solid disc, using constant C and D, as shown previously, D must be zero; therefore

$$\sigma_r = C - \frac{3+\nu}{8}\rho\omega^2 r^2$$

When r = 250 mm, σ_r = –34.5 MPa; therefore

$$-34.5\left(10^6\right) = C - \left(\frac{3+0.28}{8} \times 7.75\left(10^3\right) \times 117{,}500 \times 0.0625\right)$$

and

$$C = \left(-34.5\left(10^6\right)\right) + \left(23.3\left(10^6\right)\right) \quad = \quad -11.2 \times 10^6 \quad = \quad \underline{-11.2}\,\text{MPa}$$

But at the centre of the disc, r = 0 mm, $\sigma_r = \sigma_\theta = C$; therefore

$\sigma_r = \sigma_\theta = -11.2$ MPa ***Answer.***

Q2.

A mild steel shaft of 50 mm diameter is subjected to a bending moment of 1.9 kNm. If the yield point of the steel in simple tension is 200 MPa, find the maximum torque that can also be applied according to:

(a) The maximum principal stress theory. [09 marks]

(b) The maximum shear stress theory and [08 marks]

(c) The shear strain energy theory. [08 marks]

Solution:

Given:

Diameter, d = 50 mm

Bending moment, M = 1.9 kNm

Yield stress in tension, σ_{yield} = 200 MPa

The maximum bending stress occurs at the surface of the shaft and is given by

$$\sigma_x = \frac{32M}{\pi d^3} = \frac{32}{\pi} \times \frac{1900}{125 \times 10^{-6}} = 155 \text{ MN/m}^2$$

The maximum shear stress at the surface is

$$\tau_{xy} = \frac{16T}{\pi d^3} = \frac{16}{\pi} \times \frac{T}{125 \times 10^{-6}} = 40.7 \times 10^3 T$$

(a) Maximum Principal Stress Theory

$$\sigma_1 = \frac{\sigma_x}{2} + \frac{1}{2}\sqrt{\left(\sigma_x{}^2 + 4\tau_{xy}{}^2\right)} = 200 \times 10^6$$

Therefore:

$$\frac{155 \times 10^6}{2} + \frac{1}{2}\sqrt{\left[\left(155 \times 10^6\right)^2 + \{4(40.7 \times 10^3 T)^2\}\right]} = 200 \times 10^6$$

$$\sqrt{[(155)^2 + \{4(0.0407T)^2\}]} = 245$$

$$0.00166T^2 = 9000$$

T* = 2.33 kNm** ***Answer.

(b) Maximum Shear Stress Theory

$$\tau_{\max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{200 \times 10^6 - 0}{2}$$

Therefore:

$$\frac{\sqrt{(\sigma_x{}^2+4\tau_{xy}{}^2)}}{2} = 200 \times 10^6$$

$$\frac{\sqrt{(\sigma_x{}^2+4\tau_{xy}{}^2)}}{2} = 200 \times 10^6$$

$$155^2 + 4(0.0407T)^2 = 200^2$$

$$0.00166T^2 = 4000$$

T* = 1.55 kNm** ***Answer.

(c) ***Shear Strain Energy Theory***

$$\sigma_1{}^2 + \sigma_2{}^2 - \sigma_1\sigma_2 = (200 \times 10^6)^2$$

Putting $\sigma_x{}^2 + 4\tau_{xy}{}^2 = A$,

$$\frac{1}{4}\left(\sigma_x + \sqrt{A}\right)^2 + \frac{1}{4}\left(\sigma_x - \sqrt{A}\right)^2 - \frac{1}{4}\left(\sigma_x + \sqrt{A}\right)\left(\sigma_x - \sqrt{A}\right) = (200 \times 10^6)^2$$

$$\frac{1}{4}\sigma_x{}^2 + 2\sigma_x\sqrt{A} + A + \sigma_x{}^2 - 2\sigma_x\sqrt{A} + A - \sigma_x{}^2 + A = (200 \times 10^6)^2$$

Simplifying gives

$$\sigma_x{}^2 + 3\tau_{xy}{}^2 = (200 \times 10^6)^2$$

$$155^2 + 3(0.00166T^2)^2 = 200^2$$

$$0.00166T^2 = 5330$$

Therefore

T* = 1.79 kNm** ***Answer

Q3.

A torque of 70 Nm is to be transmitted over a length of 1 metre in a piece of machinery. Two sections, both made of carbon steel, are being considered for the shaft design – a solid shaft of 25 mm x 25 mm square section and a hollow shaft of 25 mm x 25 mm square outside with a thickness of 1 mm.

(a) Determine the maximum stress induced in each section. [10 marks]

(b) Calculate the angle of twist for each section. [10 marks]

(c) Calculate the torsional spring rate for each section. [Hint: the spring rate is defined as torque divided by the angle of twist.] [03 marks]

(d) Which section would you recommend for use? Explain your choice. [02 marks]

Solution:

It is important to take the two sections side by side.

Given:

Section I – Solid Square Shaft	**Section II – Hollow Square Shaft**
• Torque, T = 70 Nm • Length, L = 1 m • Section: Solid, 25mm × 25 mm • Assume Shear modulus, G = 77 GPa	• Torque, T = 70 Nm • Length, L = 1 m • Section: Hollow, 25mm × 25 mm outside • Thickness, t = 1 mm all round • Assume Shear modulus, G = 77 GPa

(a) Determine the maximum stress induced in each section. [10 marks]

Section I – Solid Square Shaft	**Section II – Hollow Square Shaft**
$\tau_{\max} = \frac{T}{\alpha t A}$ where α = 0.208, But $t = a = b = a$ $\tau_{\max} = \frac{T}{0.208a^3} = \frac{4.81T}{a^3}$ $= \frac{4.81 \times 70}{0.024^3}$ $= 24.356 \times 10^6$ N/m² $\boldsymbol{\tau_{\max} = 24.356}$ **MN/m²** ***Answer***	$\tau_{\max} = \frac{T}{2tA_m}$ The median square is 24mm × 24mm $\tau_{\max} = \frac{70}{2 \times 0.001 \times 0.024 \times 0.024}$ $\tau_{\max} = \frac{70}{2 \times 0.001 \times 0.024 \times 0.024}$ $= 60.764 \times 10^6$ N/m² $\boldsymbol{\tau_{\max} = 60.764}$ **MN/m²** ***Answer***

(b) Calculate the angle of twist for each section. [10 marks]

Section I – Solid Square Shaft	**Section II – Hollow Square Shaft**
$\phi = \frac{TL}{\beta b a^3 G}$	$\phi = \frac{TL}{4A_m^2 G} \oint \frac{ds}{t}$ The median square is 24mm × 24mm

where α = 0.141, But $t = a = b = a$ Assume G for carbon steel = 77 GPa $\phi = \frac{TL}{0.141ba^3G} = \frac{7.10TL}{a^4G}$ $= \frac{7.10 \times 70}{0.024^3 \times 77 \times 10^9}$ $= 0.019$ rad **$\phi = 0.019$ rad** ***Answer*** or **$\phi = 1.09^0$** ***Answer***	Assume G for carbon steel = 77 GPa $\phi = \frac{70}{4 \times 0.024^2 \times 0.024^2 \times 77 \times 10^9} \times \left[2\left(\frac{0.024}{0.001}\right) + 2\left(\frac{0.024}{0.001}\right)\right]$ $= 0.000685 \times [96]$ $= 0.066$ rad. **$\phi = 0.066$ rad** ***Answer*** or **$\phi = 3.78^0$** ***Answer***

(c) **Calculate the torsional spring rate for each section. [Hint: the spring rate is defined as torque divided by the angle of twist.]** **[03 marks]**

Section I – Solid Square Shaft	**Section II – Hollow Square Shaft**
$k = \frac{T}{\phi}$ $= \frac{70}{0.019}$ **$k = 3,684$ N/rad** ***Answer*** or $k = \frac{70}{1.09}$ **$k = 64.220$ N/deg.** ***Answer***	$k = \frac{T}{\phi}$ $= \frac{70}{0.066}$ **$k = 1,060$ N/rad** ***Answer*** or $k = \frac{70}{3.78}$ **$k = 18.518$ N/deg.** ***Answer***

(d) **Which section would you recommend for use? Explain your choice.** **[02 marks]**

The solid shaft is the preferred one because:

(i) It suffers the lesser stress

(ii) It undergoes lesser twist

(iii) And it has a superior spring rate.

Q4.

A laminated thick-walled hydraulic cylinder was fabricated by shrink-fitting a steel jacket having an outside diameter of 300 mm onto a steel tube having an inside diameter of 100 mm and an outside diameter of 200 mm. the interference was 0.15 mm.

(a) Determine the interfacial contact pressure. [05 marks]

(b) Determine the maximum tensile stress in the laminated cylinder resulting from the shrink fit. [05 marks]

(c) Determine the maximum tensile stress in the laminated cylinder after an internal pressure of 200 MPa is applied. [10 marks]

(d) Compare the stress distribution in the laminated cylinder with the stress distribution in a uniform cylinder for an internal pressure of 200 MPa. [05 marks]

Solution:

Given:

Laminated thin-walled cylinder.

Inner tube:

Outside diameter, d_o = 200 mm
Inside diameter, d_i = 100 mm

Jacket or Outer tube:

Outside diameter, d_o = 300 mm
Inside diameter, d_i = 100 mm
Interference

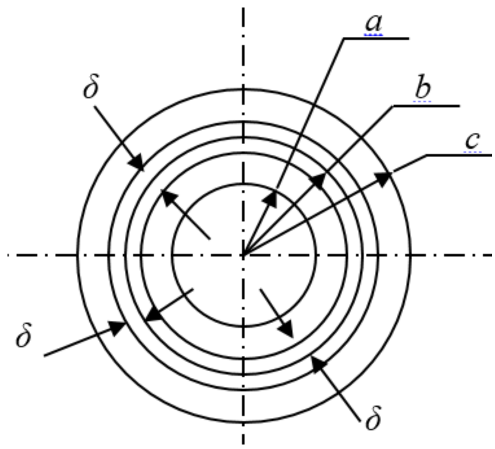
Interference = 0.15 mm
E = 200 GPa

(a) Determine the interfacial contact pressure. [05 marks]

The interfacial contact pressure (shrink pressure) can be determined by using

$$p_s = \frac{EI(c^2 - ab^2)(b^2 - a^2)}{4b^3(c^2 - a^2)}$$

where I is the interference = 0.15 mm.



Thus:

$$p_s = \frac{200 \times 10^9 \times (0.00015)(0.150^2 - 0.100^2)(0.100^2 - 0.050^2)}{4 \times 0.100^3(0.150^2 - 0.050^2)}$$

$$= 35.16 \times 10^6$$

$\boldsymbol{p_s = 35.16}$ **MPa** ***Answer.***

(b) What is the maximum tensile stress in the laminated cylinder due to the shrink fit?
[05 marks]

The maximum tensile stress resulting from this shrink pressure will be the tangential stress σ_t that occurs at the inside surface of the jacket. This stress can be determined by using the expression below for the case of internal pressure only. Thus:

$$\sigma_t = \frac{a^2 p_i - b^2 p_o}{b^2 - a^2} + \frac{a^2 b^2 (p_i - p_o)}{(b^2 - a^2)\rho^2}$$

where for the jacket:

a = 100 mm
b = 150 mm
ρ = 100 mm on the inside surface
p_i = 35.16 MPa
p_o = 0

giving:

$$\sigma_t = \frac{0.100^2 \times 35.16 \times 10^6}{0.150^2 - 0.100^2} \times \left(1 + \frac{0.150^2}{0.100^2}\right)$$

$$= 91.4 \times 10^6$$

Therefore, maximum tensile stress, which will be the principal stress will be

$\boldsymbol{\sigma_1}$ **= 91.4 MPa** ***Answer.***

(c) Determine the maximum tensile stress in the laminated cylinder and were it occurs after an internal pressure of 200 MPa is applied. [10 marks]

Stresses in the laminated cylinder after the internal pressure is applied will result from both the shrink pressure and the applied pressure. Again, the maximum tensile stress will be a tangential stress; however, it may occur either at the inside surface of the jacket or at the inside surface of the tube, depending on the magnitudes of the stresses associated with the two pressure loadings. Reduced forms of the equation used in (b) above can be used for each of the loadings.

For the tube:

$$\sigma_t = \sigma_t\ (\text{tube interference}) + \sigma_t\ (\text{laminated cylinder due to } p_i)$$

$$\sigma_t = \left\{\frac{0.100^2 (35.16 \times 10^6)}{0.100^2 - 0.050^2} \times \left(1 + \frac{0.050^2}{0.050^2}\right)\right\} + \left\{\frac{0.050^2 (200 \times 10^6)}{0.150^2 - 0.050^2} \times \left(1 + \frac{0.150^2}{0.050^2}\right)\right\}$$

$$= -93.8 \times 10^6 + 250 \times 10^6$$

$$= 156.2 \times 10^6$$

$$\boldsymbol{\sigma_{t_{\text{tube}}} = 156.2\ \text{MPa}}$$

For the jacket:

$$\sigma_t = \sigma_t \text{ (tube interference)} + \sigma_t \text{ (laminated cylinder due to } p_i)$$

$$\sigma_t = \left\{\frac{0.100^2(35.16 \times 10^6)}{0.150^2 - 0.100^2} \times \left(1 + \frac{0.150^2}{0.100^2}\right)\right\} + \left\{\frac{0.050^2(200 \times 10^6)}{0.150^2 - 0.050^2} \times \left(1 + \frac{0.150^2}{0.100^2}\right)\right\}$$

$$= 91.4 \times 10^6 + 81.3 \times 10^6$$

$$= 172.7 \times 10^6$$

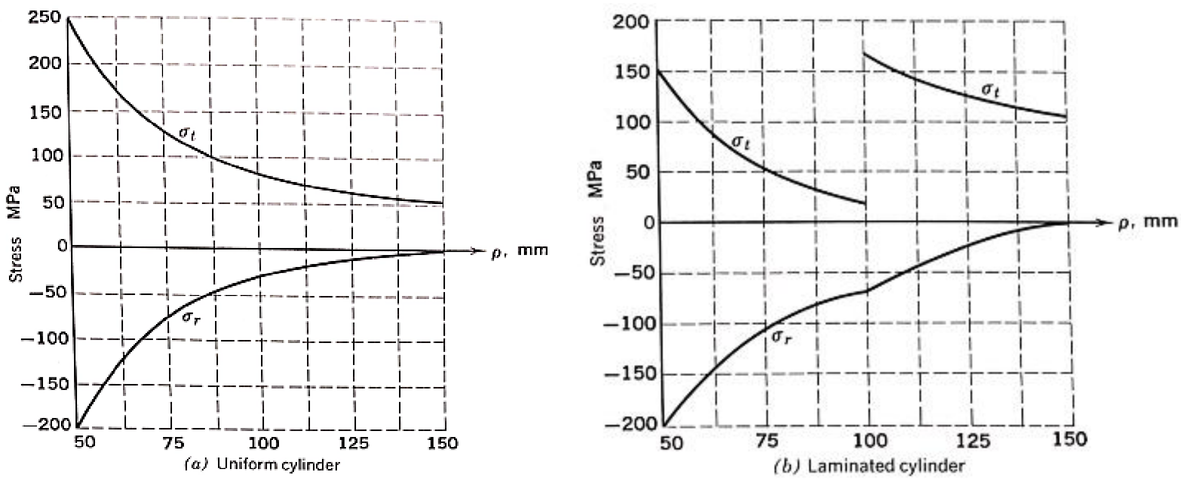
$$\boldsymbol{\sigma_{t_{\text{jacket}}} = 172.7 \text{ MPa}}$$

Therefore, taking the larger of the two values, the one for the jacket:

$\boldsymbol{\sigma_{\max} = 172.7 \text{ MPa}}$ ***Answer.***

(d) Compare the stress distribution in the laminated cylinder with the stress distribution in a uniform cylinder for an internal pressure of 200 MPa. [05 marks]

Radial and tangential stress distributions in the laminated cylinder and in the uniform cylinder for an internal pressure of 200 MPa are as shown in the figure below.



END OF MEC 3352 TERM II TEST
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