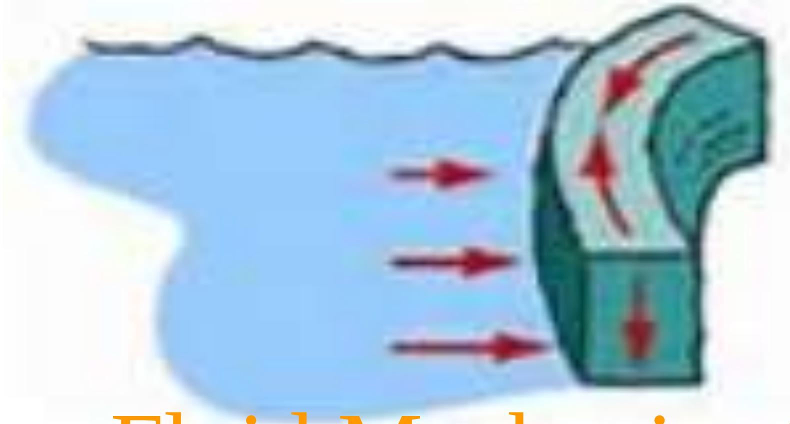
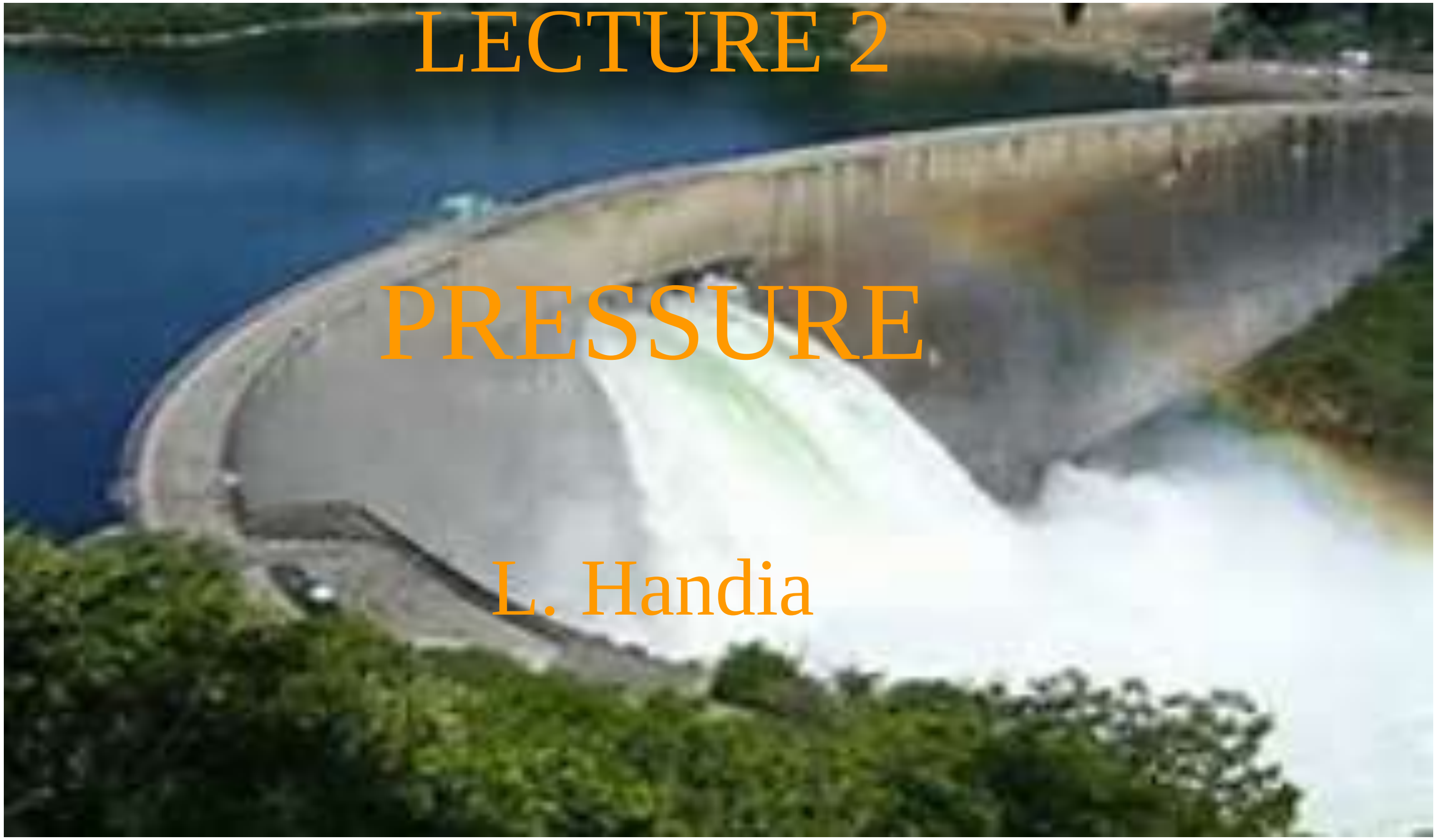


Fluid Mechanics CEE 3311



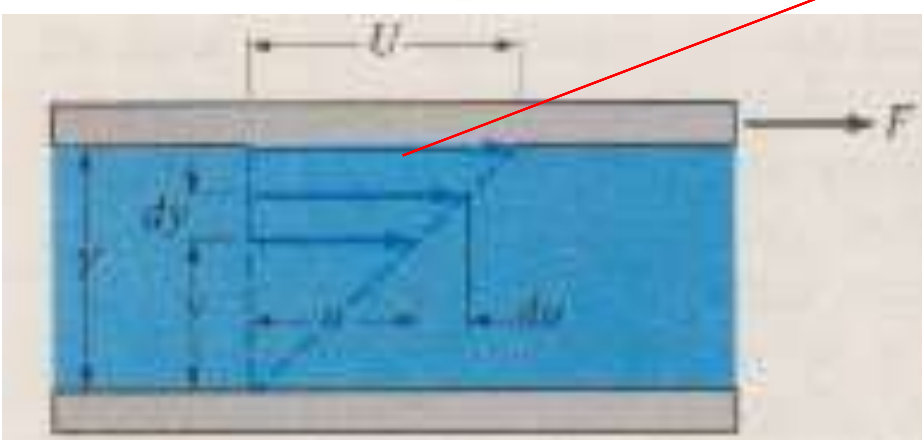
LECTURE 2

PRESSURE

L. Handia

FLUID STATICS

- There are no shear stresses in fluids at rest ($\Delta F_t = 0$);
- hence only **normal pressure forces** ($\Delta F_n \neq 0$) are present.



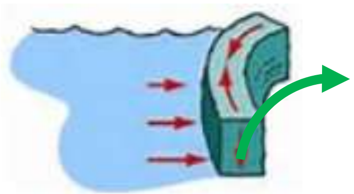
When F= 0, then velocity is zero. From Newton's Equation of Viscosity

$$\tau = \frac{F}{A} = \mu \frac{du}{dy}$$

$$\tau = \frac{0}{A} = \mu \frac{0}{dy} = 0$$

FLUID STATICS

- **Normal forces** produced by static fluids are often very important:



  - water pushing against dam wall (they tend to overturn concrete dams)

  - burst pressure vessels
  - break lock gates on canals
- Obviously, to design such facilities, we need to be able to compute the magnitudes and locations of normal pressure forces.

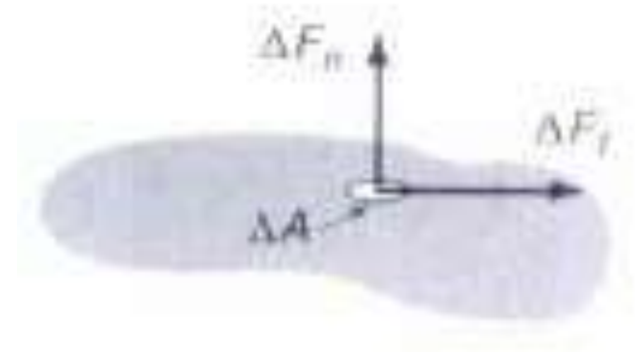
FLUID STATICS

- Note that normal pressure forces alone can occur in a moving fluid if the fluid is moving in bulk *without deformation*, i.e., as if it were solid or rigid.
- The average pressure intensity p is defined as the force exerted on a unit area.

FLUID STATICS

- If F represents the total normal pressure force on some finite area A, while dF represents the force on an infinitesimal area dA, the pressure is

$$p = \frac{dF}{dA}$$

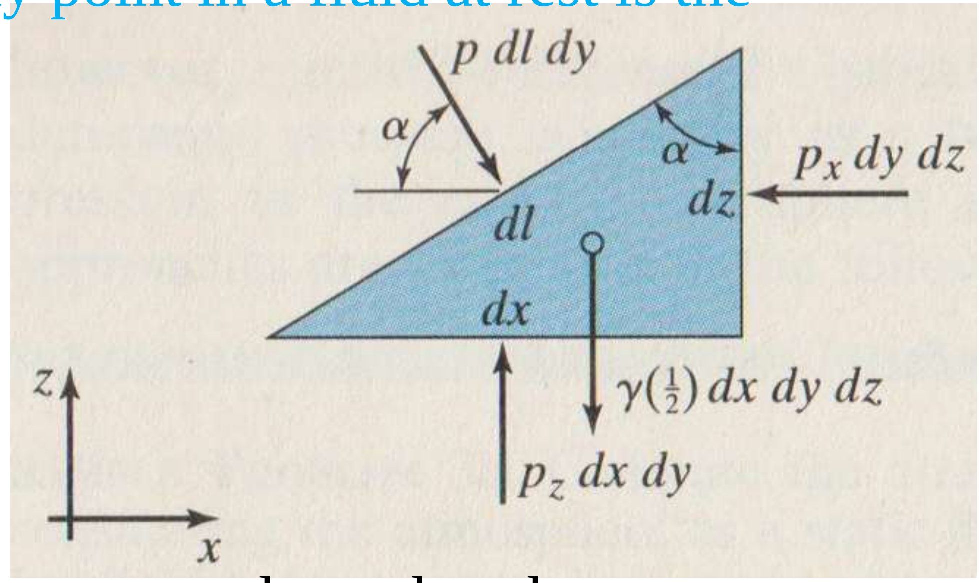


- If the pressure is uniform over the total area, then p = F /A

Pressure

Pressure at a point the same in all directions

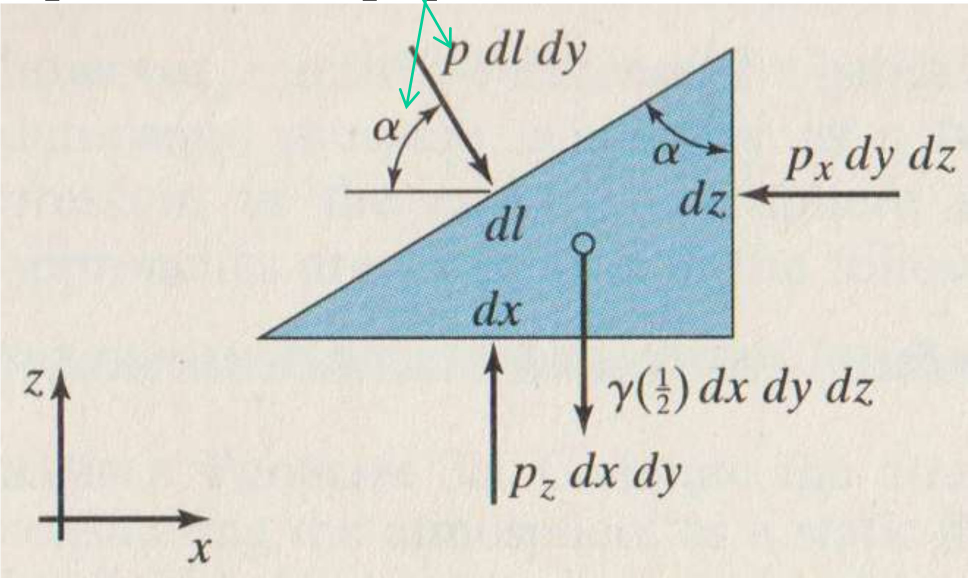
- No tangential stresses can exist in a fluid at rest, and the only forces between adjacent surfaces are pressure forces ***normal*** to the surfaces.
- Therefore the pressure at any point in a fluid at rest is the same in every direction.
- This can be proved by reference to Fig., showing a very small wedge-shaped element of fluid at rest whose thickness perpendicular to the plane of the paper is constant and equal to dy.



Pressure

Pressure at a point the same in all directions

- Let p be the average pressure in any direction in the plane of the paper, let α be defined as shown, and let p_x and p_z be the average pressures in the horizontal and vertical directions
- The forces acting on the element of fluid, with the exception of those in the y direction on the two faces parallel to the plane of the paper, are shown in the diagram.

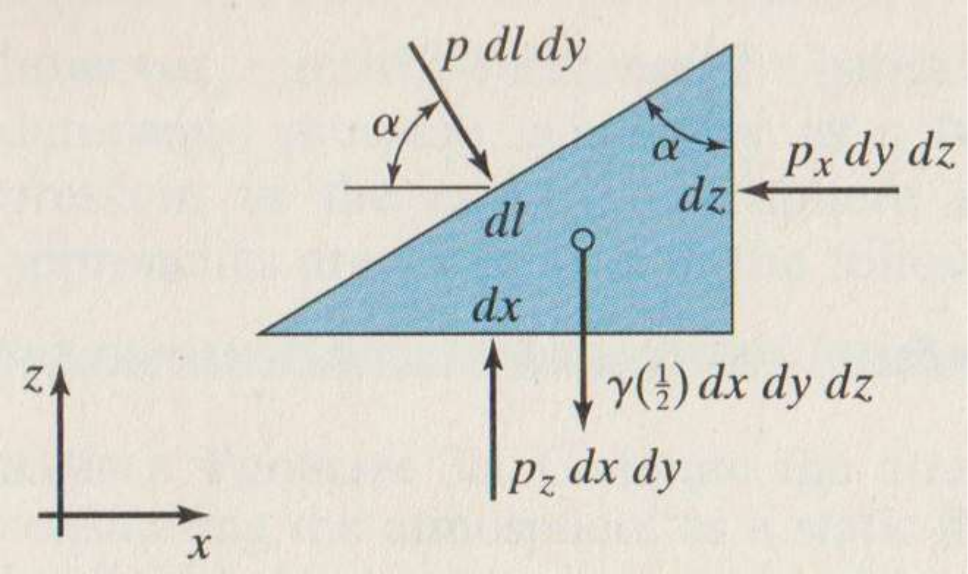


Pressure

Pressure at a point the same in all directions

➢For our purpose, forces in the y direction need not be considered because they cancel.

➢Since the fluid is at rest, *no tangential forces* are involved. As this is a condition of equilibrium, ***the sum of the force components*** on the element ***in any direction*** must be equal to **zero**.



Pressure at a point the same in all directions

- Writing such an equation for the components in the x direction

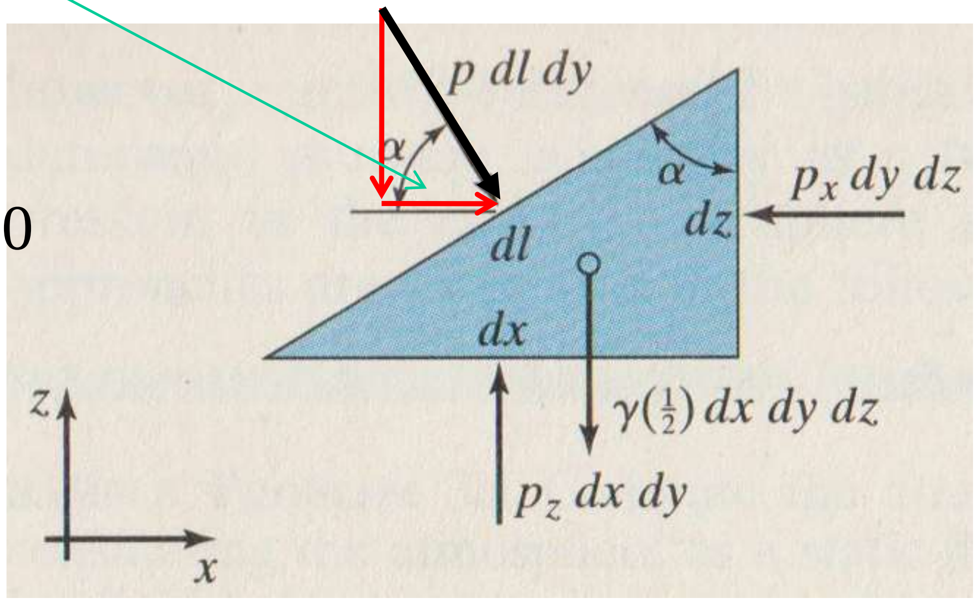
$$p\ dl\ dy \cos\alpha - p_x\ dy\ dz = 0.$$

- Since $dz = dl \cos\alpha$, it follows from above eqn. that $p = p_x$ as shown below:

$$p\ dl\ dy \cos\alpha - p_x\ dy\ dz = 0$$

$$p\ dl\ dy \cos\alpha - p_x\ dy\ dl \cos\alpha = 0$$

$$p = p_x$$



Pressure

Pressure at a point the same in all directions

- Similarly, summing forces in the z direction gives

$$p_z\, dx\, dy - p\, dl\, dy \sin\alpha - \tfrac{1}{2}\gamma\, dx\, dy\, dz = 0$$

- The third term (weight) is of higher order than the other two terms and so may be neglected. It follows from this that $p = p_z$ as shown below

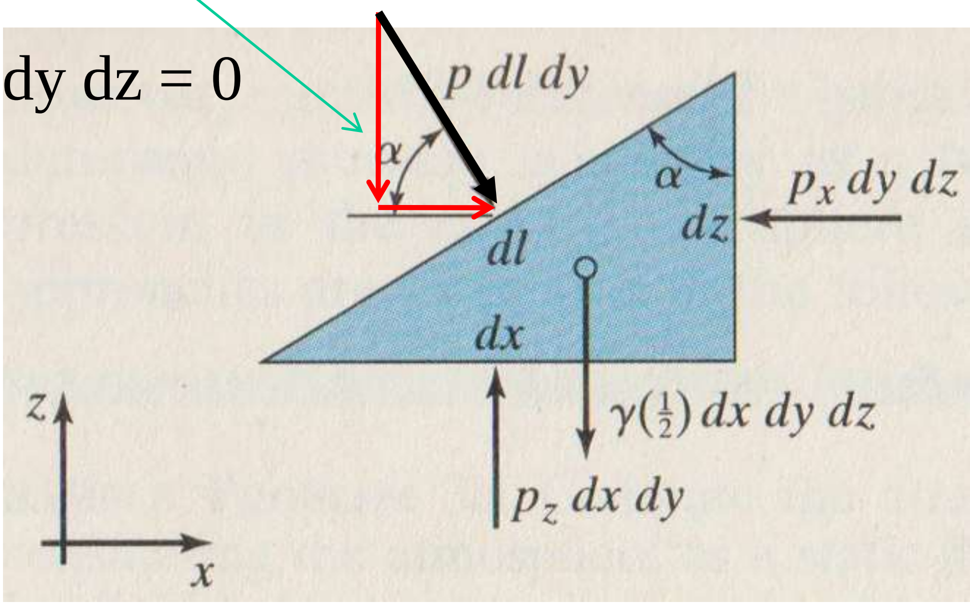
$$p_z\, dx\, dy - p\, dl\, dy \sin\alpha - \tfrac{1}{2}\gamma\, dx\, dy\, dz = 0$$

$$p_z\, dx\, dy - p\, dl\, dy \sin\alpha - 0 = 0$$

Since $dx = dl \sin\alpha$.

$$p_z\, dl \sin\alpha\, dy - p\, dl\, dy \sin\alpha = 0$$

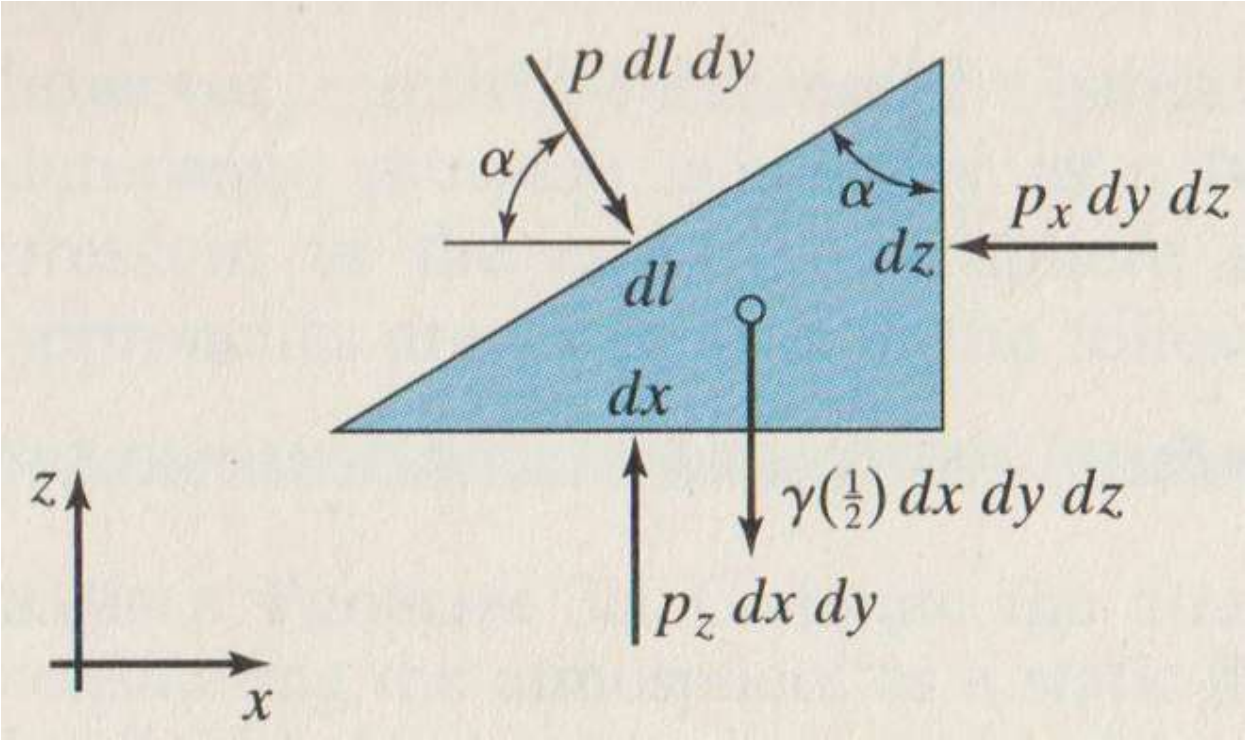
$$p = p_z$$



Pressure

Pressure at a point the same in all directions

- It can also be proved that $p = p_y$ by considering a three-dimensional case.
- The results are ***independent of α***; ***hence*** the pressure at any point in a fluid at rest is the same in all directions (i.e., $p_x = p_y = p_z$)



Pressure

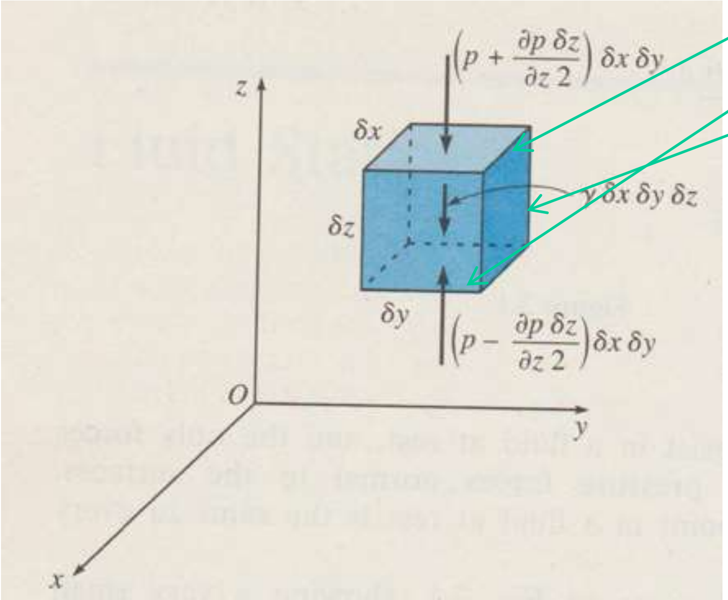
Pressure at a point the same in all directions

- **Pascal's Law** states that the pressure at a point in a fluid at rest is the same in all directions ***independent of the orientation*** of the surface ***around*** that point.

Pressure

Variation of pressure in a static fluid

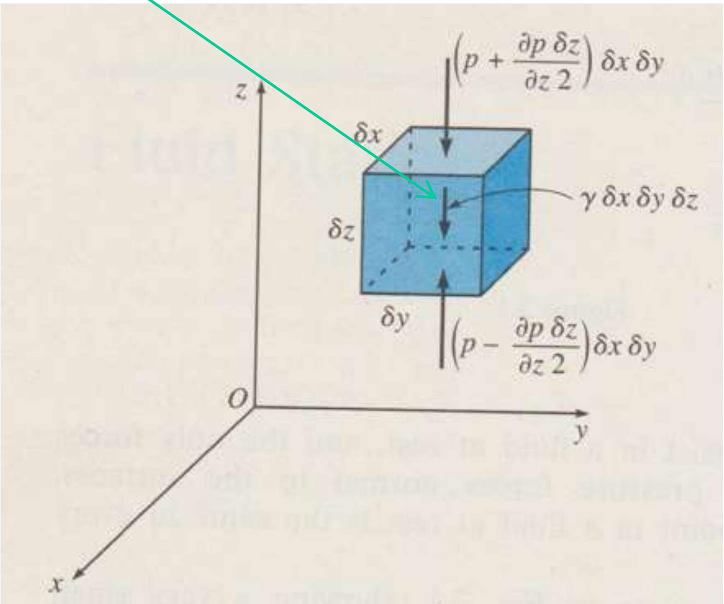
- Consider the differential element of static fluid shown in Fig below.
- Since the element is ***very small***, we can ***assume*** that the density of the fluid within the element is constant.
- Assume that the pressure at the center of the element is p and that the dimensions of the element are δx, δy and δz.



Pressure

Variation of pressure in a static fluid

- The forces acting on the fluid element in the vertical direction are
  - the ***body force***, the action of gravity on the mass within the element, and
  - the ***surface forces***, transmitted from the surrounding fluid and acting at right angles against the top, bottom, and sides of the element.



Pressure

Variation of pressure in a static fluid

➢Because the fluid is at *rest*, the element *is in equilibrium* and the *summation of forces* acting on the element in any direction must be *zero*.

➢If forces are summed in the *horizontal* direction, that is, x or y, the *only forces* acting are the pressure forces on the *vertical faces* of the element.

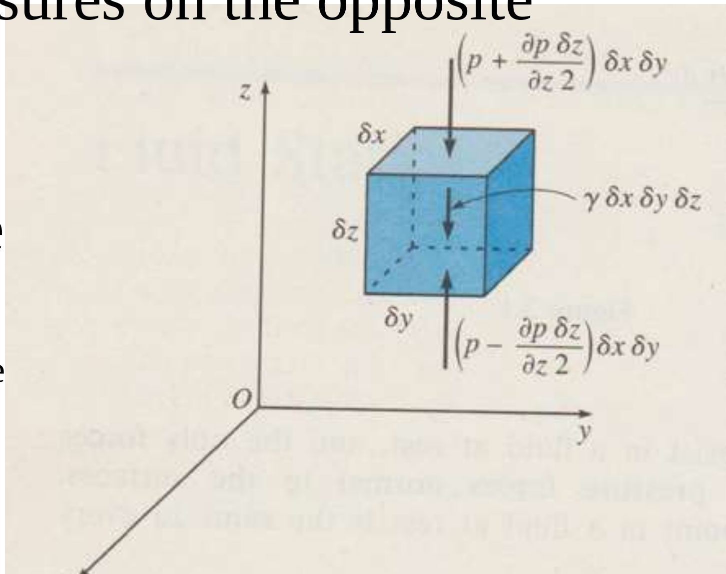
➢To satisfy $\sum F_x = 0$ and $\sum F_y = 0$, the pressures on the opposite vertical faces must be equal. Thus

$$\frac{\partial p}{\partial x} = \frac{\partial p}{\partial y} = 0$$

for the case of the fluid at rest

What is δp/δx in words?

(i.e., there is no change in pressure in the x-direction and also in the y-direction, the horizontal direction)



Variation of pressure in a static fluid

Summing forces in the vertical direction and setting the sum equal to zero

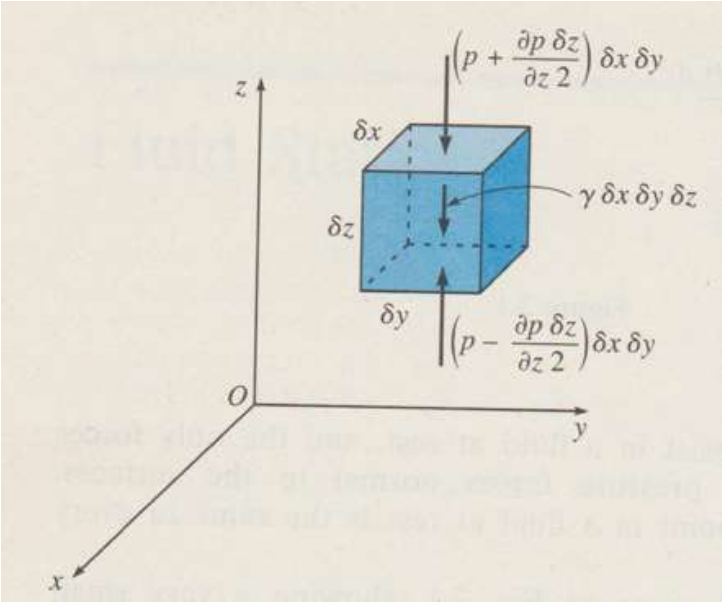
$$\sum F_z = \left(p - \frac{\partial p}{\partial z}\frac{\delta z}{2} \right)\delta x \delta y - \left(p + \frac{\partial p}{\partial z}\frac{\delta z}{2} \right)\delta x \delta y - \gamma \delta x \delta y \delta z = 0$$

Negative because it is opposite direction to z direction

Rate of change of p with respect to z direction multiplied by distance over which change occurs i.e., from the centre to the bottom surface= δz/2

Positive because it is in the same direction to z direction

Note that for a force, it is negative and positive whenever it is in the opposite and same direction, respectively to z direction



This results in $\partial p / \partial z = -\gamma$, which, since p is independent of x and y, can be written as $\dfrac{dp}{dz} = -\gamma$

Pressure

Variation of pressure in a static fluid

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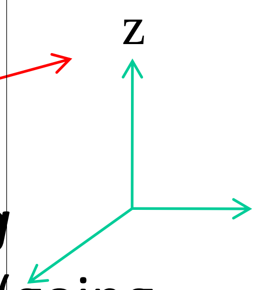
$$\frac{dp}{dz} = -\gamma$$



- This is the general expression that relates *variation of pressure* in a static fluid to *vertical position*.
- The minus sign indicates that ***as z gets larger (increasing elevation), the pressure gets smaller***. Or as z gets smaller (going down), the pressure gets larger (scuba diving suit, see next slide)
- To evaluate pressure variation in a fluid at rest, one must integrate the above equation between appropriately chosen limits.
- For incompressible fluids (γ = constant. This is earlier assumption), above equation can be integrated directly. For compressible fluids, however, γ must be expressed algebraically as a function of z or p if one is to determine pressure accurately as a function of elevation.

Google

Why should we wear diving suits when scuba diving?

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As the depth of the sea increases, the pressure increases since pressure increases with depth. The pressure exerted by water deep under the sea is much greater than at the sea level. Hence, the deep sea divers wear special suits which **protect them from extreme pressure of water**. These suits are called diving suits. 3 Jul 2018

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Pressure

Variation of pressure in a static fluid

$$\frac{dp}{dz} = -\gamma; dp = -\gamma dz; \int_{p_1}^{p} dp = -\gamma \int_{z_1}^{z} dz$$

Where subscript 1 indicates conditions at a reference elevation. Integration gives

$$p - p_1 = -\gamma (z - z_1)$$

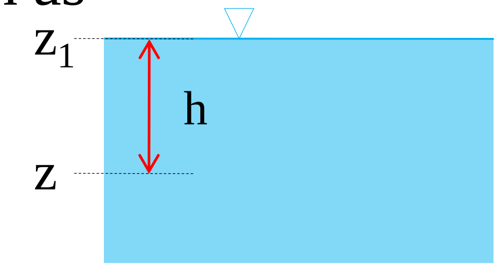
where p is the *pressure at* an elevation *z*. This expression is generally applicable to liquids, since they are *only slightly compressible*. Only where there are *large changes in elevation*, as in the ocean, need the *compressibility of the liquid be considered*, to arrive at an accurate determination of pressure variation. For small changes in elevation, equation will give accurate results

Pressure

Variation of pressure in a static fluid

For the case of a liquid at rest, it is convenient to measure distances vertically downward *from the free liquid surface*. If h is the distance below the free liquid surface and if the pressure of air and vapour on the surface is arbitrarily taken to be zero, equation $p - p_1 = -\gamma(z - z_1)$ can be written as

$$p = \gamma h$$



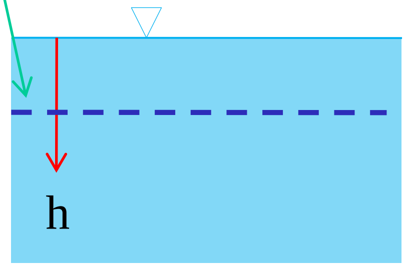
As there must always be some pressure on the surface of any liquid, the *total pressure* at any depth h is given by the above equation *plus the pressure on the surface*.

In many situations this surface pressure may be disregarded.

Pressure

Variation of pressure in a static fluid

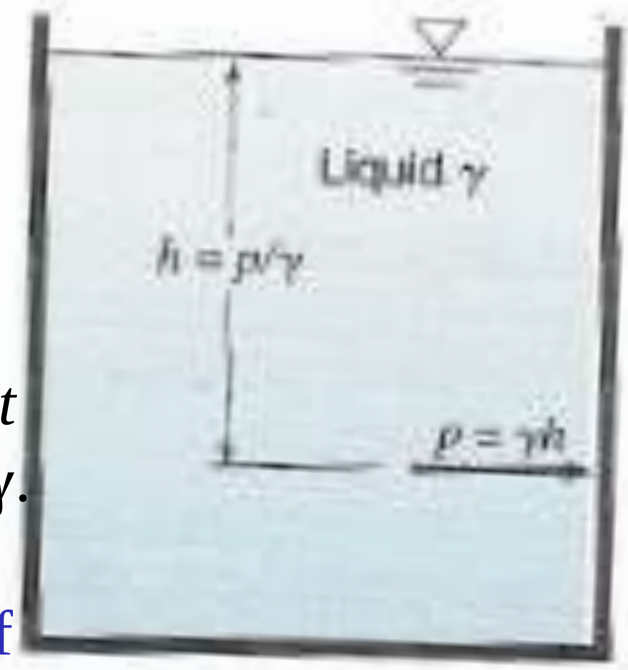
- From $p = \gamma h$, it may be seen that all points in a connected body of constant density fluid at rest are under the *same pressure* if they are at the *same depth* below the liquid surface (Pascal's law).
- This indicates that a *surface of equal pressure* for a liquid at rest is a *horizontal plane*.
- Strictly speaking, it is a surface everywhere normal to the direction of gravity and is approximately a spherical surface concentric with the earth. For *practical* purposes, a *limited portion* of this surface may be considered a *plane area*.



Pressure

Pressure expressed in height of fluid

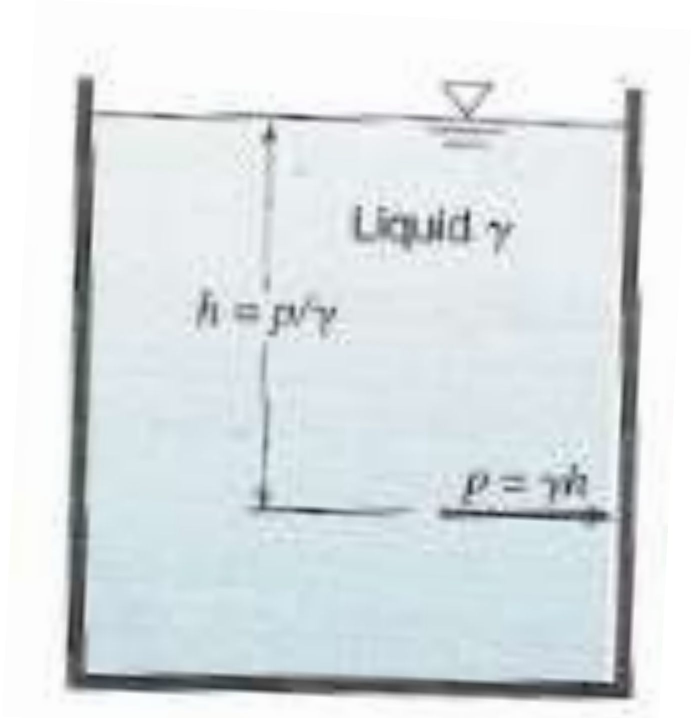
- Imagine an open tank of liquid upon whose surface there is no pressure, though in reality the minimum pressure upon any liquid surface is the pressure of its own vapor.
- Disregarding this for the moment, by the previous equation, the pressure at any depth h is $p = \gamma h$.
- If γ is assumed constant, there is a definite relation between p and h. That is, pressure (force per unit area) is equivalent to a *height h of some fluid* of constant specific weight γ.
- It is often more convenient to express pressure in terms of a height of a column of fluid rather than in pressure per unit area e.g., pressure in the pipeline is 10m of water.



Pressure

Pressure expressed in height of fluid

- Even if the surface of the liquid is *under some pressure*, it is necessary only to convert this pressure into an equivalent height of the fluid in question and *add this to the value of h* (Fig) to obtain the total pressure.

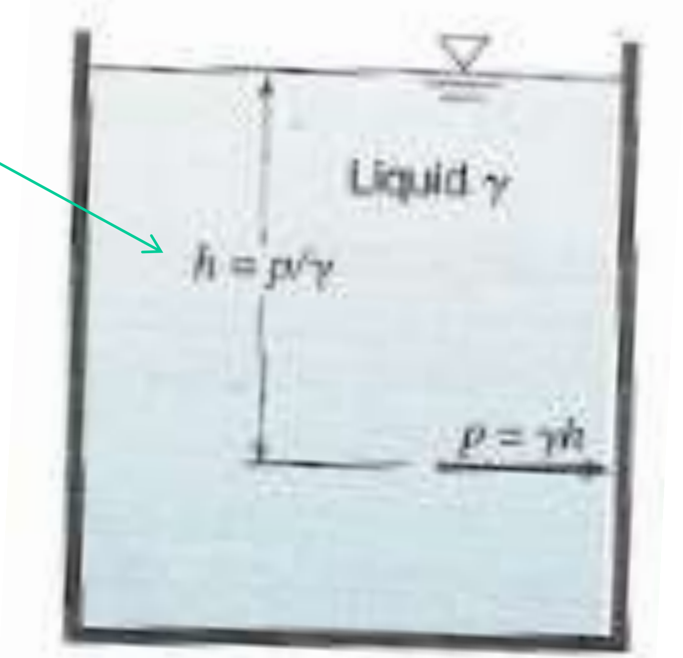


Pressure

Pressure expressed in height of fluid

➢The preceding discussion has been applied to a liquid, but it is equally possible to use it for a *gas or vapour* by specifying some *constant specific weight* γ for the gas or vapour in question. Thus pressure p may be expressed in the height of a column of any fluid by the relation

$$h = \frac{p}{\gamma}$$



➢When pressure is expressed *in meters*, it is commonly referred to as ***pressure head*** e.g., pressure head in the pipeline is 10m of water.

Pressure

Pressure expressed in height of fluid

- It is convenient to express pressures occurring in one fluid in terms of height of another fluid, e.g., barometric pressure in millimeters of mercury.
- Equation $p - p_1 = - \gamma (z - z_1)$ may be expressed as follows:

$$\frac{p}{\gamma} + z = \frac{p_1}{\gamma} + z_1 = \text{constant}$$

- This shows that for an incompressible fluid at rest, at any point in the fluid the sum of the elevation z and the pressure head p/γ is *equal* to the sum of these two quantities *at any other point*. The significance of this statement is that, in a fluid at rest, with an *increase in elevation there is a decrease in pressure head, and vice versa*. This concept is depicted in Fig below.

Pressure

Pressure expressed in height of fluid

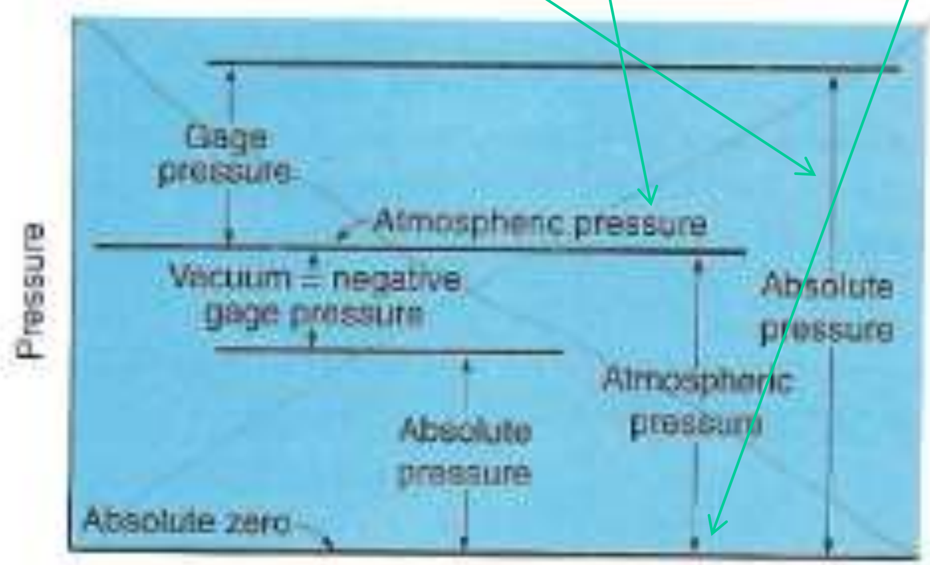
$$\frac{p}{\gamma} + z = \frac{p_1}{\gamma} + z_1 = \text{constant}$$

$$\frac{p_A}{\gamma} + z_A = \frac{p_B}{\gamma} + z_B = \text{constant}$$

Pressure

Absolute and gauge pressures

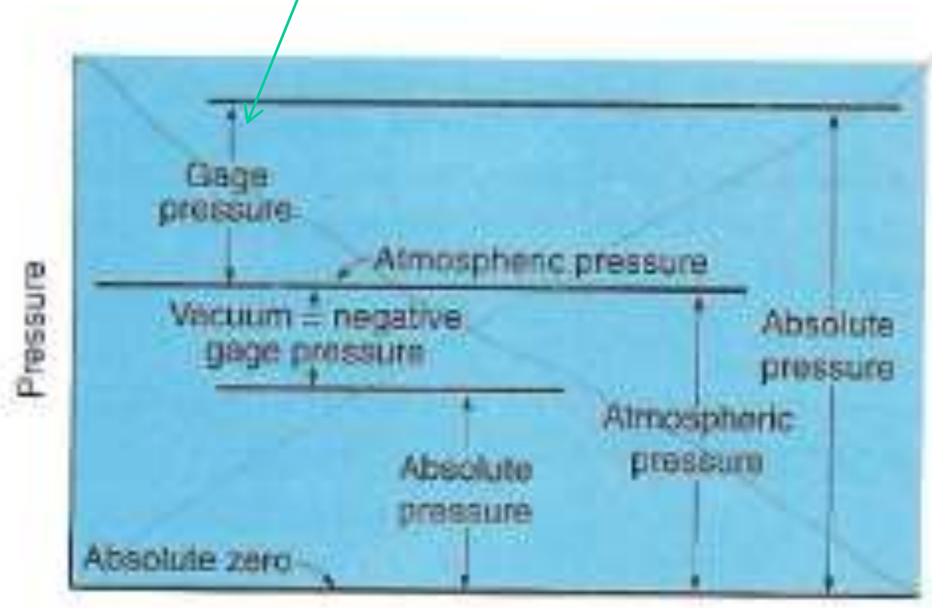
- If pressure is measured relative to absolute zero, it is called absolute pressure
- The definition of absolute pressure is the pressure of having no matter inside a space, or a perfect vacuum. Measurements taken in absolute pressure use this absolute zero as their reference point. The best example of an absolute referenced pressure is the measurement of barometric pressure.



Pressure

Absolute and gauge pressures

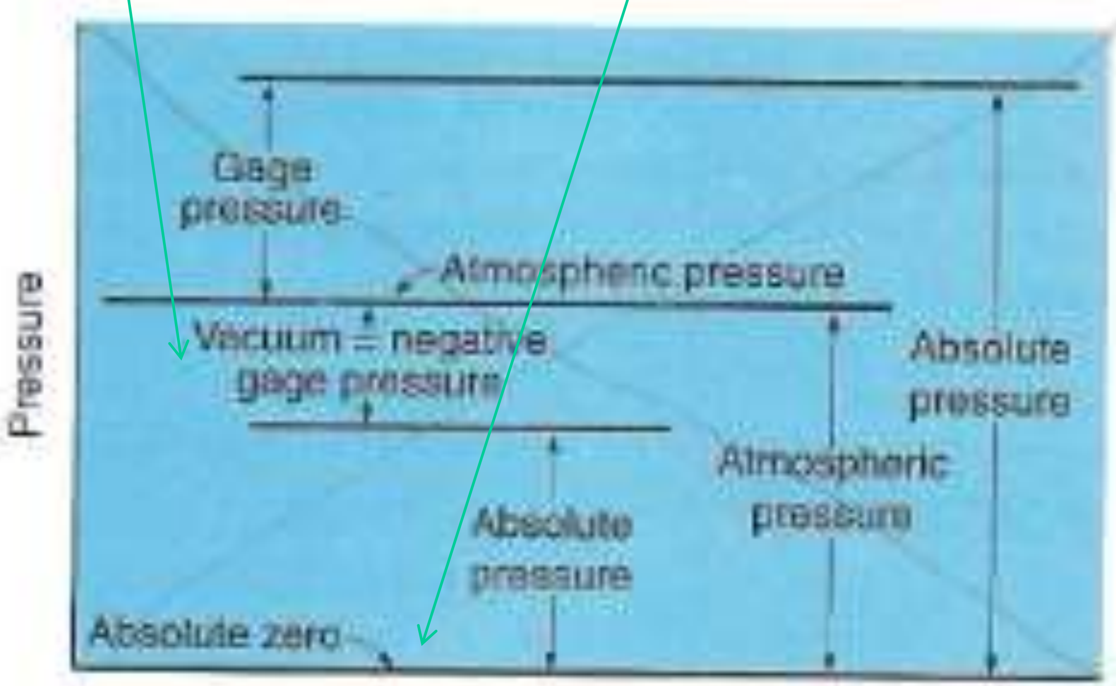
- When measured relative to atmospheric pressure as a base, it is called gauge pressure.
- This is because practically all pressure gages register zero when open to the atmosphere, and hence measure the difference between the pressure of the fluid to which they are connected and that of the surrounding air.



Pressure

Absolute and gauge pressures

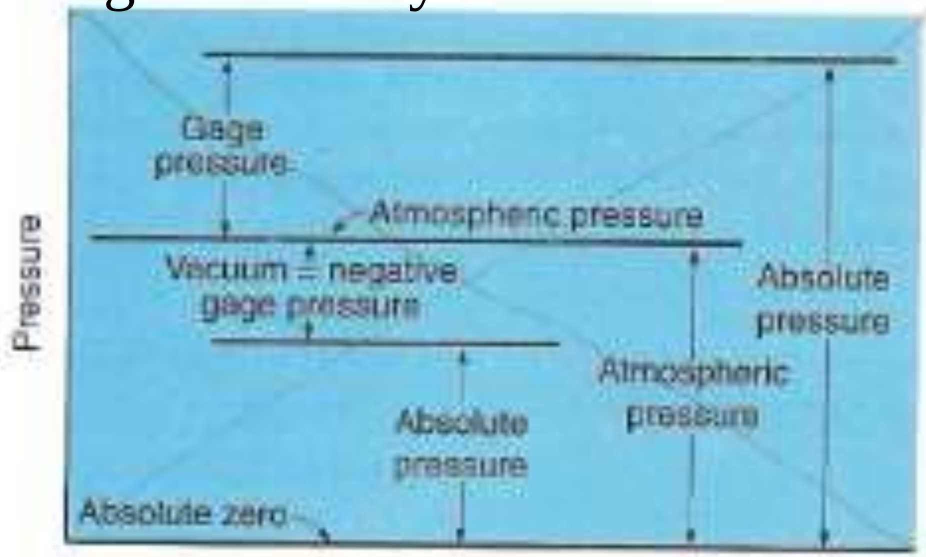
- If the pressure is below that of the atmosphere, it is designated as a vacuum, and its gauge value is the amount by which it is below that of the atmosphere. What is called a "high vacuum" is really a low absolute pressure. A perfect vacuum would correspond to absolute zero pressure.



Pressure

Absolute and gauge pressures

- Gauge pressures are positive if they are above that of the atmosphere and negative if they are vacuum.



- It can be seen from the foregoing discussion that the following relation holds: $p_{abs} = p_{atm} + p_{gauge}$

where p_{gauge} may be positive or negative (vacuum).

Pressure

Measurement of *absolute pressure*

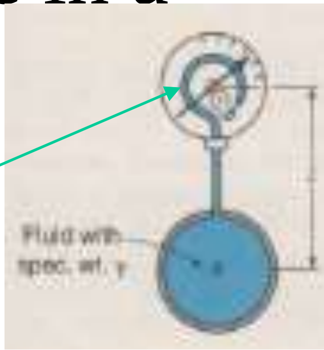
BAROMETER

- The *absolute pressure* of the *atmosphere* is measured with a barometer.
- Types of barometers. (a) Mercury barometer. (b) Aneroid barometer.

Pressure

Pressure measurement *in a fluid*

There are many ways by which pressure in a fluid may be measured.



1. Bourdon gauge

- Pressures or vacuums are commonly measured by the bourdon gage
- A curved tube of elliptical cross section will change its curvature with changes in pressure within the tube

Pressure

Pressure measurement

2. Pressure transducer (a device that converts variations in a physical quantity, such as pressure, into an electrical signal, or vice versa)

CS456

Titanium SDI-12/RS-232 Pressure Transducer

Water Level, Stage, and Flow Sensors / CS456

The CS456 is a pressure transducer for water-level measurements in canals, wells, ponds, harbors, lakes, streams, and tanks. It has a rugged titanium case that allows it to be used in saltwater and other harsh environments. The CS456 outputs either a digital SDI-12 or RS-232 signal to indicate observed pressure and temperature. This output can be read by many of our dataloggers. The CS456 replaces the CS455 transducer. The new transducers have a smaller gap between the water ports and the diaphragm so that less air is trapped that the user must remove during deployment. Trapped air causes the transducer's readings to drift as the air slowly dissolves into the water.

Pressure

Pressure measurement

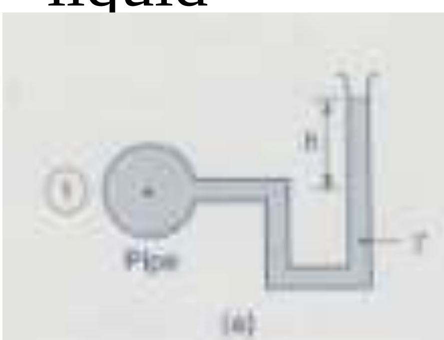
2. Pressure transducer (a device that converts variations in a physical quantity, such as pressure, into an electrical signal, or vice versa)

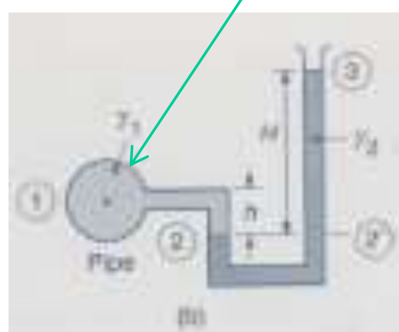
Pressure

Pressure measurement

3. Manometers

- Manometers are instruments that use columns of liquids to measure pressures
- They can be connected to a pipe used to transport a liquid





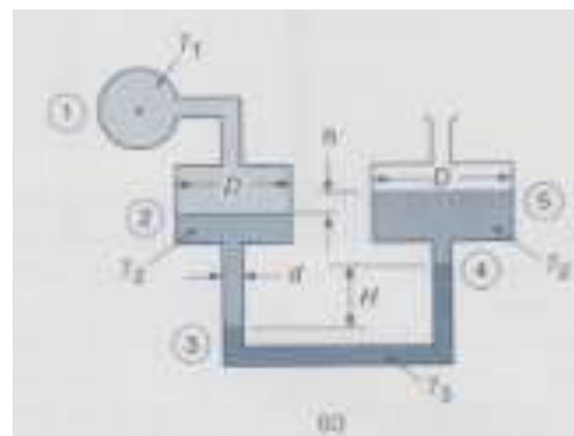


Figure 2.7 Manometers: (a) U-tube manometer (small pressures); (b) U-tube manometer (large pressures); (c) micromanometer (very small pressure changes).

Pressure

Pressure measurement

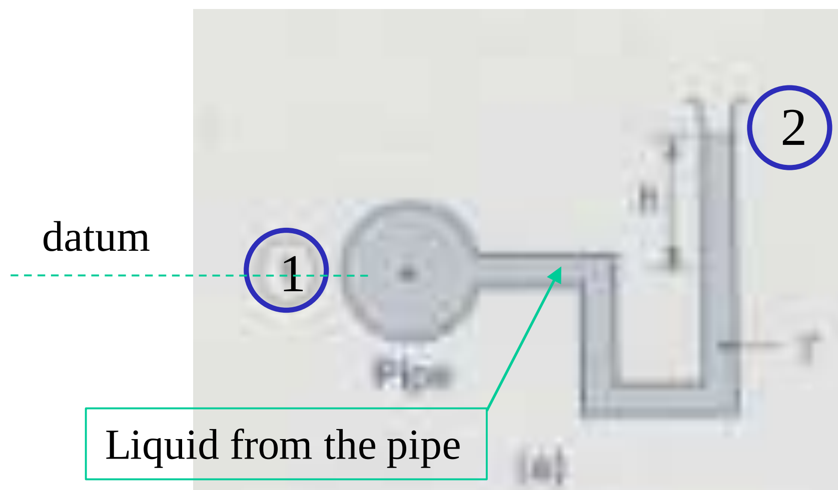
a. U-tube Manometer (small pressures)

- The datum from which z_1 and z_2 are measured is located at any desired position such as through point 1.
- Since $p_2 = 0$ (gauge pressure is selected; if absolute pressure is desired, we would select $p_2 = p_{atm}$) & $z_2 - z_1 = h$

$$\frac{p_1}{\gamma} + z_1 = \frac{p_2}{\gamma} + z_2$$

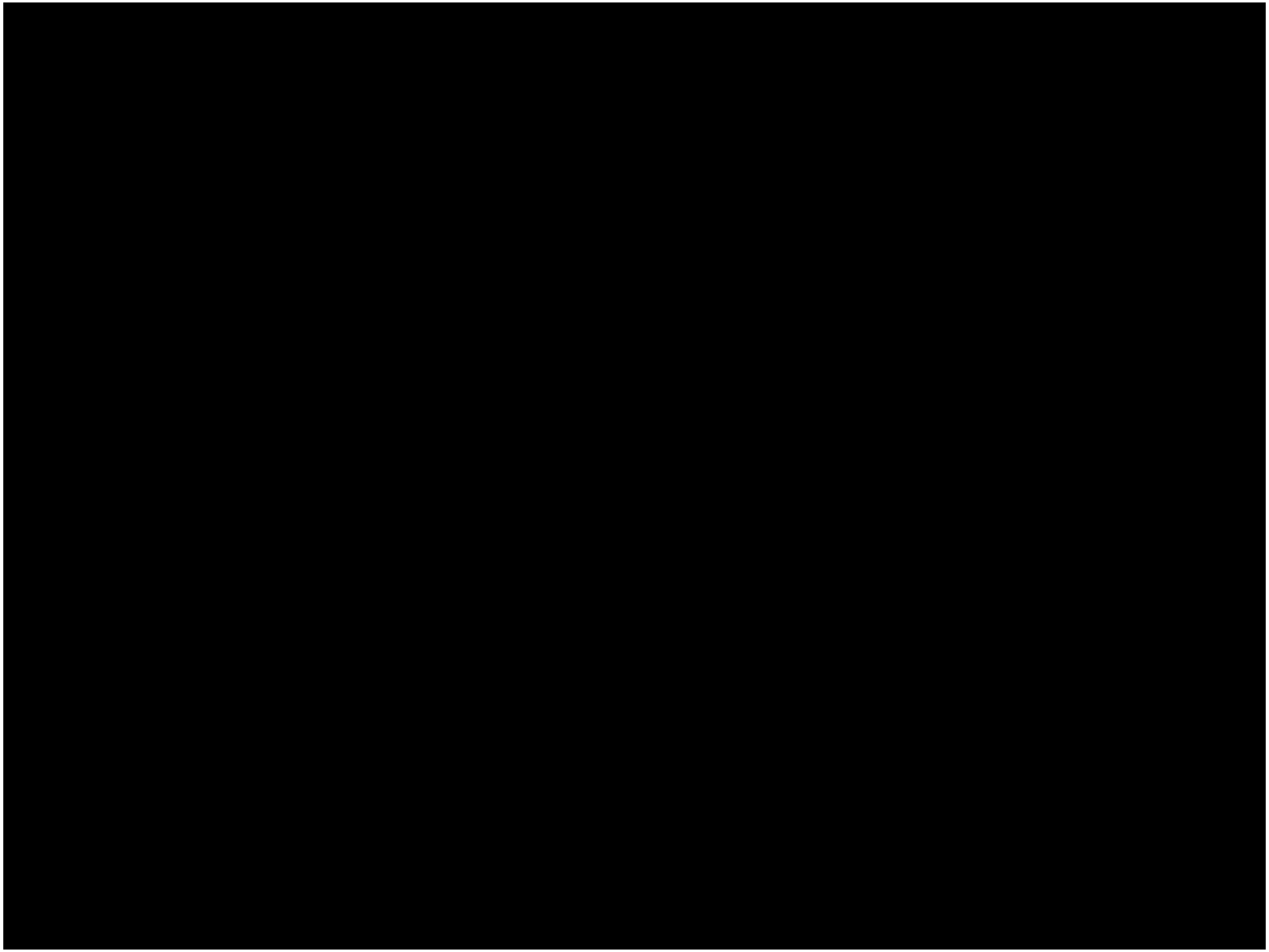
$$\frac{p_1}{\gamma} + 0 = \frac{0}{\gamma} + h$$

$$p_1 = \gamma h$$



By measuring h, one computes p_1, the pressure in the pipe.

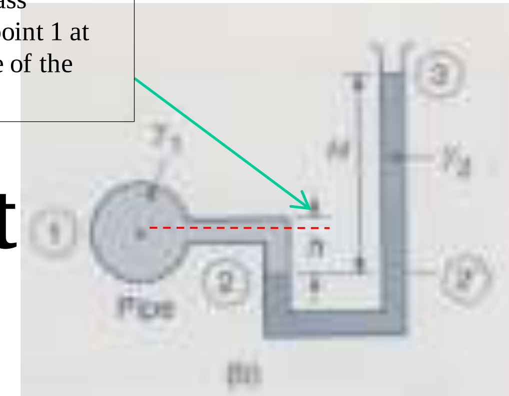
https://www.youtube.com/watch?v=9SQ2FPHCZJk



Pressure

Pressure measurement

Should pass through point 1 at the centre of the pipe



b. U-tube Manometer (large pressures)

- Manometer used to measure relatively large pressures since we can select γ_2 to be quite large (e.g., mercury $\gamma_2 = 13.6\,\gamma_{water}$
- The pressure can be determined by introducing three points as indicated. This is necessary because $\frac{p_1}{\gamma} + z_1 = \frac{p_2}{\gamma} + z_2$ *applies throughout* ***one fluid***; γ must be constant. The value of γ changes abruptly at point 2; hence we write

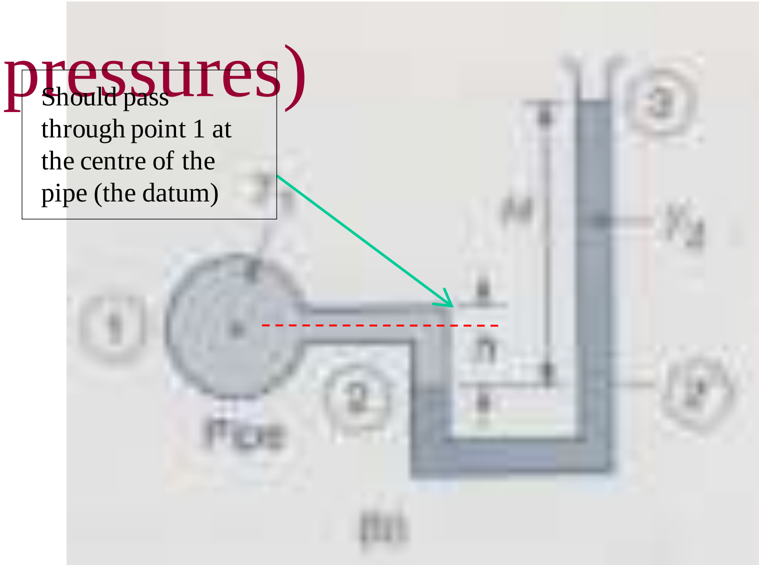
$$\frac{p_1}{\gamma_1} + z_1 = \frac{p_2}{\gamma_1} + z_2 \qquad \text{for } \gamma_1$$

$$\frac{p_2}{\gamma_2} + z_2 = \frac{p_3}{\gamma_2} + z_3 \qquad \text{for } \gamma_2$$

Pressure

Pressure measurement

b. U-tube Manometer (large pressures)



$$\frac{p_1}{\gamma_1} + z_1 = \frac{p_2}{\gamma_1} + z_2 \qquad 1$$

$$\frac{p_2}{\gamma_2} + z_2 = \frac{p_3}{\gamma_2} + z_3 \qquad 2$$

where γ_1 and γ_2 are for liquids 1 and 2, respectively.

- Equation 1 becomes

$$p_1 = \gamma_1\left(\frac{p_2}{\gamma_1} + z_2 - z_1\right) = p_2 + \gamma_1(z_2 - z_1) \qquad 3$$

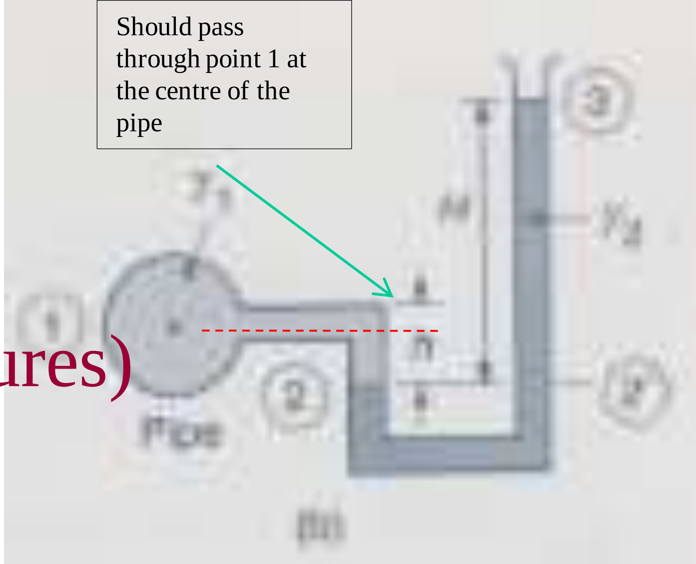
- Setting $p_3 = 0$ (gauge pressure is used), Equation 2 yields

$$p_2 = \gamma_2\left(\frac{p_3}{\gamma_2} + z_3 - z_2\right) = \gamma_2(0 + z_3 - z_2) = \gamma_2(z_3 - z_2) \qquad 4$$

Pressure

Pressure measurement

b. U-tube Manometer (large pressures)



$$p_1 = \gamma_1\left(\frac{p_2}{\gamma_1} + z_2 - z_1\right) = p_2 + \gamma_1(z_2 - z_1) \qquad 3$$

$$p_2 = \gamma_2\left(\frac{p_3}{\gamma_2} + z_3 - z_2\right) = \gamma_2(0 + z_3 - z_2) = \gamma_2(z_3 - z_2) \qquad 4$$

- Putting Eqn 4 into Eqn 3 results in (z_1-z_2=h, z_3-z_2=H see Fig)

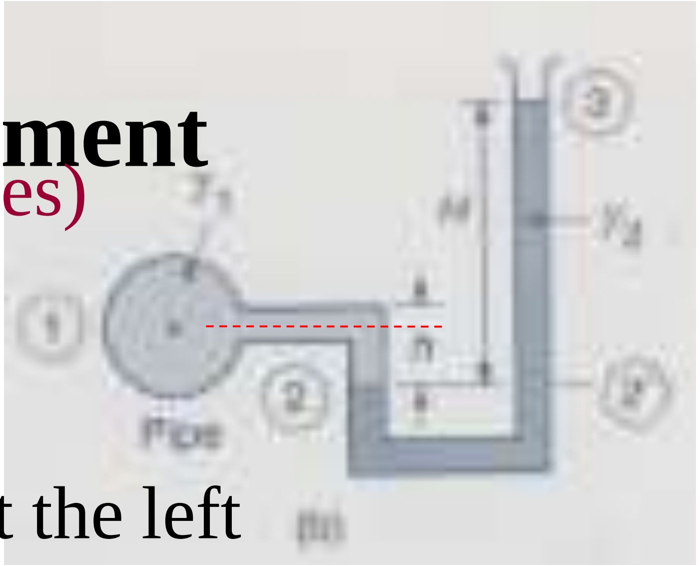
$$p_1 = \gamma_2(z_3 - z_2) + \gamma_1(z_2 - z_1) = -\gamma_1 h + \gamma_2 H$$

- In the analysis above we recognize that the pressure at point 2' is equal to the pressure at point 2, since points 2 and 2' are at the *same elevation* in the *same fluid* above a datum passing through the horizontal bottom of the manometer.

Pressure

Pressure measurement

b. U-tube Manometer (large pressures)



$$p_1 = \gamma_1(z_2 - z_1) + \gamma_2(z_3 - z_2) = -\gamma_1 h + \gamma_2 H$$

- Interpret the manometer by starting at the left water **pipe**, add pressure when the elevation decreases, and subtract pressure when the elevation increases until the end is encountered.

$$p_1 = \gamma_1(z_2 - z_1) + \gamma_2(z_3 - z_2) = -\gamma_1 h + \gamma_2 H \Rightarrow p_1 + \gamma_1 h - \gamma_2 H = p_3$$

$$p_1 + \gamma_1 h - \gamma_2 H = p_3$$

$h = -(z_2 - z_1)$

Pressure

Pressure measurement

- c. Micromanometer (very small pressure changes)

- Example

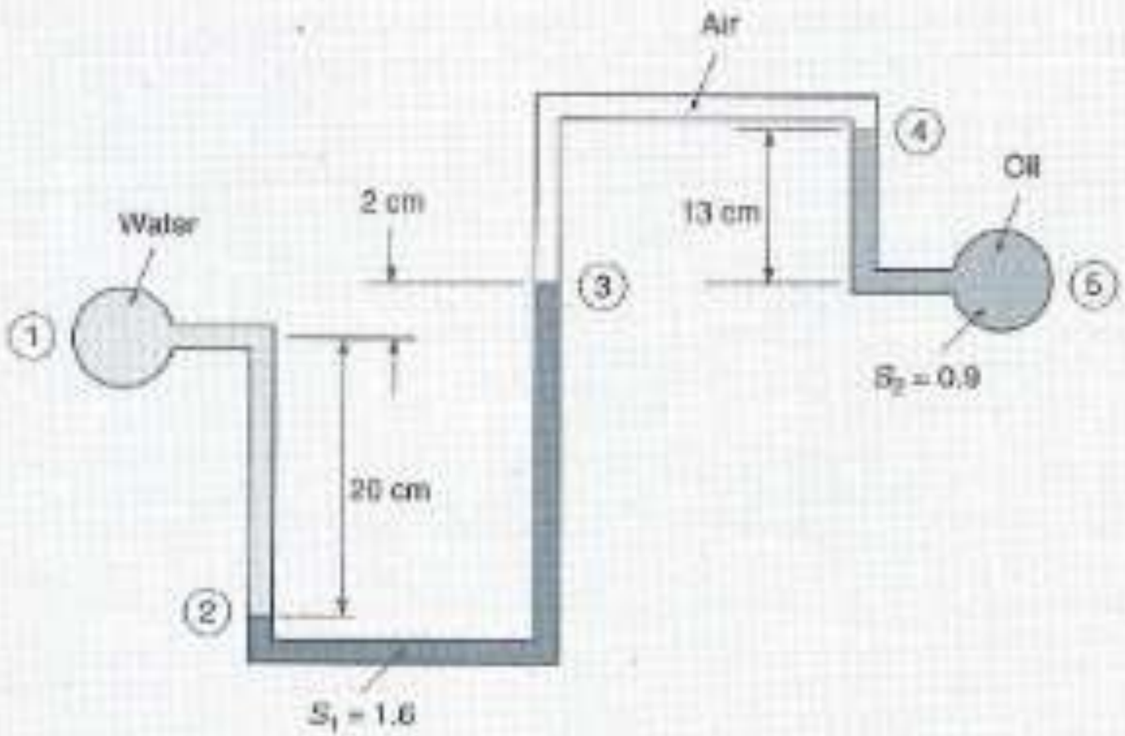


Figure E2.3

$$p_1 - p_5 = (p_1 - p_2) + (p_2 - p_3) + (p_3 - p_4) + (p_4 - p_5)$$
$$= (z_2 - z_1) + \gamma S_1(z_3 - z_2) + \gamma S_{air}(z_4 - z_3) + \gamma S_2(z_5 - z_4)$$

where $\gamma = 9810$ N/m^3, $S_1 = 1.6$, $S_2 = 0.9$, and $S_{air} = 0$. Thus

$$p_1 - p_5 = 9810(-0.20 + 1.6 \times 0.22 + 0 \times 0.13 + 0.9 \times -0.13)$$
$$= 343 \text{ Pa}$$

Note that by neglecting the weight of the air, the pressure at point 3 is equal to the pressure at point 4.

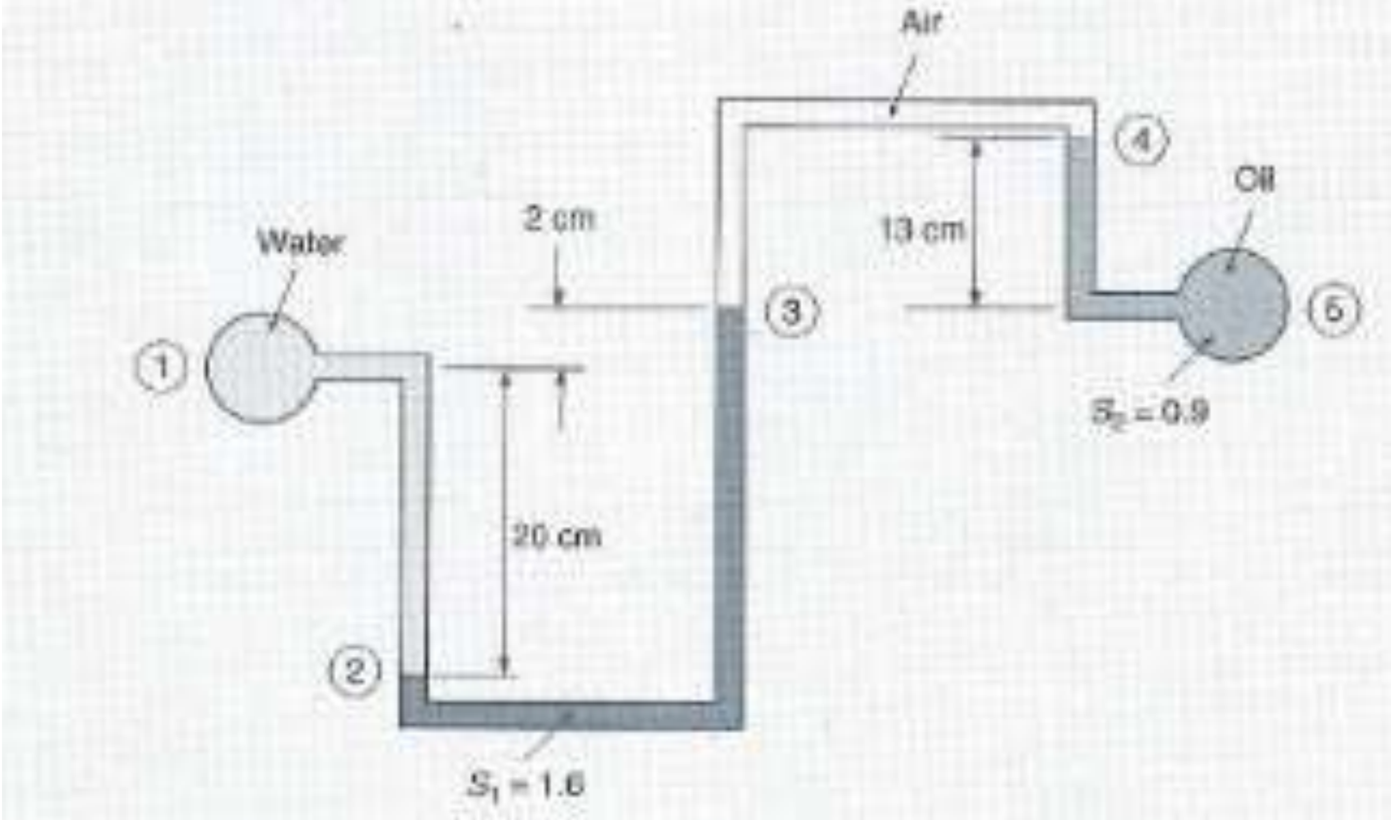
Pressure

Pressure measurement

- An alternative method of interpreting the manometer is to start at the left water pipe, add pressure when the elevation decreases, and subtract pressure when the elevation increases until the pipe at the right is encountered (From

$$p_1 = \gamma_1(z_2 - z_1) + \gamma_2(z_3 - z_2) = -\gamma_1 h + \gamma_2 H \Rightarrow p_1 + \gamma_1 h - \gamma_2 H = p_3 \quad)$$

$$p_1 + \gamma_1 h - \gamma_2 H = p_3$$



$$p_1 + \gamma_1(z_2 - z_1) - \gamma_2(z_3 - z_2) - \gamma_3(z_4 - z_3) + \gamma_4(z_5 - z_4) = p_5$$

- Note that by neglecting the weight of the air (, the pressure at point 3 is equal to that at point 4 i.e., $\gamma_3(z_4 - z_3) = 0$ (if you want, you can include it)

$$p_1 + 9810 \times 0.2 - (1.6 \times 9810) \times (0.2 + 0.02) + (0.9 \times 9810) \times 0.13 = p_5$$

$p_1 - p_5 = 343Pa$ » This yields the same answer

https://www.youtube.com/watch?v=gxrkLkJybnA

- ..\..\illustration videos\Manometer Explained _ Working Principle.mp4

[redacted]