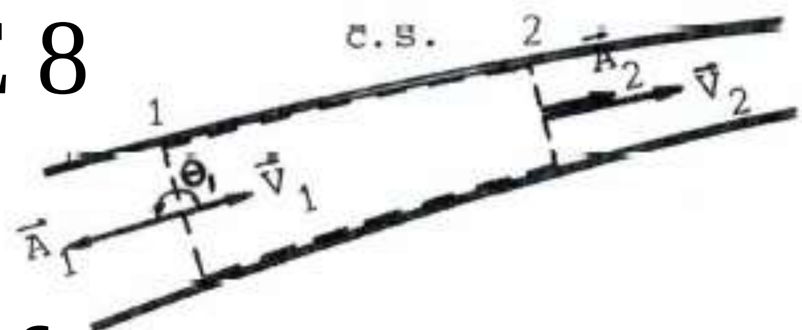
Fluid Mechanics CEE 3311

LECTURE 8



Conservation of mass

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When a fluid is in motion, it must move in such a way that mass is conserved.

The continuity principle is based on the conservation of mass. In this case: extensive property N=mass

intensive property η=1

Then control volume equation $\frac{dN_{sys}}{dt} = \frac{d}{dt}\int_{cv} \eta\,\rho\,dV + \int_{cs} \eta\,\rho\,(\vec{v}\,d\vec{A})$

becomes (with η=1):

$$\frac{d(\text{Mass})}{dt} = \frac{d}{dt}\int_{cv} \rho dV + \int_{cs} \rho \vec{v}\,\overrightarrow{dA}$$

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If the flow is steady, there results

Conservation of mass $\frac{d(\text{Mass})}{dt} = 0$

$$0 = 0 + \int_{cs} \rho \vec{v}\, \overrightarrow{dA}$$

$$\int_{cs} \rho \vec{v}\, \overrightarrow{dA} = 0$$

For one dimensional flow $\int_{cs} \rho \vec{v}\, \overrightarrow{dA}$ can be written as $\sum \rho \vec{v} \vec{A}$

Therefore, for steady one dimensional flow the formula diminishes to

$$\sum_{cs} \rho \vec{v} \vec{A} = 0$$

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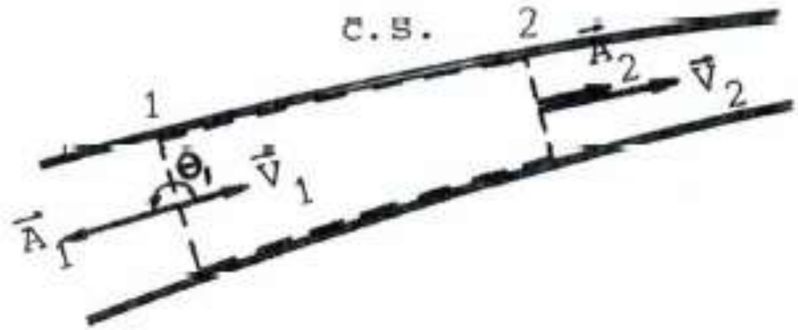
This formula may be used, e.g., in a steady flow case in a conduit

$$\sum \rho \vec{v} \vec{A} = -\rho_1 A_1 v_1 + \rho_2 A_2 v_2 = 0$$

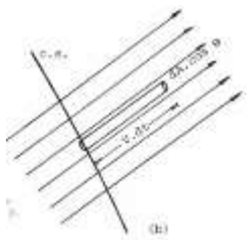
for $\vec{v_1} \vec{A_1} = A_1 v_1 \cos 180° = -A_1 v_1$

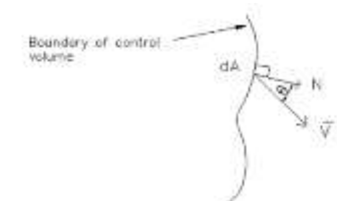
and $\vec{v_2} \vec{A_2} = A_2 v_2 \cos 0° = +A_2 v_2$



The **continuity equation** takes the form

$$\rho_2 A_2 v_2 = \rho_1 A_1 v_1$$





For a steady flow through a control volume with many inlets and outlets, the net mass flow must be zero, where inflows are negative and outflows are positive.

$$\rho_2 A_2 v_2 = \rho_1 A_1 v_1$$

If the density is constant in the control volume, the **continuity equation** then reduces to

$$A_1 v_1 = A_2 v_2$$

This form of the equation is used quite often, particularly with liquids and low speed gas

This equation is simply

$$Q_1 = Q_2$$

The mass of fluid at 1 and 2 is the same hence conservation of mass

I like to state the continuity principle as: The rate of change of storage with respect to time is equal to inflow minus outflow $\frac{\Delta S}{\Delta t} = I - O$

EXAMPLE 4.1

Water flows at a uniform velocity of 3 m/s into a nozzle that reduces the diameter from 10 cm to 2 cm (Fig. E4.1). Calculate the water's velocity leaving the nozzle and the flow rate.

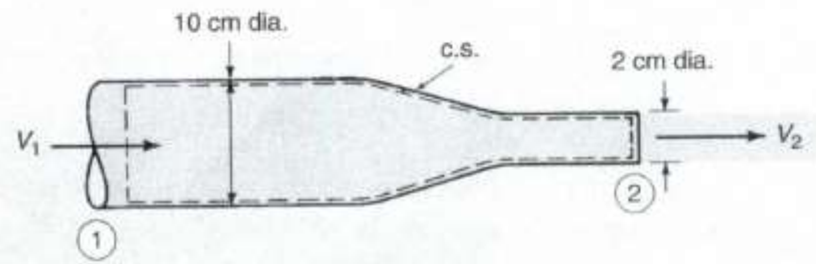


Figure E4.1

Solution: The control volume is selected to be the inside of the nozzle as shown. Flow enters the control volume at section 1 and leaves at section 2. The simplified continuity equation (4.3.6) is used:

$$A_1V_1 = A_2V_2$$

$$\therefore\ V_2 = V_1\frac{A_1}{A_2}$$

$$= 3\,\frac{\pi \times 0.1^2/4}{\pi \times 0.02^2/4} = 75\ \mathrm{m/s}$$

The flow rate, or discharge, is found to be

$$Q = V_1A_1$$

$$= 3 \times \pi \times 0.1^2/4 = 0.0236\ \mathrm{m^3/s}$$