Fluid Mechanics CEE 3311
LECTURE 9

Conservation of energy

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The energy equation will be derived starting from the first law of thermodynamics which states that for steady flow, the external work done on any system plus the thermal energy transferred into or out of the system is equal to the change of energy of the system.

Another definition of the first law of thermodynamics:
It states that the change in the internal energy ΔU of a closed system is equal to the amount of heat Q supplied *to* the system, minus the amount of work W done *by* the system on its surroundings.

In thermodynamics, the **internal energy** of a system is the total energy contained within the system. It is the energy necessary to create or prepare the system in any given state, but does not include the kinetic energy nor the potential energy of the system.

In other words, for steady flow during time Δt

$$\Delta E = Q - W \quad 9.1$$

where ΔE is change of energy of the system, Q is thermal energy or heat transferred into a system and W is external work done

The total energy E consists of kinetic energy E_K, potential energy E_P and internal energy E_u. Thus the total energy of the system is

$$E = E_k + E_p + E_u \quad 9.2$$

E is an extensive property of the system. Then the corresponding intensive property is given by e, which is made up of e_k, e_p, and u .

In applying the control volume equation

$$\frac{dN_{sys}}{dt} = \frac{d}{dt}\int_{cv} \eta\, \rho\, dV + \int_{cs} \eta\, \rho\, (\vec{v}\, d\vec{A}) \qquad 9.3$$

$$\frac{dE}{dt} = \frac{dQ}{dt} - \frac{dW}{dt} = \frac{d}{dt}\int_{cv} e\, \rho\, dV + \int_{cs} e\, \rho\, (\vec{v}\, d\vec{A}) \qquad 9.4$$

$$\Delta E = Q - W$$

$E = E_k + E_p + E_u$

Or

$$\frac{dQ}{dt} - \frac{dW}{dt} = \frac{d}{dt}\int_{cv} (e_k + e_p + u)\rho \, dV + \int_{cs} (e_k + e_p + u)\rho \, (\vec{v}\, d\vec{A}) \quad 9.5$$

in which $\frac{dQ}{dt}$= rate of flow of heat into the system

$\frac{dW}{dt}$= rate of work done by the system on its surrounding

The kinetic energy per unit mass e_k can be written as

$$e_k = \frac{\frac{1}{2}mv^2}{m} = \frac{1}{2}v^2 \text{ while } e_p = \frac{mgz}{m} = gz \quad 9.6$$

Substituting these into eq.

$$\frac{dQ}{dt} - \frac{dW}{dt} = \frac{d}{dt}\int_{cv} (\frac{1}{2}v^2 + gz + u)\rho \, dV + \int_{cs} (\frac{1}{2}v^2 + gz + u)\rho \, (\vec{v}\, d\vec{A}) \quad 9.7$$

For convenience of analysis, work W is divided into flow work W_f and shaft work W_s

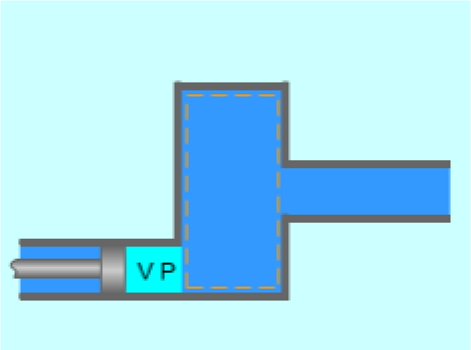
$$W = W_f + W_s$$

Flow Work

- Work is needed to push the fluid into or out of the boundaries of a control volume if mass flow is involved. This work is called the flow work (flow energy). Flow work is necessary for maintaining a continuous flow through a control volume.

Flow Work

- Consider a fluid element of volume V, pressure P, and cross-sectional area A as below.



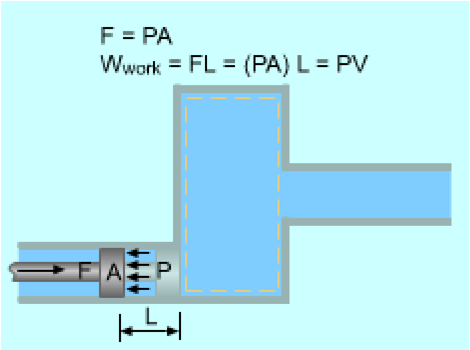
A Flow Element

- The flow immediately upstream will force this fluid element to enter the control volume, and it can be regarded as an imaginary piston.

Flow Work

- The force applied on the fluid element by the imaginary piston is:

  $$F = PA$$



Flow Work with Imaginary Piston

- The work done due to pushing the entire fluid element across the boundary into the control volume is

  $$W_{flow} = FL = PAL = PV$$

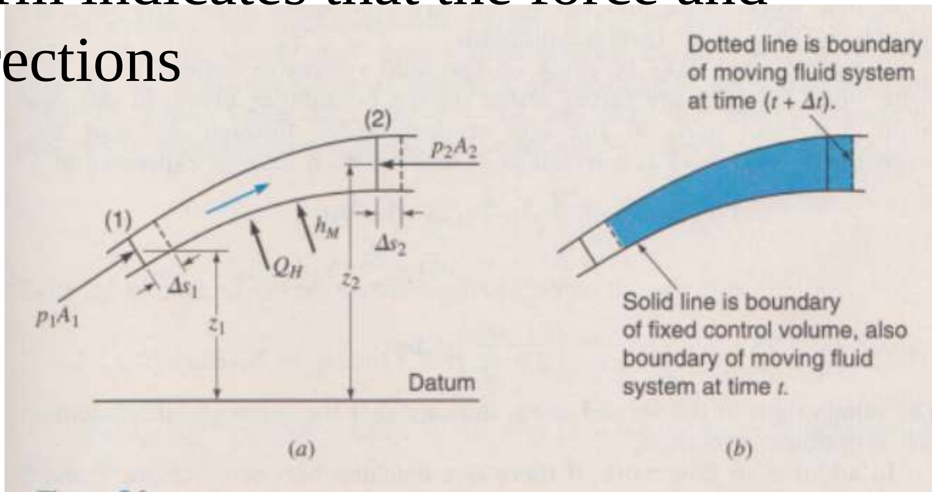
- The work done due to pushing the fluid element out of the control volume is the same as the work needed to push the fluid element into the control volume.

Flow work

- Another way of looking at the same flow work is given below:
- External work may be done on the fluid system in various ways.
- It is done when the pressure forces acting on the boundaries move, in our case, when P_1A_1 and P_2A_2 at the end sections move through Δs_1 and Δs_2, respectively. This work is referred to as flow work. It may be expressed as

$$\text{Flow work} = P_1 A_1 \Delta s_1 - P_2 A_2 \Delta s_2$$

- The minus sign in the second term indicates that the force and displacement are in opposite directions



- Since Δs = v Δ t, the flow work done by the system on the surrounding fluid in time can also be presented as

$$W_{f,1} = P_1 A_1 \Delta s_1 = P_1 A_1 v_1 \Delta t \qquad 9.8$$

- The rate of flow work (W/t) $$\frac{dW_{f,1}}{dt} = P_1 A_1 v_1 \qquad 9.9$$

- And similarly $$\frac{dW_{f,2}}{dt} = -P_2 A_2 v_2 \qquad 9.10$$

- In terms of vector dot product, eqn can be written as

$$\frac{dW_f}{dt} = P\left(\vec{v}\vec{A}\right) \qquad 9.11$$

- Then the rate at which flow work is done on the system's surroundings is obtained by summing eqn 10.11 for all streams passing the control surface; in general

$$\frac{dW_f}{dt} = \sum_{cs} P\left(\vec{v}\vec{A}\right) \qquad 9.12$$

Shaft work

Shaft work is defined as any work other than flow work. It is usually in the form of a shaft that either

- Takes energy out of the system (work on a mechanism like a turbine blade)
- Puts energy into the system (shaft attached to a mechanism like a pump that does work on the system)

In the latter case, the fluid system is doing negative work on its surrounding.

- Rate of change of work dW/dt is the sum of the shaft work rate dWs/dt and the flow work rate $$\frac{dW}{dt}=\frac{dW_f}{dt}+\frac{dW_s}{dt}$$

- If we substitute for dW/dt into Eq. 9.7 $\frac{dQ}{dt}-\frac{dW}{dt}=\frac{d}{dt}\int_{cv}\left(\frac{1}{2}v^2+gz+u\right)\rho dV+\int_{cs}\left(\frac{1}{2}v^2+gz+u\right)\rho\left(\vec{v}d\vec{A}\right)$ we get

$$\frac{dQ}{dt}-\left(\frac{dW_f}{dt}+\frac{dW_s}{dt}\right)=\frac{d}{dt}\int_{cv}\left(\frac{1}{2}v^2+gz+u\right)\rho dV+\int_{cs}\left(\frac{1}{2}v^2+gz+u\right)\rho\left(\vec{v}d\vec{A}\right)$$

- Putting Eq 9.12 $\frac{dW_f}{dt}=\sum_{cs}P\left(\vec{v}\vec{A}\right)$ into above Eq we obtain

$$\frac{dQ}{dt}-\left(\sum_{cs}P\left(\vec{v}\vec{A}\right)+\frac{dW_s}{dt}\right)=\frac{d}{dt}\int_{cv}\left(\frac{1}{2}v^2+gz+u\right)\rho dV+\int_{cs}\left(\frac{1}{2}v^2+gz+u\right)\rho\left(\vec{v}d\vec{A}\right)$$

- Replacing summation by integration and introducing ρ/ρ

$$\frac{dQ}{dt}-\left(\int_{cs}P\frac{\rho}{\rho}\left(\vec{v}d\vec{A}\right)+\frac{dW_s}{dt}\right)=\frac{d}{dt}\int_{cv}\left(\frac{1}{2}v^2+gz+u\right)\rho dV+\int_{cs}\left(\frac{1}{2}v^2+gz+u\right)\rho\left(\vec{v}d\vec{A}\right)$$

or

$$\frac{dQ}{dt}-\frac{dW_s}{dt}=\frac{d}{dt}\int_{cv}\left(\frac{1}{2}v^2+gz+u\right)\rho dV+\int_{cs}\left(\frac{1}{2}v^2+gz+u+\frac{P}{\rho}\right)\rho\left(\vec{v}d\vec{A}\right) \qquad 9.13$$

$$\frac{dQ}{dt} - \frac{dW_s}{dt} = \frac{d}{dt}\int_{cv}\left(\frac{1}{2}v^2 + gz + u\right)\rho dV + \int_{cs}\left(\frac{1}{2}v^2 + gz + u + \frac{P}{\rho}\right)\rho\left(\vec{v}d\vec{A}\right) \qquad 9.13$$

- Equation 9.13 is the basic form of the energy equation.
- Usually some simplifications are allowed to be made.