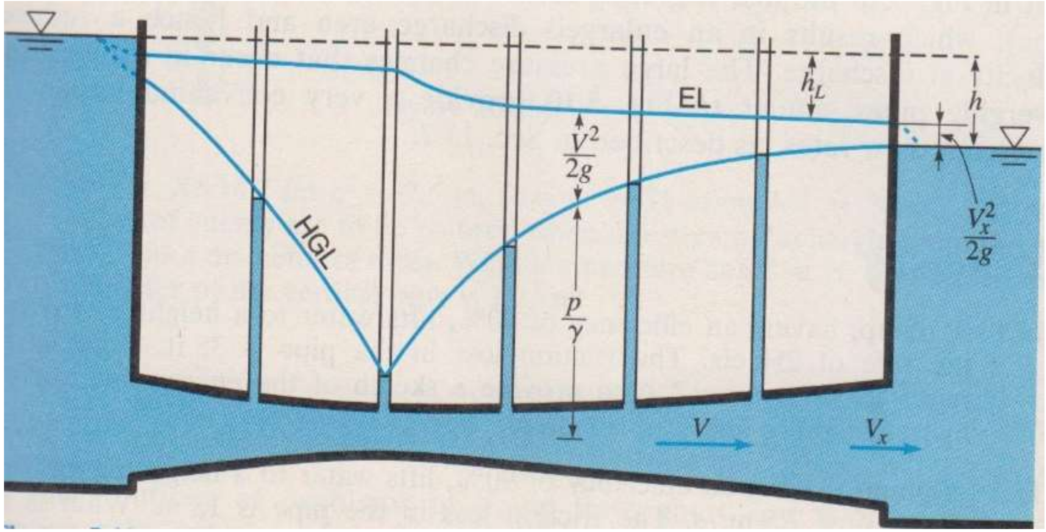
Fluid Mechanics CEE 3311

LECTURE 11



Conservation of energy

Concept of hydraulic and energy grade lines

L. Handia

Since each element of the energy equation or the Bernoulli equation has the dimension of length, it may be presented graphically with reference to an arbitrary datum level:

The **hydraulic grade line** (HGL) represents the locus of the piezometric head $h = p/\rho.g + z$ along the flow, while

the **energy grade line** (EGL) represents the locus of the total head $H = p/\rho.g + z + \alpha.(V^2/2g)$ along the flow.

$$\propto_1 \frac{v_1^2}{2g} + \frac{p_1}{\rho g} + z_1 + h_p = \propto_2 \frac{v_2^2}{2g} + \frac{p_2}{\rho g} + z_2 + h_t + h_l$$

The locus of points of the piezometric head and total head, respectively, are set out vertically above the centre line of the pipe. The total head in the system is greater than the piezometric head by an amount $\alpha.(V^2/2g)$. Consequently, the EGL is above the HGL by a distance $\alpha.(V^2/2g)$ (see Fig. 7.3). Strictly speaking, each streamline has its own HGL and EGL. When "one-dimensional flow" is assumed, it is usual to consider only the streamline in the centre of the pipe. Then the z measurements are taken to the centre line and the pressure head $p/\rho g$ is measured upwards from there.

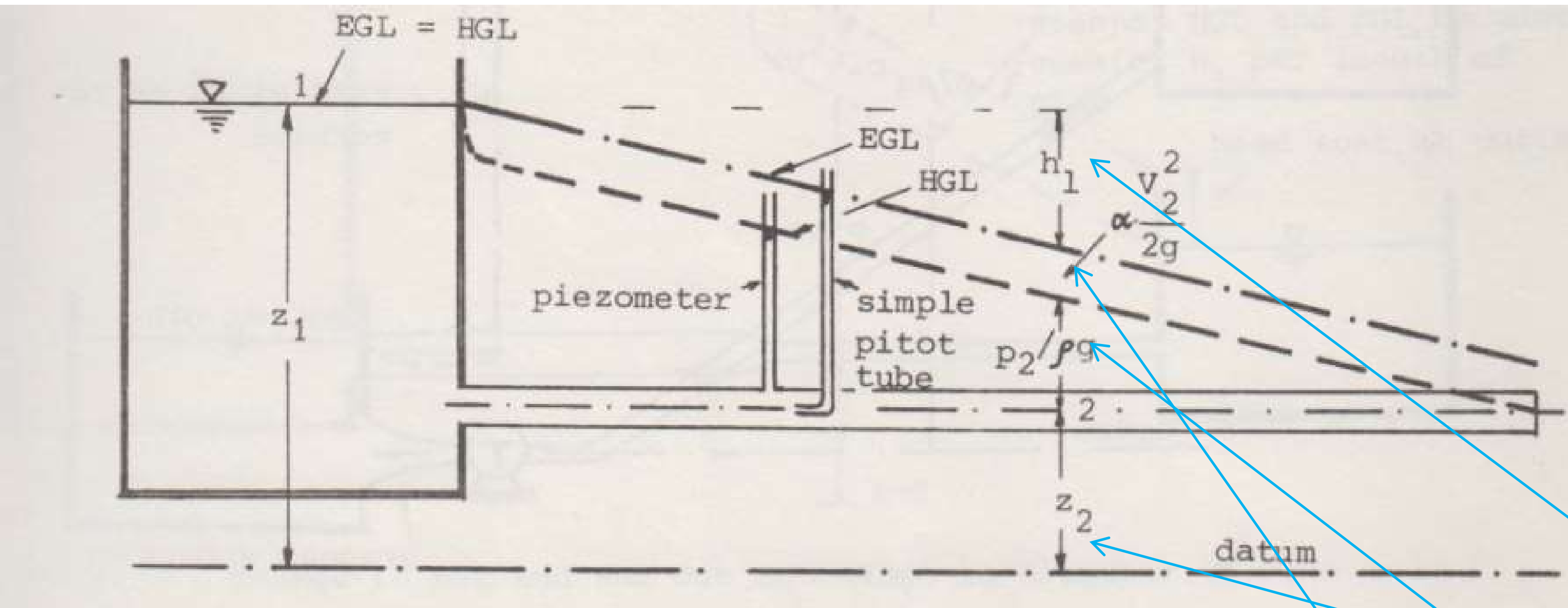


Fig. 7.3 EGL and HGL in a straight pipe

$$\propto_1 \frac{v_1^2}{2g} + \frac{p_1}{\rho g} + z_1 + h_p = \propto_2 \frac{v_2^2}{2g} + \frac{p_2}{\rho g} + z_2 + h_t + h_l$$

Some helpful hints for drawing hydraulic grade lines and energy grade lines are enumerated below:

a) By definition, the EGL is positioned above the HGL an amount $\alpha.(V^2/2g)$. Thus, if the velocity is zero as in a lake or reservoir, the EGL and HGL will coincide with the liquid surface (see Fig. 7.3).

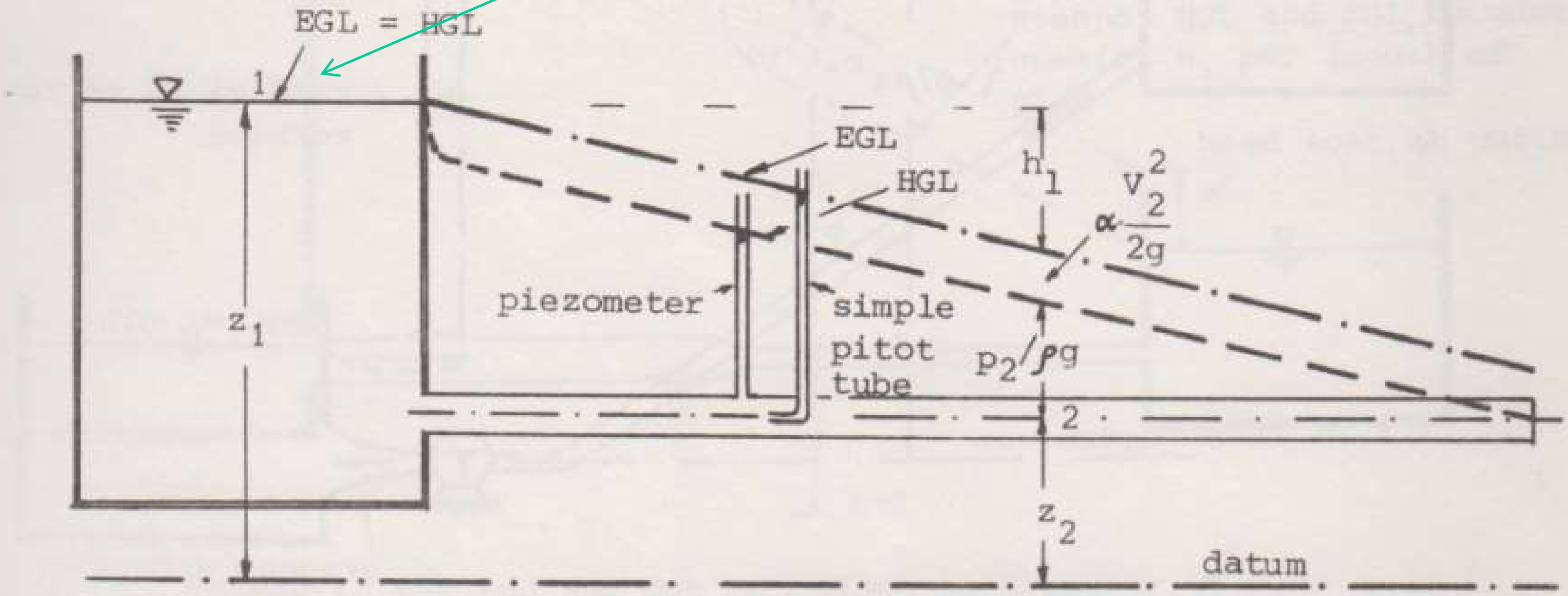


Fig. 7.3 EGL and HGL in a straight pipe

b) Head loss h_L for flow in a pipe or cannel always means that the EGL will slope **downward** in the direction of flow (Fig. 7.3). The only exception to this rule occurs when a pump supplies energy (and pressure) to the flow. Then an abrupt rise in the EGL (and the HGL) occurs from the upstream side to the downstream side of the pump (see Fig. 7.4). In this figure the pipe diameter is constant. Therefore, according to the continuity equation, $V_1 = V_2 = V_3 = V_4 = V$. Moreover, the HGL also rises an amount h_p at the pump.

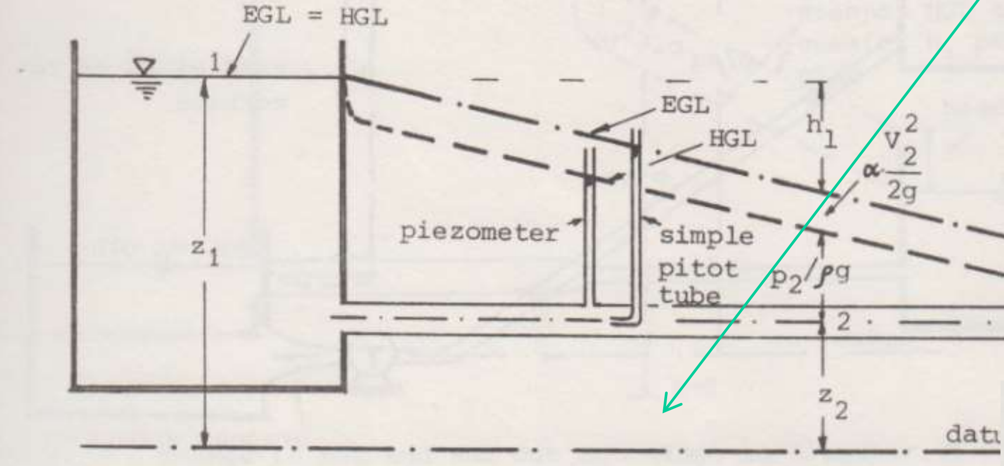


Fig. 7.3 EGL and HGL in a straight pipe

Pump head is like what happened to Lazarus when Jesus resurrected him. His blood pressure rose from zero to a high value due to the pump (heart) just like the pump head (heart head)

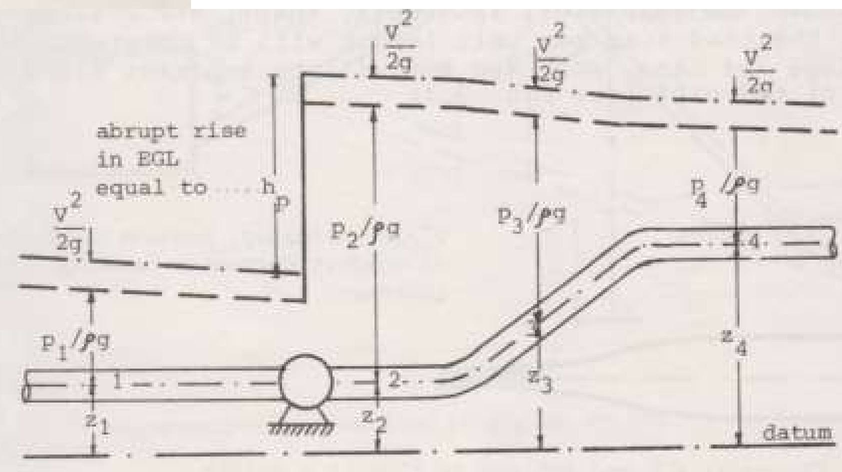


Fig. 7.4 Rise in EGL and HGL due to pump

c) A turbine takes energy abruptly out of the flow. Then the EGL and HGL will drop abruptly as in Fig. 7.5. The figure also shows that much of the kinetic energy can be converted to pressure if there is a gradual expansion such as at the outlet. Thus the head loss at the outlet is reduced, making the turbine installation more efficient. If the outlet is an abrupt expansion as in Fig. 7.7 , all the kinetic energy is lost. Thus the EGL will drop an amount $V^2/2g$ at the outlet.

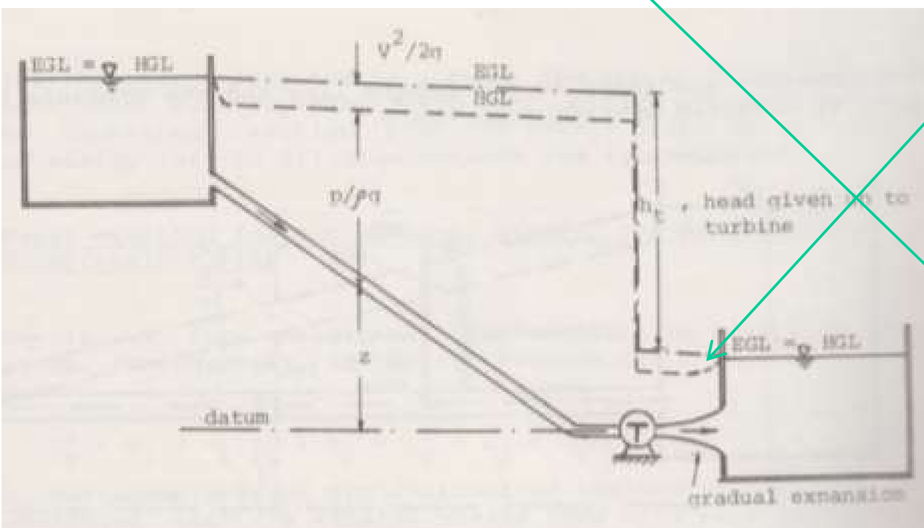
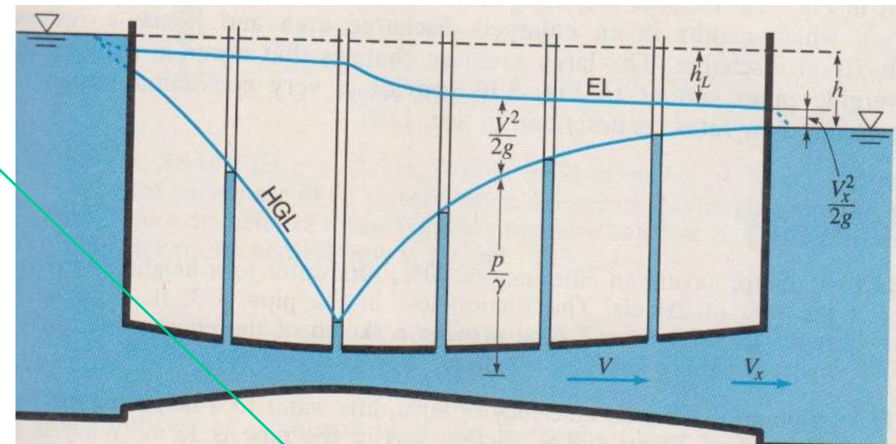


Fig. 7.5 Drop in EGL and HGL due to turbine

$V^2/2g$ reducing (due to increase of cross sectional area A) while $P/\rho g$ is increasing as the outlet expands



Turbine operates opposite to a pump. Some machines can be operated as a turbine or pump depending on the direction of flow of the water.

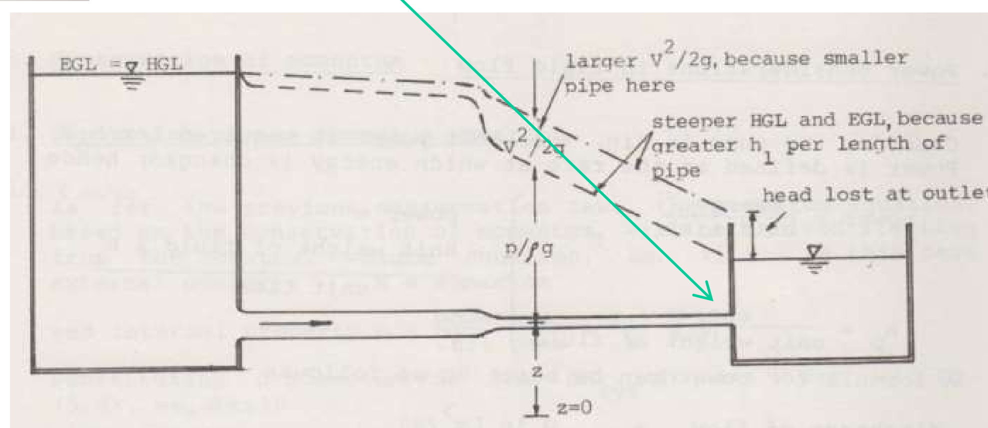


Fig. 7.7 Change in EGL and HGL due to change in diameter of pipe

d) In a pipe or channel where the pressure is zero, the HGL is coincident with the system, because $p/\rho g = 0$ at these points. This fact can be used to locate the HGL at certain points, such as at the outlet end of a pipe, where the liquid discharges into the atmosphere or at the upstream end where the gauge pressure is zero at the reservoir surface (see Fig. 7.3).

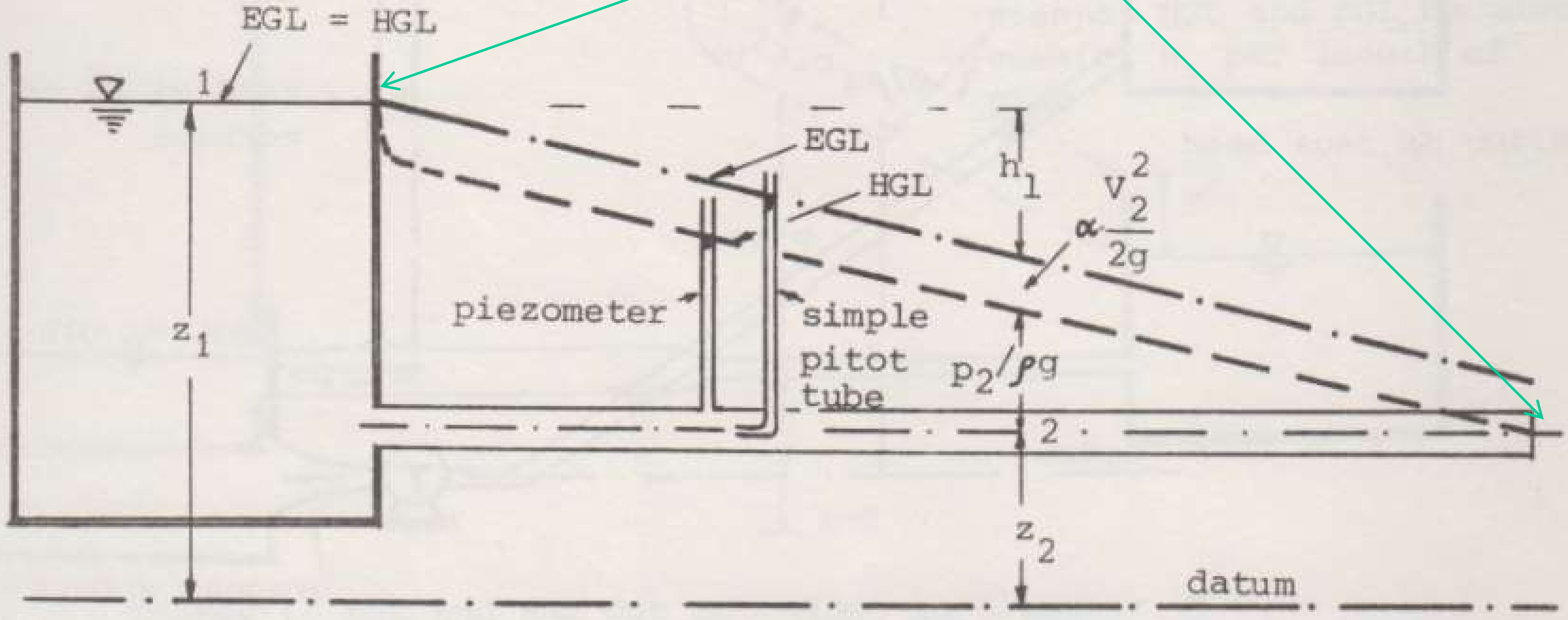


Fig. 7.3 EGL and HGL in a straight pipe

e) For steady flow in a pipe that has uniform physical characteristics (diameter, roughness, shape, etc.) along its length, the head loss per unit length will be constant. Thus the slope of the EGL and HGL will be constant along the length of the pipe (see Fig. 7.3)

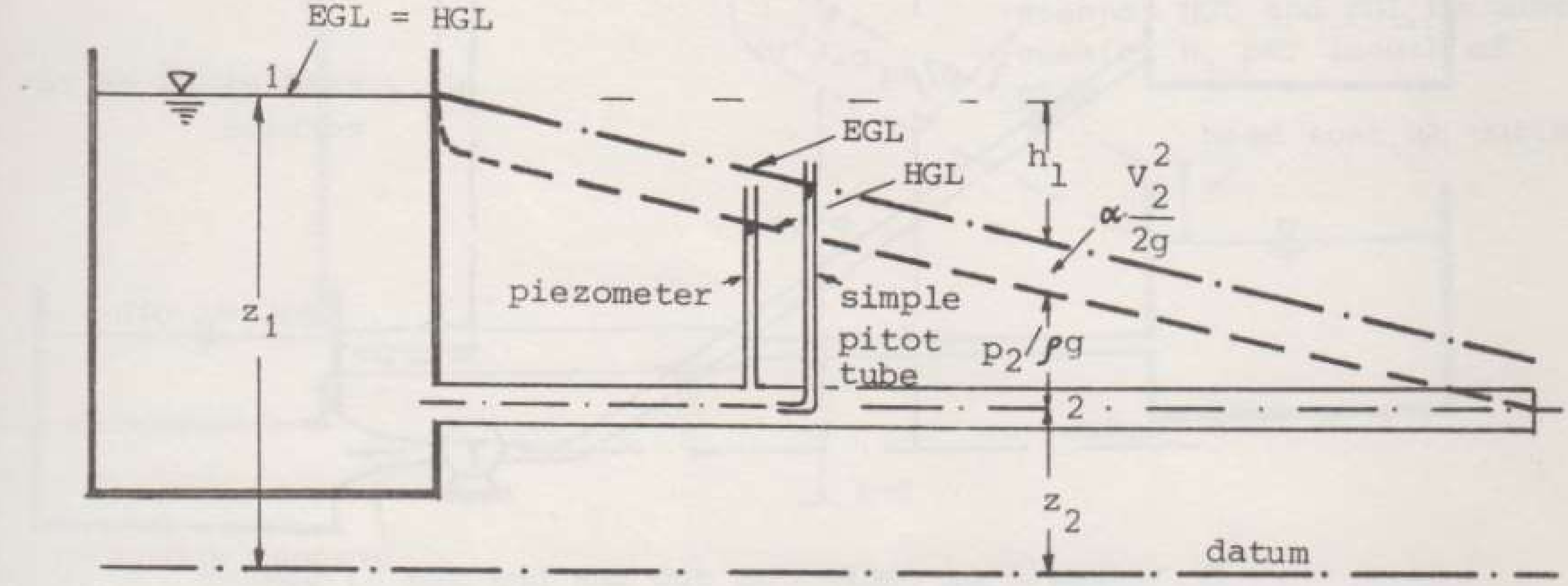


Fig. 7.3 EGL and HGL in a straight pipe

f) If a flow passage changes diameter, such as in a nozzle or by means of a change in pipe size, the velocity in there will also change. Hence, the distance between the EGL and the HGL will change (Fig. 7.6). In addition, the slope of the EGL will change, because the head loss per unit length will be larger in the conduit with the larger velocity (see Fig. 7.7). In Chapter 12 you will learn that $h_l :: V^2$.

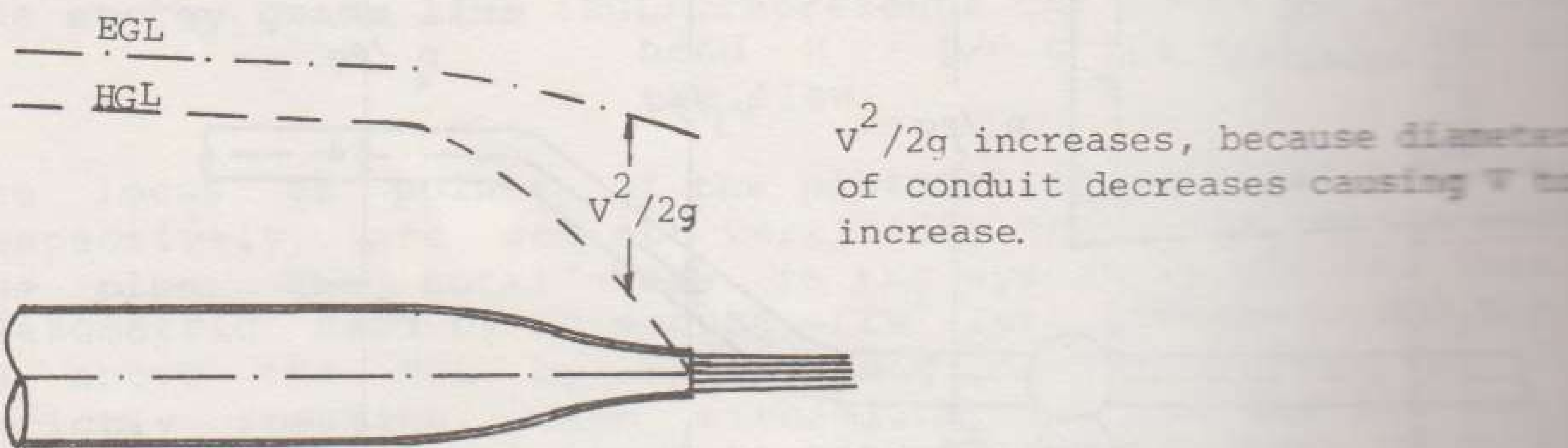
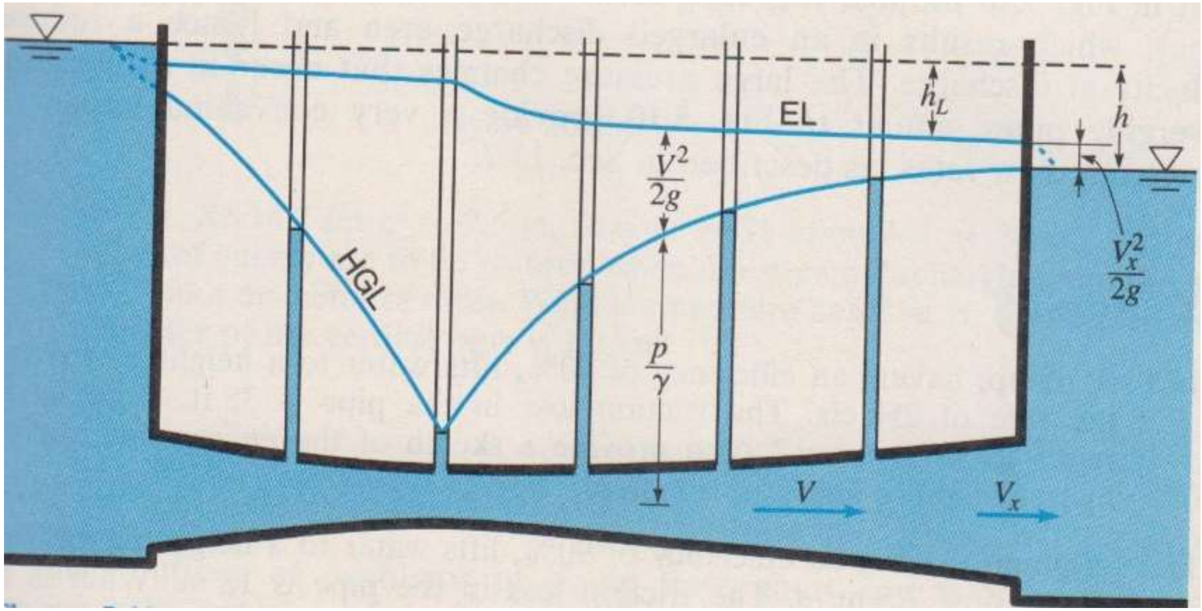


Fig. 7.6 Change in EGL and HGL due to flow in a nozzle



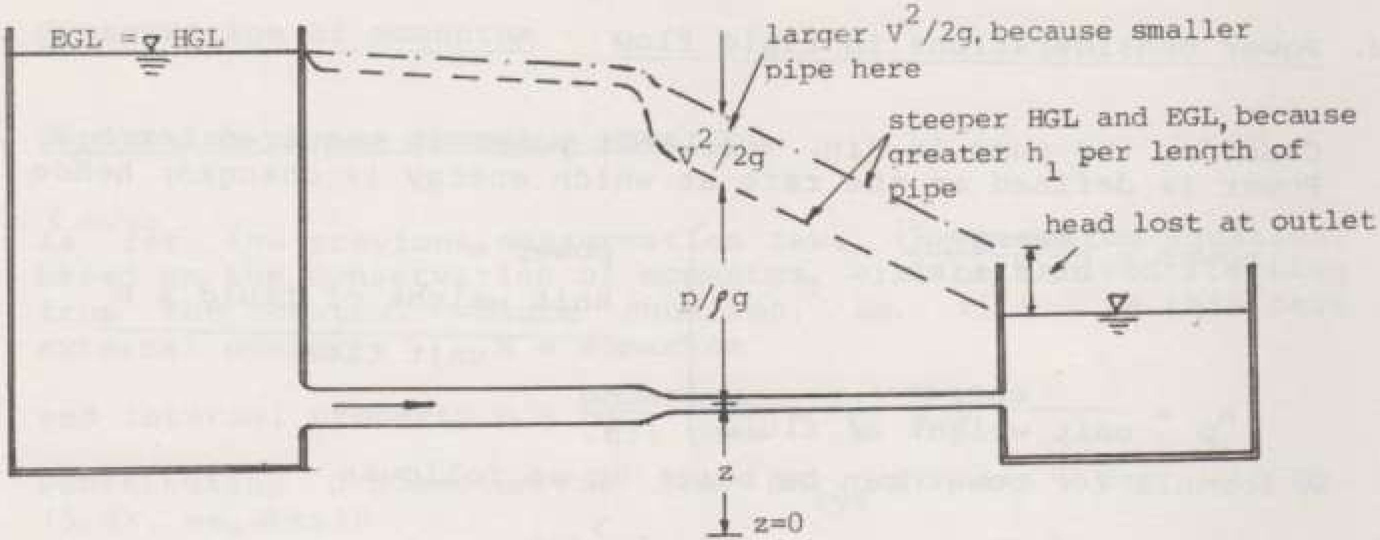


Fig. 7.7 Change in EGL and HGL due to change in diameter of pipe

g) If the HGL falls below the pipe, then $p/\rho g$ is negative, thereby indicating sub-atmospheric pressure (see Fig. 7.8). By means of the drawn HGL, places where cavitation may occur, are easily determined. At sea level, cavitation of water will occur at appr. $p/\rho g = -10$ m. For safety reasons, the minimum allowable pressure head in a pipe system is kept at -7.5 m, thus $p/\rho g > -7.5$ m.

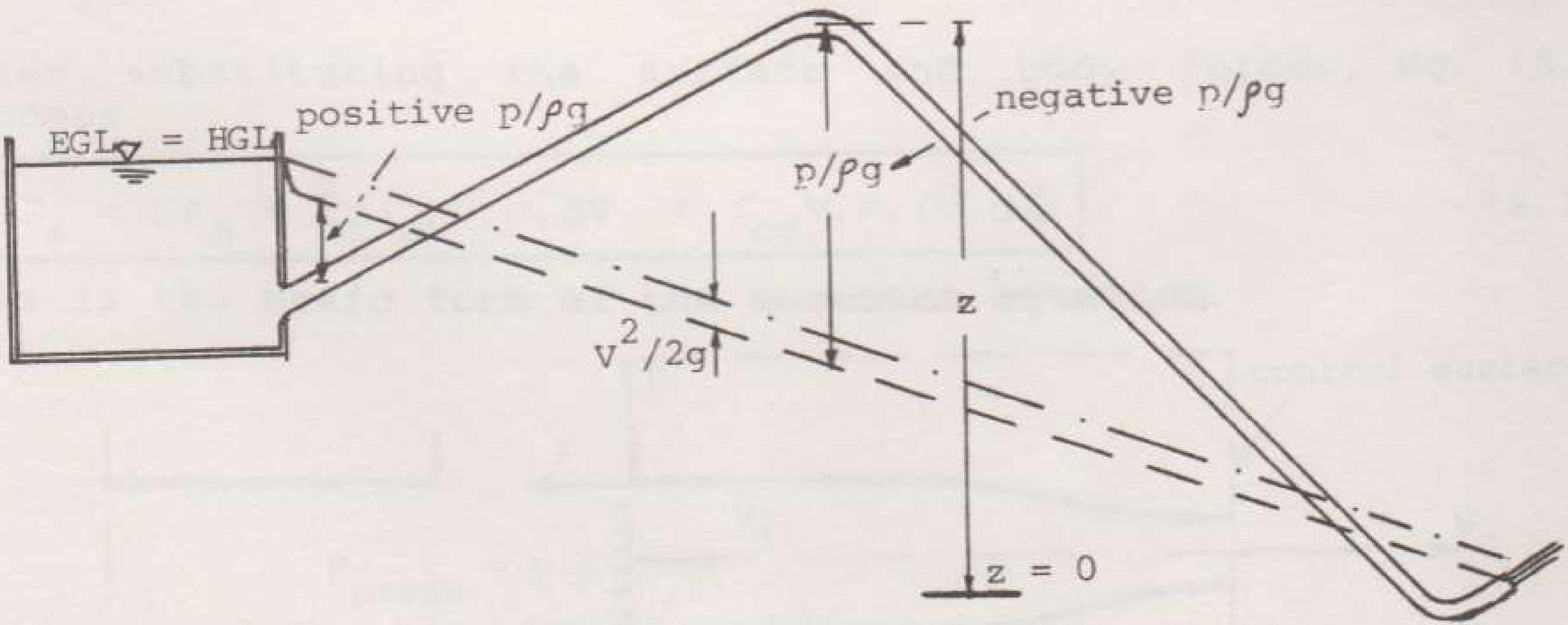


Fig. 7.8 Subatmospheric pressure in a pipe system.

Power considerations in fluid flow

Consider the pump. What power is required for h_p?
Power is defined as the rate at which energy is changed, hence

$$power = \frac{energy}{unit\ \ time}$$

=unit weight of fluid x h_p/unit time

$$P = \frac{(mg)h_p}{t} = \frac{(\rho V)g}{t} h_p = \rho g \frac{V}{t} h_p = \rho g Q h_p$$

$P = \rho g Q h_p$ power required for h_p

$P = \rho g Q h_t$ power delivered to a turbine

See Lecture 10 example below

Water flows from a reservoir through a 0.8-m-diameter pipeline to a turbine-generator unit and exits to a river that is 30 m below the reservoir surface (Fig. E4.7). If the flow rate is

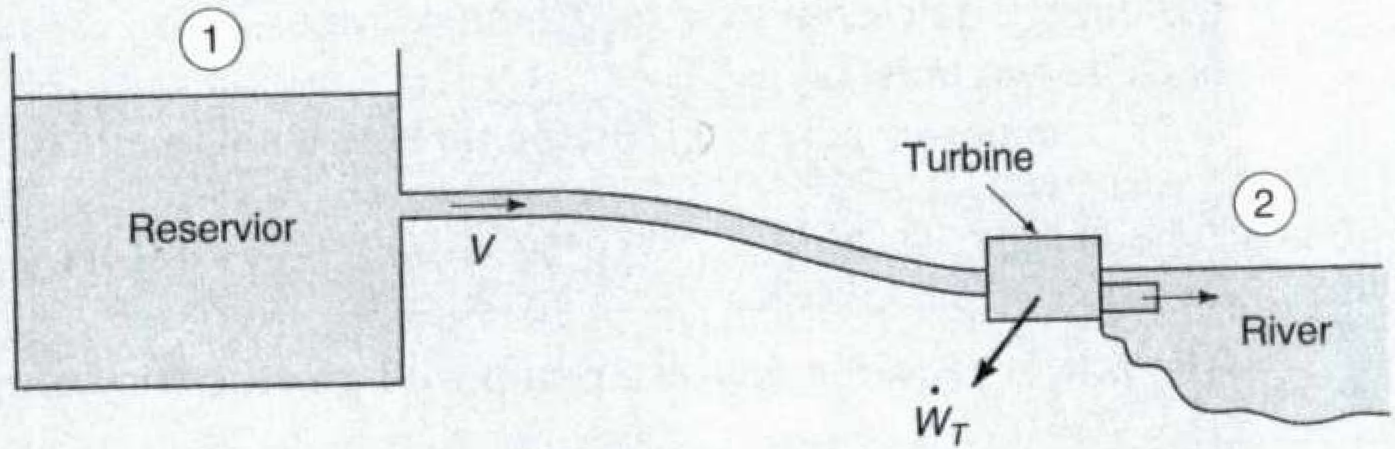


Figure E4.7

3 m³/s and the turbine-generator efficiency is 80%, calculate the power output. Assume the loss coefficient in the pipeline (including the exit) to be $K = 2$.

Solution: The control volume to be used extends from section 1 to section 2; we consider the water surface of the left reservoir to be the entrance and the water surface of the river to be the exit. Because we assume the water surfaces to be large, the velocities at the surfaces are negligible. The velocity in the pipe is

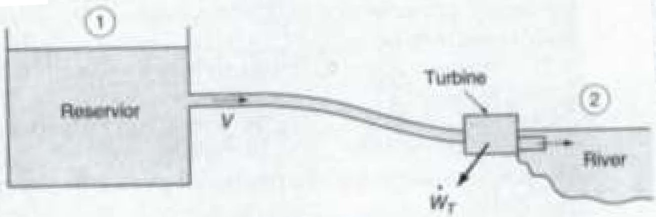


Figure E4.7

$$V = \frac{Q}{A}$$

$$= \frac{3}{\pi \times 0.8^2/4} = 5.968 \text{ m/s}$$

Now, consider the energy equation. We will use gage pressures so that $p_1 = p_2 = 0$; the datum is placed through the lower section 2 so that $z_2 = 0$; the velocities V_1 and V_2 are negligibly small; K is assumed to be based on the 0.8-m-diameter pipe velocity. The energy equation (4.4.24) then becomes

$$\cancelto{0}{H_P} + \cancelto{0}{\frac{V_1^2}{2g}} + \cancelto{0}{\frac{p_1}{\gamma}} + z_1 = H_T + \cancelto{0}{\frac{V_2^2}{2g}} + \cancelto{0}{\frac{p_2}{\gamma}} + \cancelto{0}{z_2} + K\frac{V^2}{2g}$$

$$30 = H_T + 2\frac{5.968^2}{2 \times 9.81}$$

$$\therefore H_T = 26.4 \text{ m}$$

From this the power output is found, using Eq. 4.4.25, to be

$$\dot{W}_T = Q\gamma H_T \eta_T$$

$$= 3 \times 9810 \times 26.4 \times 0.8 = 622\,000 \text{ W} \quad \text{or} \quad 622 \text{ kW}$$

In this example we have used gage pressure; the potential-energy datum was assumed to be placed through section 2, V_1 and V_2 were assumed to be insignificantly small, and K was assumed to be based on the 0.8-m-diameter pipe velocity.