

Fluid Mechanics CEE 3311

LECTURE 15

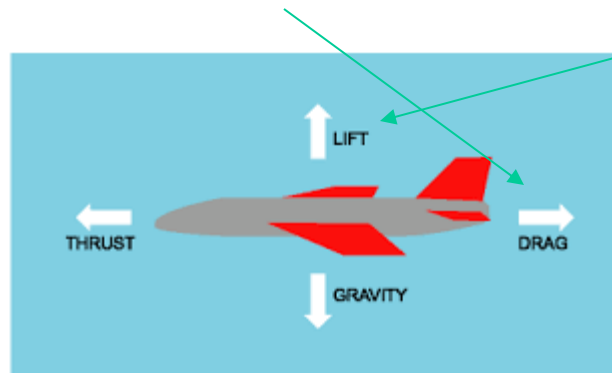
Flow of a Real Fluid
(External Flows)

L. Handia

Real fluid

Forces on immersed bodies

- When bodies are completely *surrounded by a flowing fluid*, as for example are airplanes, birds, automobiles, raindrops, submarines, and fish, the flows are known as ***external flows***.
- Such a body, wholly immersed in a homogeneous fluid, may be subject to two kinds of forces arising from *relative motion* between the body and the fluid.
- These forces are termed the ***drag*** and the ***lift***, depending on whether the force is *parallel* to the motion or *perpendicular* to it, respectively.



Forces on immersed bodies

- Fluid mechanics draws *no distinction* between two cases of relative motion, namely, when a *body moves* rectilinearly at constant speed *through a stationary fluid* or when a *fluid travels* at constant velocity past a *stationary body*.
- Thus it is possible to test *airplane* models in wind tunnels and *torpedo* models in water tunnels and predict with confidence the behaviour of their prototypes when moving through still fluid.
- For instructional purposes, it is somewhat simpler to fix our ideas on the stationary body in the moving fluid, while the *practical result desired* is more frequently associated with a body moving through still fluid such as a flying plane & moving vehicle.

Forces on immersed bodies

- In this lecture we shall first consider the drag, or resistance forces. As we shall not be concerned with wave action at a free surface, gravity does not enter the problem, and the forces involved are those due to inertia and viscosity.
- The drag forces on a submerged body can be viewed as having two components: a ***pressure drag*** F_p , and a ***friction drag*** (or surface drag) F_f .
- The pressure drag, often referred to as the ***form drag*** because it *depends on the form or shape* of the body, is equal to the integration of the components in the direction of motion of all the pressure forces exerted on the surface of the body. It may be expressed as

$$F_p = C_p \rho \frac{V^2}{2} A$$

where C_p is coefficient dependent on the geometric form of the body, A is the projected area of the body normal to the flow

Forces on immersed bodies

- The friction drag along a body surface is equal to the integral of the components of the shear stress along the surface in the direction of motion.
- For convenience, the friction drag is commonly expressed in the same general form as for pressure drag. Thus,

$$F_f = C_f \rho \frac{V^2}{2} BL$$

where C_f = friction-drag coefficient, dependent on viscosity among other factors

L = length of surface parallel to flow

B = transverse width, conveniently approximated for irregular shapes by dividing total surface area by L

Boundary layer

- The boundary layer is a very thin layer of fluid adjacent to surface, in which **viscosity** is important, while **outside this layer** the fluid can be considered as **frictionless** or ideal.
- This concept, originated by Ludwig Prandtl in 1904, is one of the important advances in modern fluid mechanics.
- In 1904 Ludwig Prandtl published a key paper, proposing that the flow fields of **low-viscosity fluids** be divided into two Zones, namely
 - ❑ a thin, viscosity-dominated boundary layer near solid surfaces, and
 - ❑ an effectively inviscid outer zone away from the boundaries.

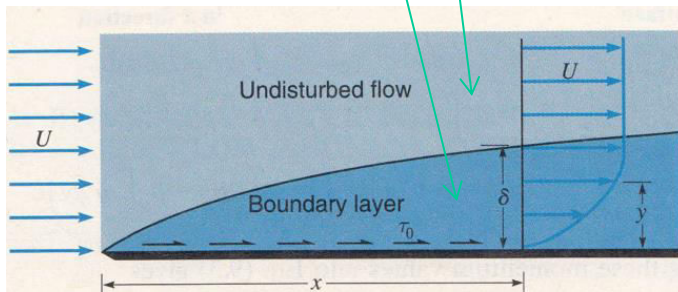


Figure 9.1

Growth of a boundary layer along a smooth plate (vertical scale exaggerated).



Ludwig Prandtl	
Born	4 February 1875 Freising, Upper Bavaria, German Empire
Died	15 August 1953 (aged 78) Göttingen, West Germany
Nationality	German
Alma mater	Technical University of Munich, RWTH Aachen
Known for	Boundary layer Mixing length theory Lifting-line theory Membrane analogy Prandtl condition Prandtl number Prandtl–Meyer expansion fan Prandtl–Meyer function Prandtl–Batchelor theorem Prandtl–Glauert transformation Prandtl–Glauert singularity Prandtl–Tomlinson model
Awards	Ackermann–Teubner Memorial Award (1918) Daniel Guggenheim Medal (1930) Wilhelm Exner Medal 1951
Scientific career	
Fields	Aerodynamics
Institutions	University of Göttingen, Technical University of Hannover
Thesis	Tilting Phenomena, A case of unstable elastic balance (1899)
Doctoral advisor	August Föppl
Doctoral students	Ackeret, Blasius, Busemann, Munk, Nikuradse, Pohlhausen, Schlichting, Tietjens, Tollmien, von Kármán, Timoshenko, Vishnu Madav Ghatage and many others (85 in total).

Boundary layer

- It means that the mathematical theory of *ideal fluid flow*, including the flow net method discussed in Lecture 14, *can actually be used* to determine the streamlines *in the real fluid* at a short distance from a solid boundary.
- The Bernoulli theorem may then be used to determine the normal pressures on the surface, for such pressures are practically the same as those outside this thin layer.

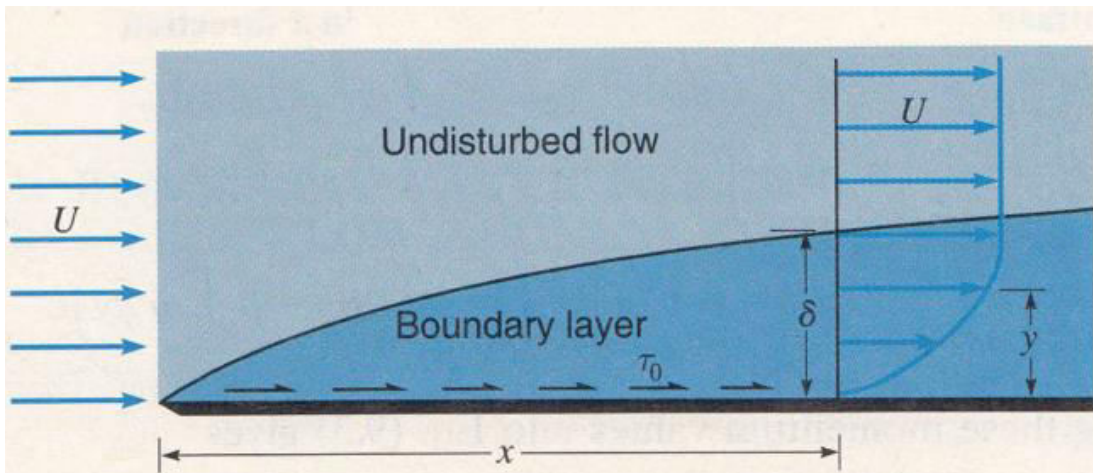


Figure 9.1

Growth of a boundary layer along a smooth plate (vertical scale exaggerated).

Boundary layer

- The boundary layer may be entirely laminar, or it may be primarily turbulent with a viscous sublayer. This is a layer where shear is predominantly due to viscosity alone.
- The thickness δ of the boundary layer is usually defined as the distance from the boundary to the point *where the velocity is 99%* of the undisturbed velocity, i.e., to where $u = 0.99U$.
- δ increasing with x

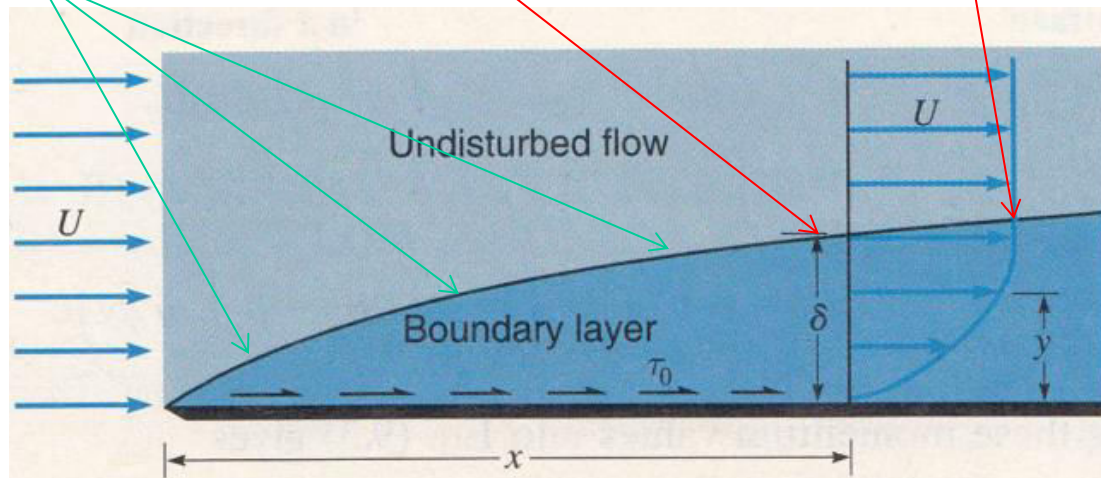


Figure 9.1

Growth of a boundary layer along a smooth plate (vertical scale exaggerated).

Turbulent boundary layer for flow along a smooth flat plate

- Comparing the laminar and turbulent boundary layers in Fig. 9.4, the velocity distribution in the *turbulent boundary layer* shows a *much steeper gradient near the wall* and a *flatter gradient* throughout the remainder of the layer.
- As would be expected, then, the wall *shear stress is greater* (due to higher du/dy) in the *turbulent boundary layer* than in the laminar layer at the same Reynolds number.

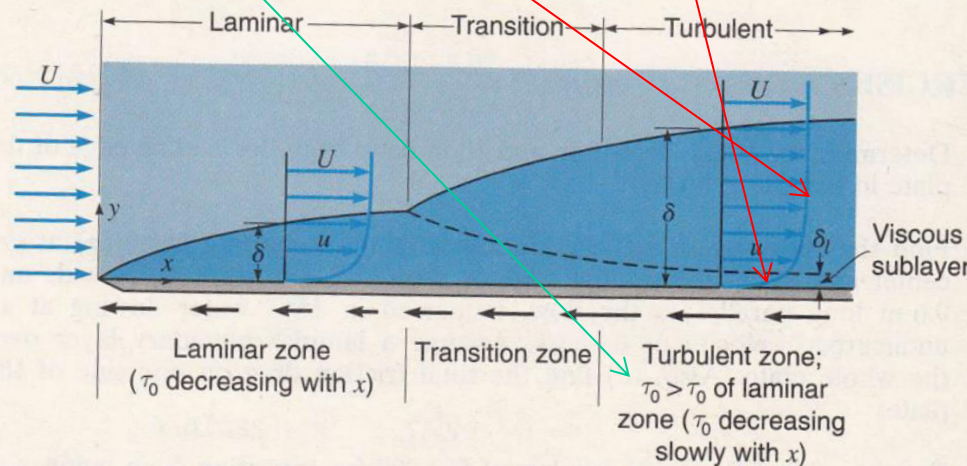


Figure 9.4

Laminar and turbulent boundary layers along a smooth flat plate (vertical scale greatly exaggerated).

$$\tau = \mu \frac{du}{dy}$$

Turbulent boundary layer for flow along a smooth flat plate

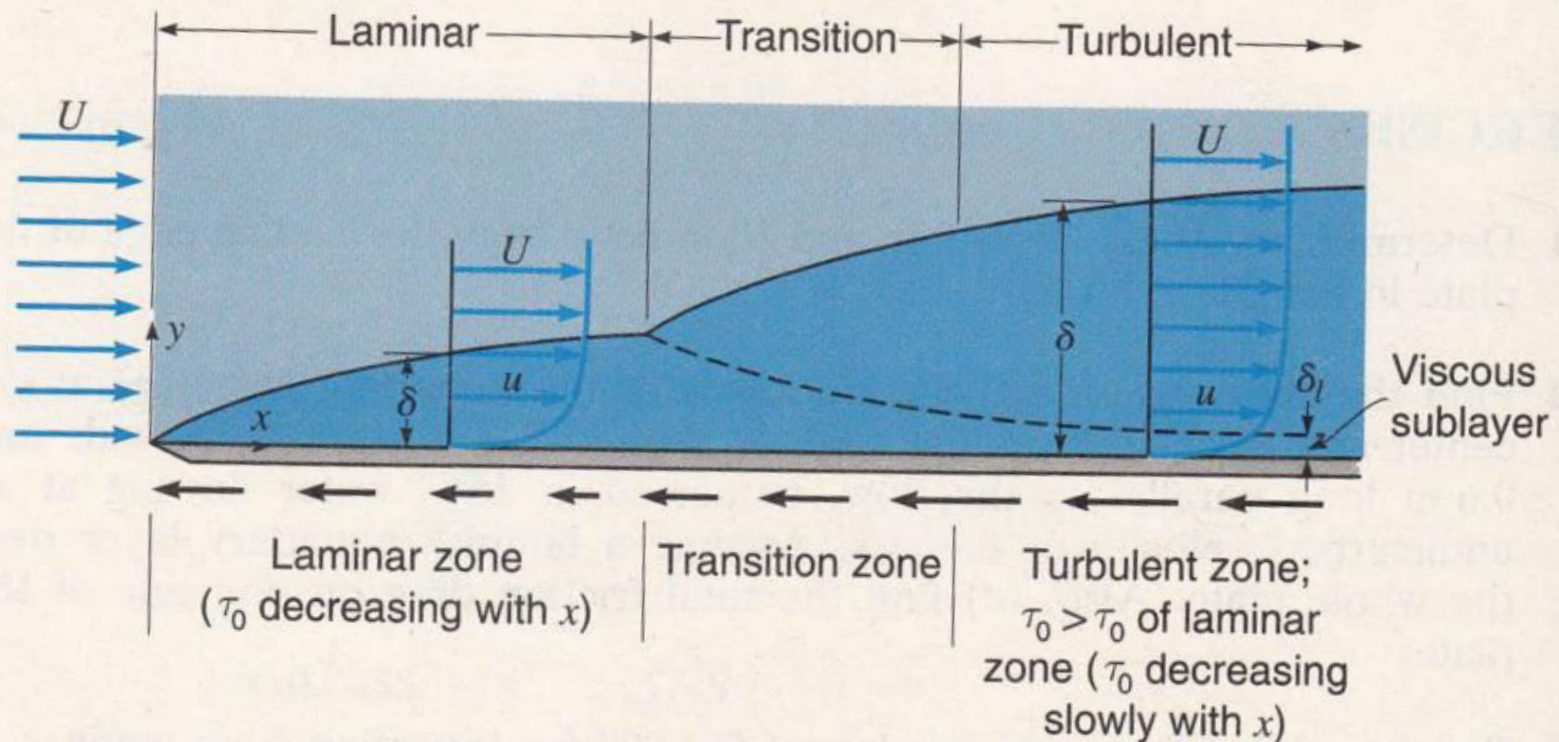


Figure 9.4

Laminar and turbulent boundary layers along a smooth flat plate (vertical scale greatly exaggerated).

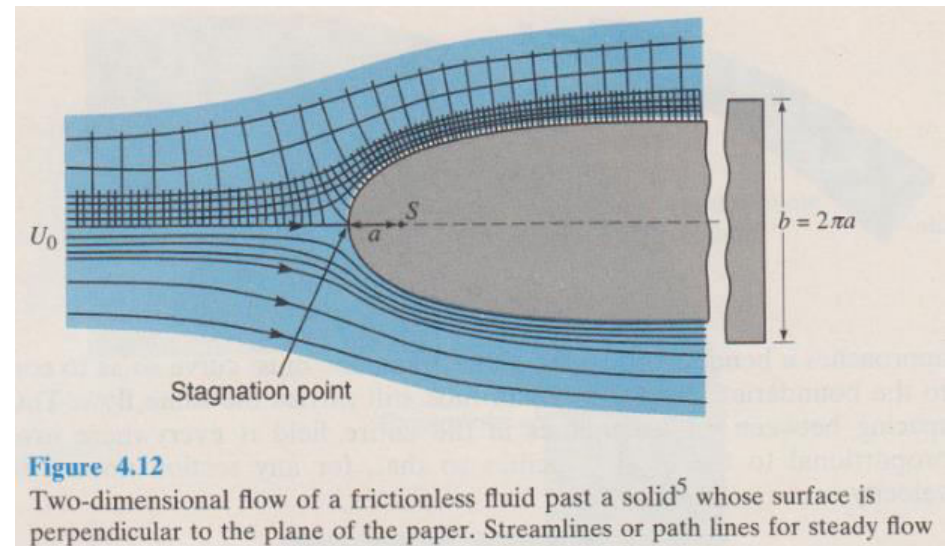
Boundary layer separation and pressure drag

The motion of a thin stratum of fluid lying wholly **inside the boundary layer** is determined by **three** forces:

1. The **forward pull of the outer free-moving fluid**, transmitted through the laminar boundary layer **by viscous shear** and through the turbulent boundary layer **by momentum transfer**;
2. The **viscous retarding effect of the solid boundary**, which must, by definition, hold the fluid stratum immediately adjacent to it at rest;
3. The pressure gradient along the boundary: the stratum is **accelerated** by a pressure gradient whose **pressure decreases** in the direction of flow and is retarded by an adverse gradient.

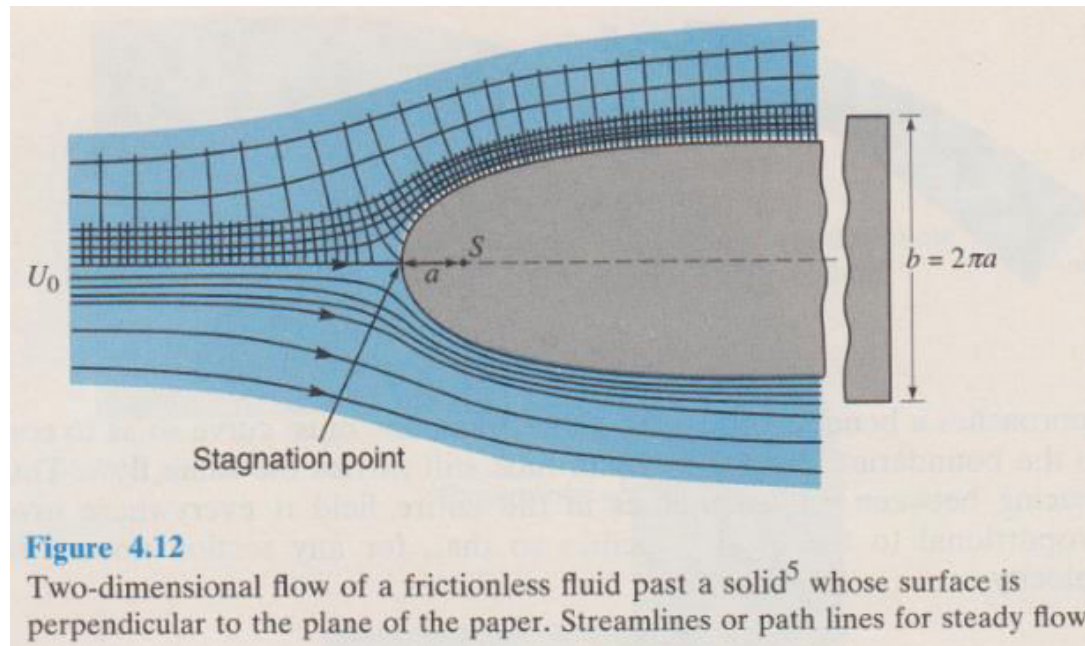
Boundary layer separation and pressure drag

- The treatment of fluid resistance in the foregoing sections has been restricted to the drag of the boundary layer along a smooth flat plate located in an unconfined fluid, that is to say, in the **absence of a pressure gradient**.
- In the *presence* of a favorable *pressure gradient*, the boundary layer is “held” in place. This is what occurs in the accelerated flow around the forebody, or upstream portion, of a cylinder, sphere, or other object, such as that of Fig. 4.12.



Boundary layer separation and pressure drag

- If a particle enters the boundary layer near the forward stagnation point *with a low velocity and high pressure*, its *velocity will increase* as it flows *into the lower-pressure* region along the side of the body.
- But there will be *some retardation* from wall friction (force 2 above), so that its total useful energy will be reduced by a corresponding *conversion into thermal energy*.



Boundary layer separation and pressure drag

- What happens next may best be explained by reference to Fig. 9.8.
- Let A represent a point in the region of accelerated flow with a normal velocity distribution in the boundary layer (either laminar or turbulent), while B is the point where the velocity **outside the boundary layer** reaches a maximum.

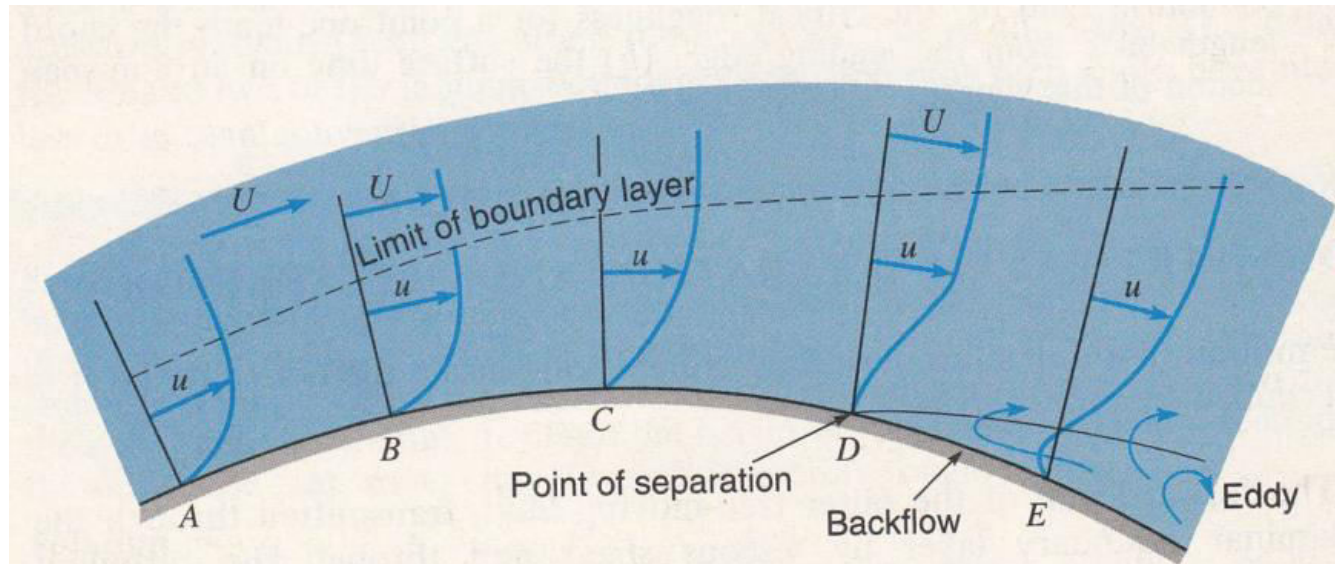


Figure 9.8

Growth and separation of boundary layer owing to increasing pressure gradient. Note that U has its maximum value at B and then gets smaller.

Boundary layer separation and pressure drag

- Then C, D, and E are points downstream where the velocity **outside the boundary layer** decreases, **resulting** in an increase in pressure in accordance with ideal-flow theory (when velocity head reduces, pressure head increases since sum of terms in the Bernoulli Eq, are constant).
- Thus the velocity of the layer **close to the wall** is reduced at C and finally brought to a *stop* at D. Now the *increasing pressure calls for further retardation; but this is impossible*, and so the boundary layer actually **separates** from the wall.
- At E there is a **backflow** **next to the wall**, driven in the *direction of decreasing pressure* (pressure increases downstream)-upstream in this case-and *feeding fluid* into the boundary layer that has left the wall at D.

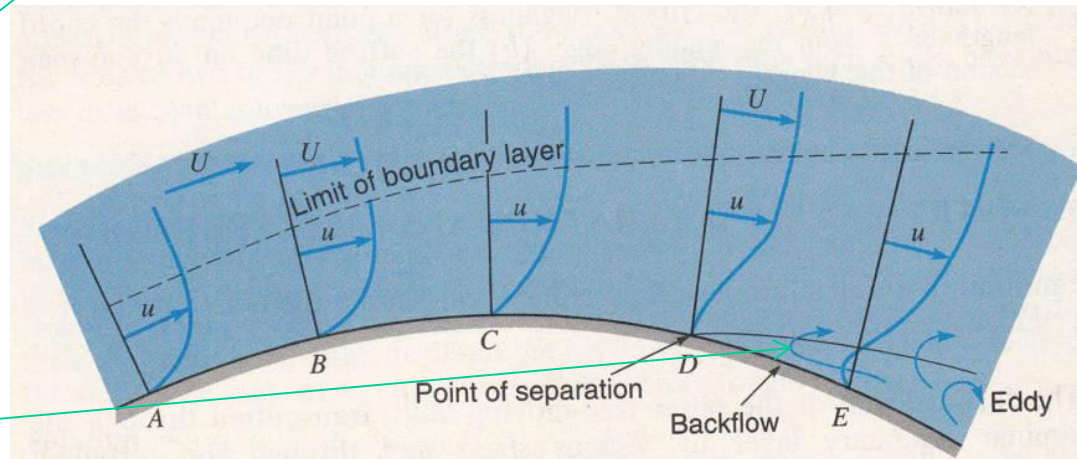


Figure 9.8

Growth and separation of boundary layer owing to increasing pressure gradient. Note that U has its maximum value at B and then gets smaller.

Boundary layer separation and pressure drag

- Downstream from the point of separation, the flow is characterised by irregular turbulent eddies, formed as the separated boundary layer becomes *rolled up* in the reversed flow. This condition generally extends for some distance downstream until the eddies are worn away by viscous attrition. The whole disturbed region is called the **turbulent wake** of the body (Fig. 9.9).

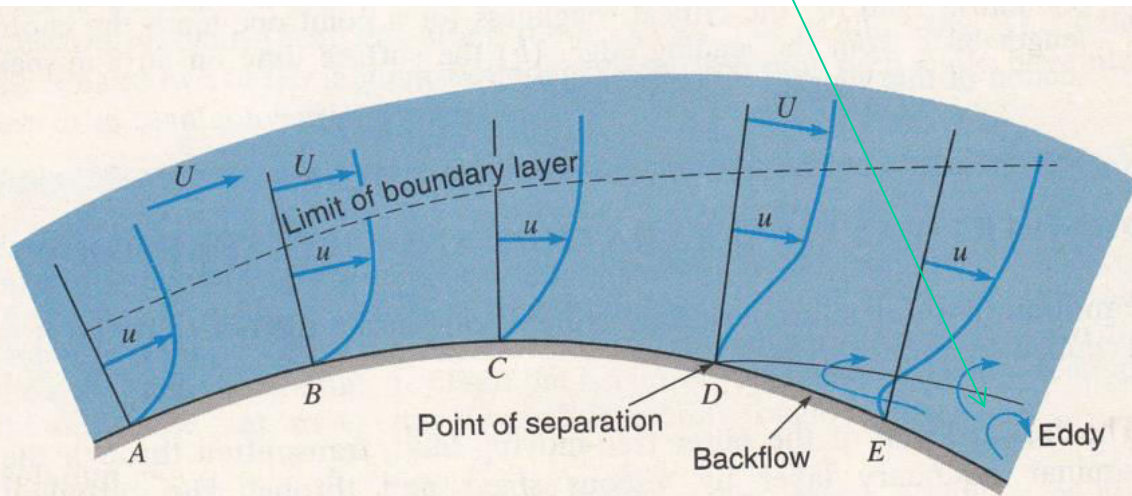


Figure 9.8

Growth and separation of boundary layer owing to increasing pressure gradient. Note that U has its maximum value at B and then gets smaller.

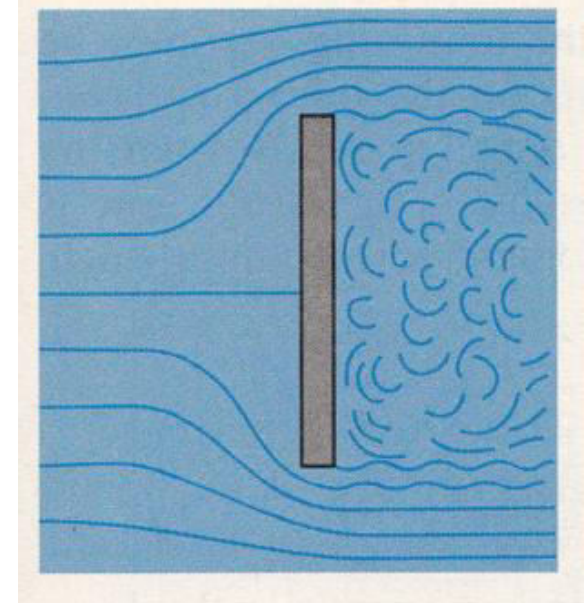
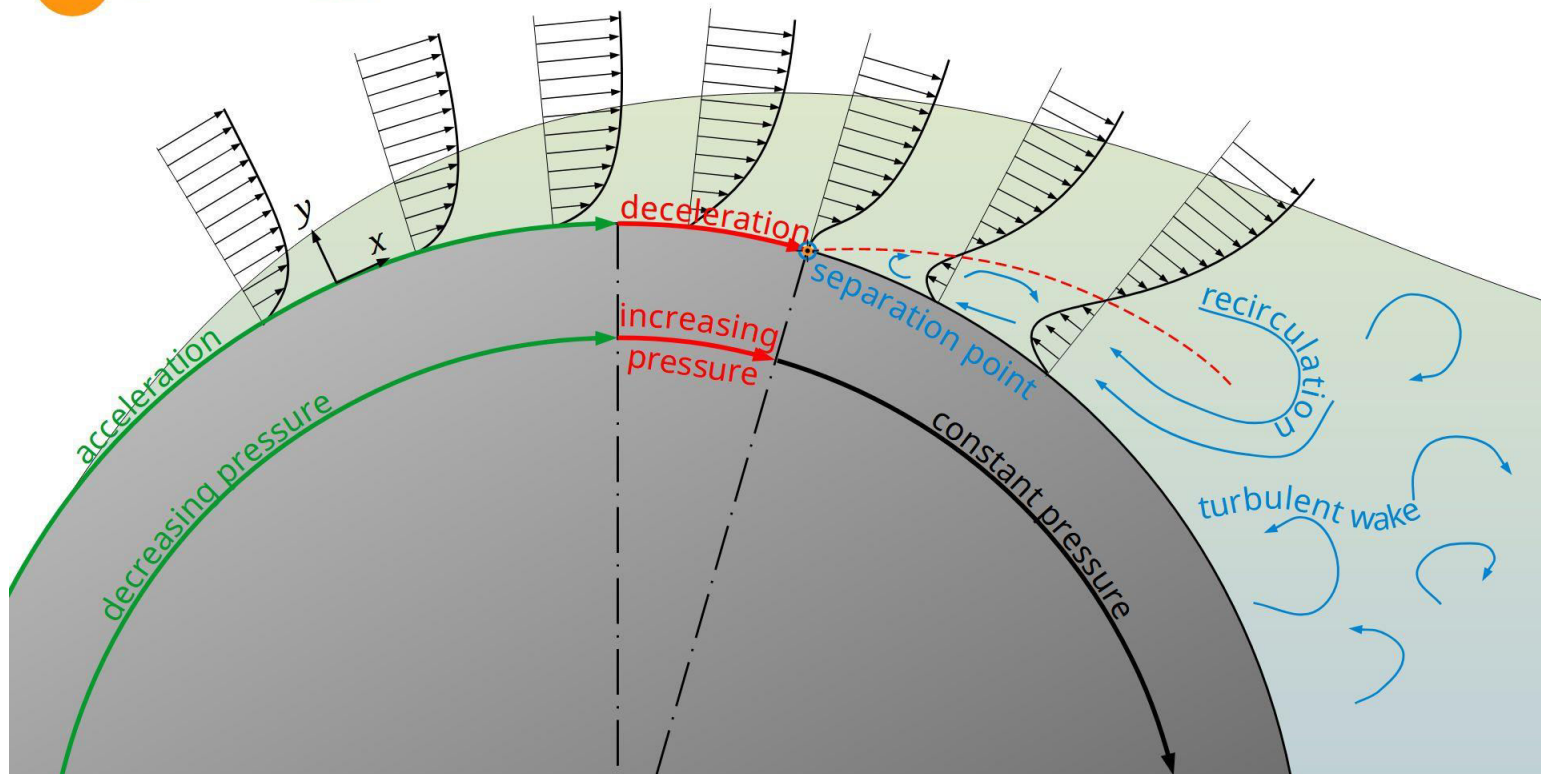


Figure 9.9 Turbulent wake behind a flat plate held normal to the flow

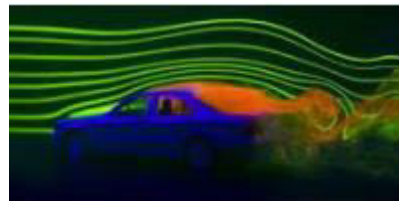
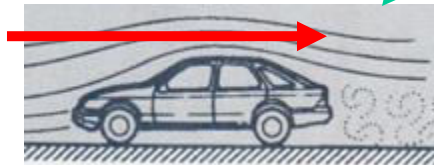
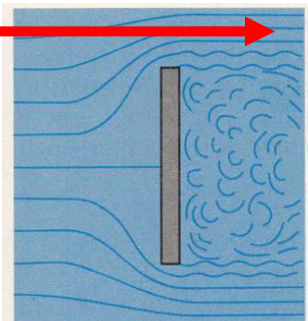
Boundary layer separation and pressure drag

- Because the eddies cannot convert their kinetic energy of rotation into an increased pressure, as the ideal-fluid theory would dictate, the *pressure within the wake remains close to that at the separation point*.



Boundary layer separation and pressure drag

- Because the eddies cannot convert their kinetic energy of rotation into an increased pressure, as the ideal-fluid theory would dictate, the *pressure within the wake remains close to that at the separation point*.
- Since this is always less than the pressure at the forward stagnation point, there results a *net pressure difference* tending to *move the body with the flow*, and this force is the **pressure drag**.
- For a moving body such as a plane or car, the front has higher pressure than the rear and therefore the drag is opposite to the plane or car movement and therefore **pressure drag tends to resist motion of the plane/car**. It drags/resists the plane/car.



Drag on three –dimensional bodies

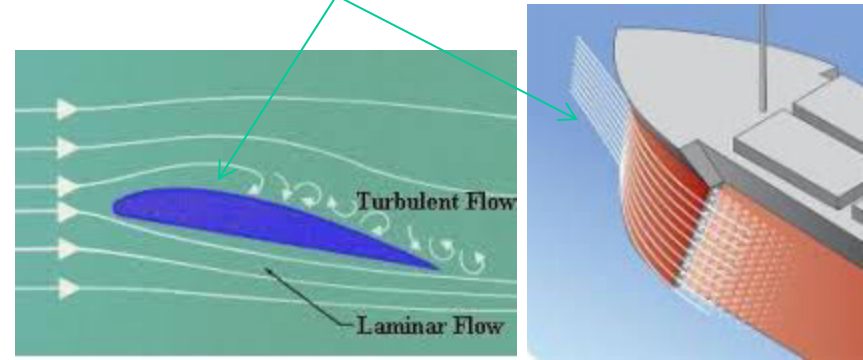
- The total drag on a body is the sum of the friction drag and the pressure drag:

$$F_D = F_f + F_p$$

- In the case of a well-streamlined body, such as an airplane wing or the hull of a submarine, the friction drag is the major part of the total drag, and may be estimated by the methods of the preceding sections on the boundary layer. Only rarely is it desired to compute the pressure drag separately from friction drag. Usually, when the wake resistance becomes significant, one is interested in the total drag only. Indeed, it is customary to employ a single equation that gives the total drag

$$F_D = C_D \rho \frac{V^2}{2} A \quad 9.32$$

where C_D is overall drag coefficient



Drag on three –dimensional bodies

Drag coefficients

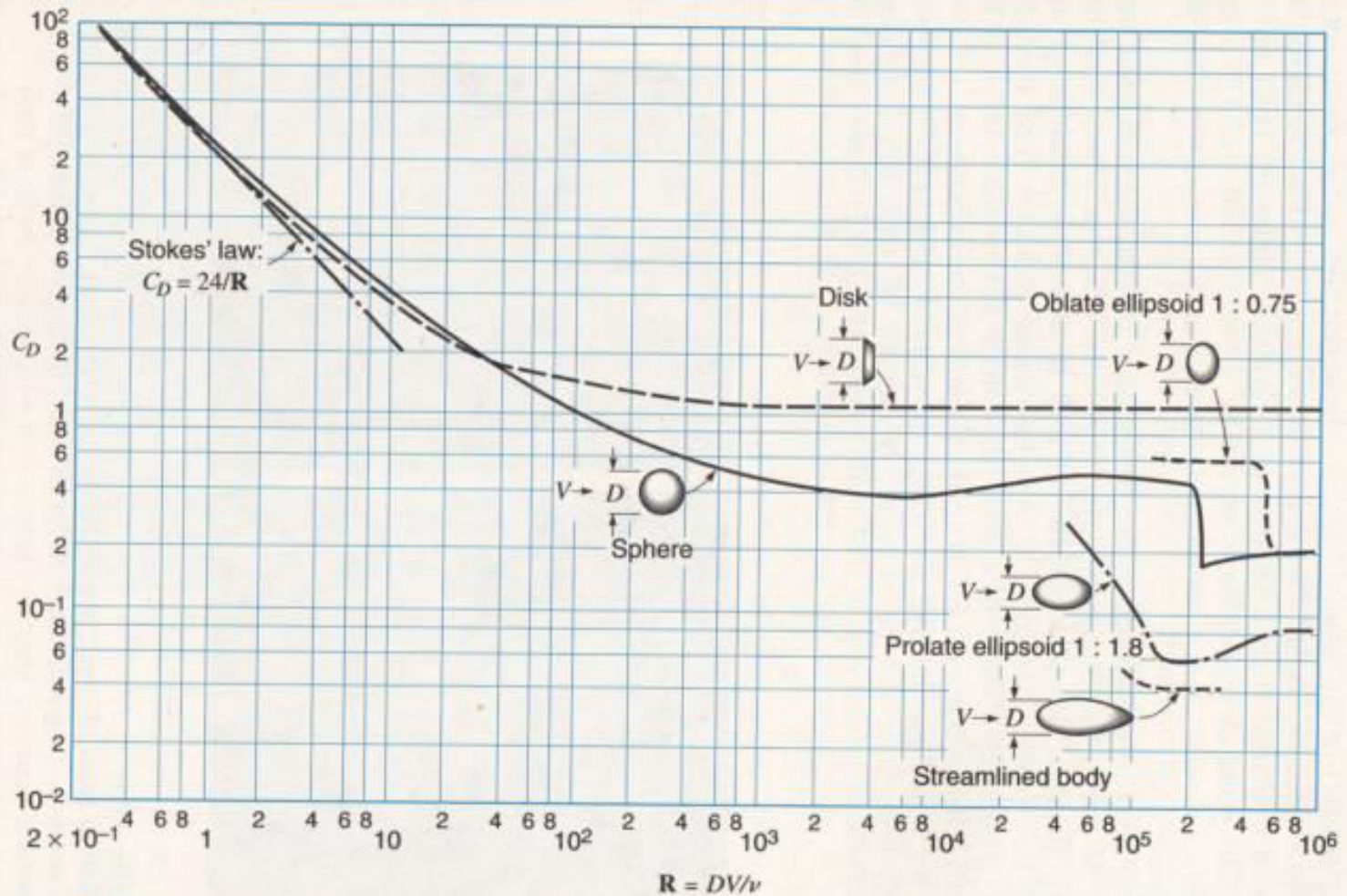


Figure 9.10

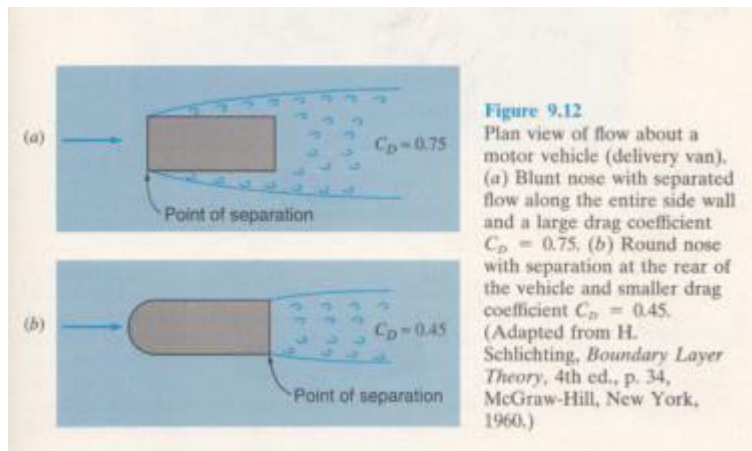
Drag coefficients for axisymmetric bodies.

Drag on three –dimensional bodies

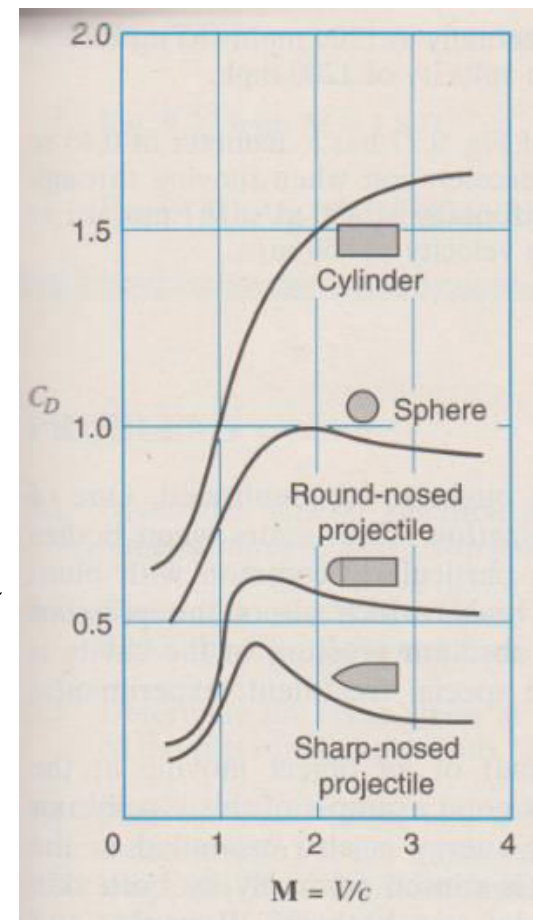
- It may be pointed out here that the ***object*** of *streamlining a body* is to move the point of separation *as far back as possible* and thus to produce the *minimum size of turbulent wake*. This decreases the pressure drag, but by making the body longer so as to promote a gradual increase in pressure, the friction drag is increased. The *optimum amount of streamlining*, then, is that for which the sum of the friction and pressure drag is a minimum.
- Attention in streamlining must be given to the *rear end*, or downstream part, of a body as well as to the *front*.
- The shape of the forebody is important principally to the extent that it *governs the location of the separation point(s) on the afterbody*. A rounded nose produces the least disturbance in the streamlines, and is therefore the best for incompressible or compressible flow at *subsonic* velocities (less than speed of sound (in air) 1,200km/h).

Drag on three –dimensional bodies

This is illustrated in Fig. 9.12, where flow about a blunt-nosed motor vehicle is compared with that about a round-nosed vehicle.



For *supersonic flow* ($>1200\text{km/h}$), a sharp-nosed body has less drag than a round-nosed body (Fig. 9.27).



Drag on three –dimensional bodies

Subsonic vs supersonic



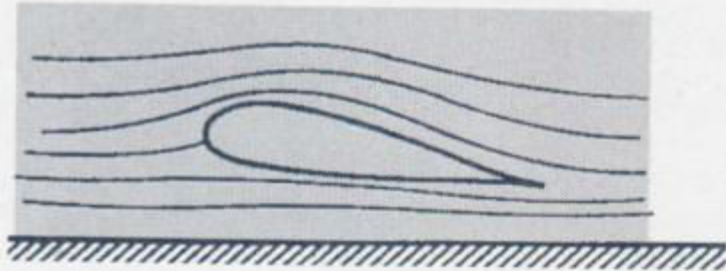
Concorde



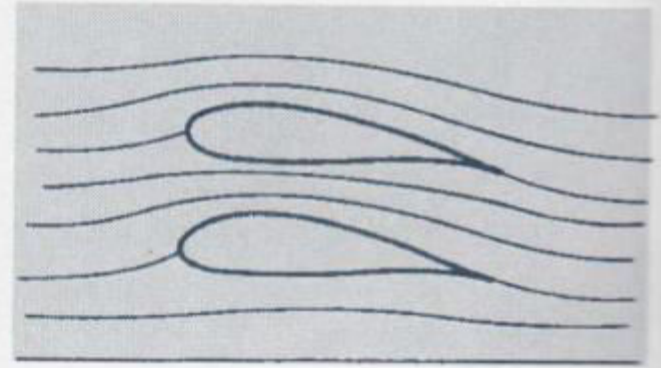
British Airways Concorde in 1986

Role	Supersonic transport
National origin	United Kingdom and France
Manufacturer	BAC (later BAe and BAE Systems) Sud Aviation (later Aérospatiale and Airbus)
First flight	2 March 1969
Introduction	21 January 1976
Retired	24 October 2003 ^[1]
Status	Retired
Primary users	British Airways Air France See Operators below for others
Produced	1965–1979
Number built	20 (inc. 6 non-commercial aircraft) ^{[2][3]}
Program cost	£1.3 billion ^[4] (£7.67 billion in 2016 pounds ^[5])
Unit cost	US\$33.8M (1972) ^[6] (\$157 million in 2018 dollars ^[7]) £23 million in 1977 (£119 million in 2016 pounds ^[5])

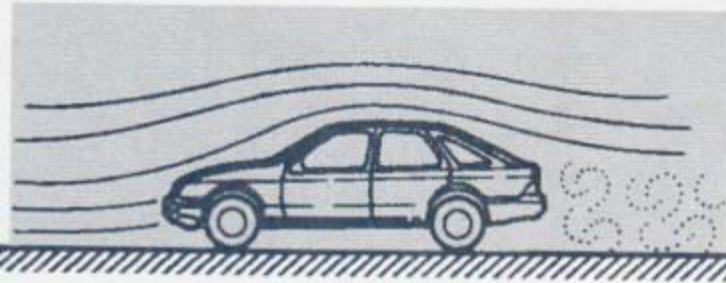
Drag on three –dimensional bodies



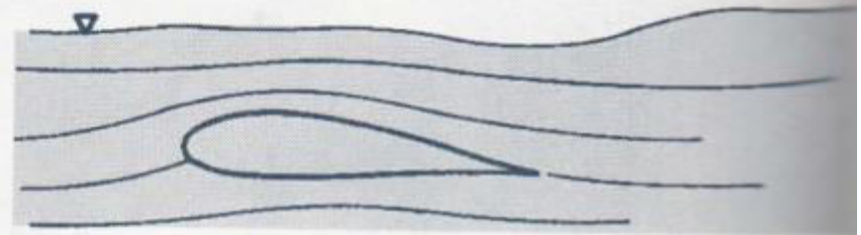
(a)



(b)



(c)



(d)

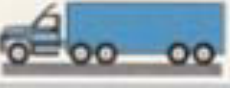
Figure 8.1 Examples of immersed flows: (a) flow near a solid boundary; (b) flow between two turbine blades; (c) flow around an automobile; (d) flow near a free surface.

Drag on three –dimensional bodies

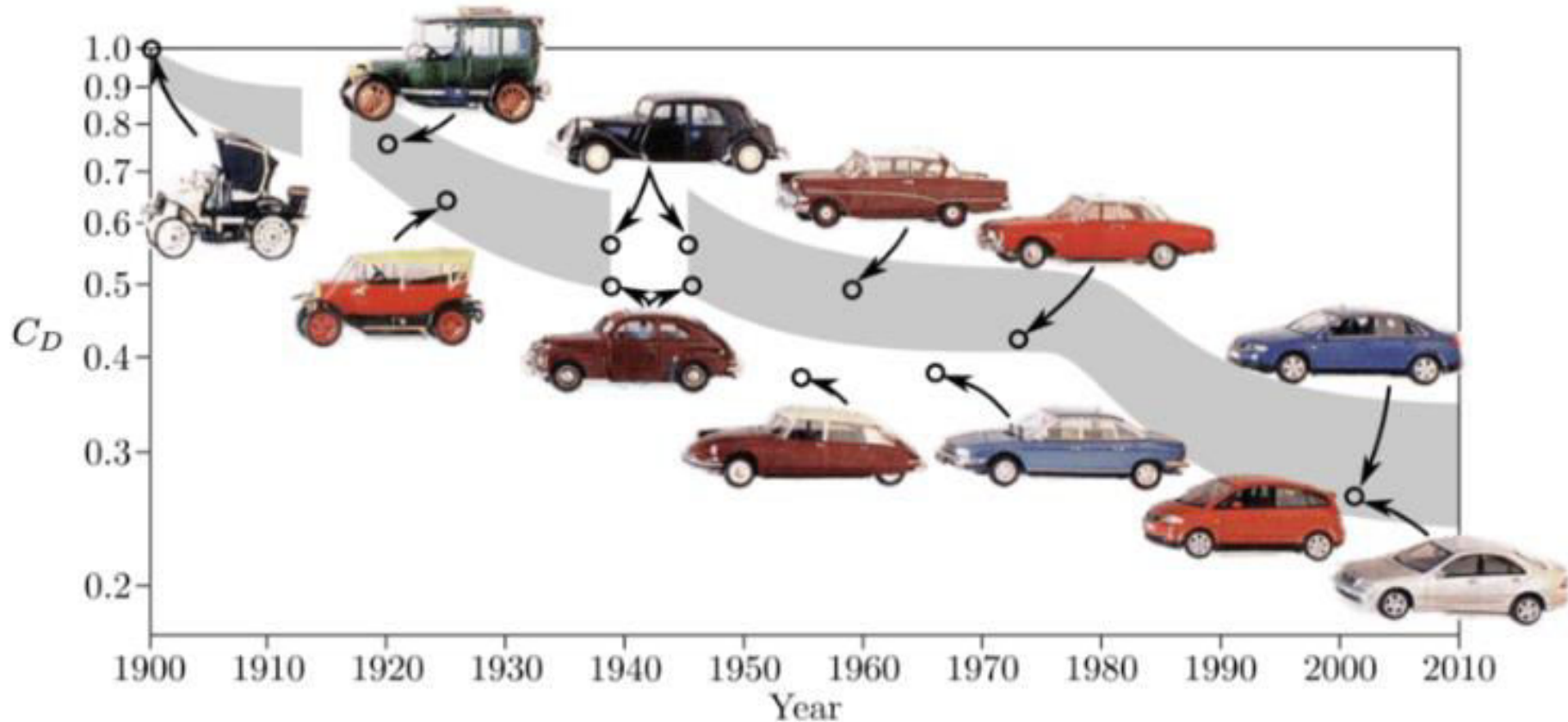
Efforts to reduce the drag force on automobiles and trucks, and thereby improve their **fuel efficiency**, have strongly **influenced body design**. Examples of historical body shapes and the resulting drag coefficients are given in the figure below

Table 9.1
The effects of automobile body shapes on aerodynamic drag

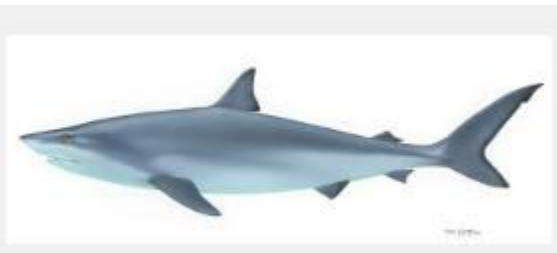
		Body shape	C_D based on frontal area
1920	WW 1		0.80
1940–45	WW 2		0.54–0.58

1968–69		0.36–0.38
1990–92		0.29–0.30
Tractor-trailer		0.75–0.95
With rounded cab, fairing		0.55–0.75

Drag on three –dimensional bodies

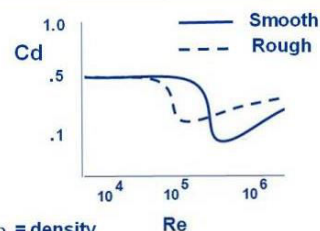
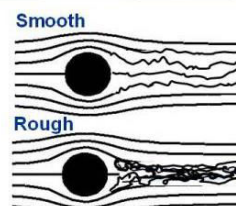






National Aeronautics and Space Administration

Drag on a Soccer Ball



d = diameter D = drag ρ = density
 A = Area V = velocity μ = viscosity

$$\text{Reynolds Number} = Re = \frac{\rho V d}{\mu} \sim 3.0 \times 10^5$$

$$\text{Drag Coefficient} = Cd = \frac{2 D}{\rho V^2 A} = .25$$

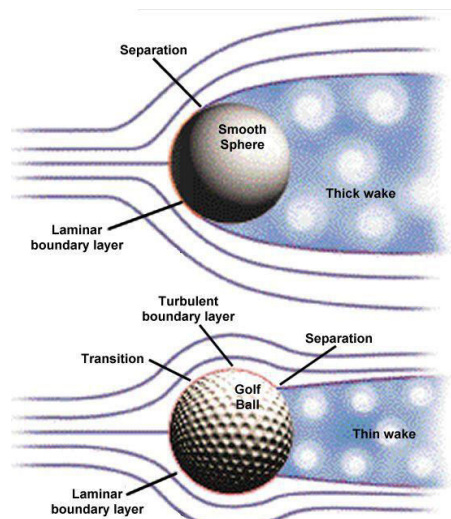


Fig. 2a. Distribution of aerodynamic drag coefficient (FLA of 20°)



Fig. 2b. Distribution of aerodynamic drag coefficient (FLA of 30°)



Fig. 2c. Distribution of aerodynamic drag coefficient (FLA of 40°)



Fig. 2d. Distribution of aerodynamic drag coefficient (FLA of 60°)

Drag on two –dimensional bodies

- Two dimensional bodies are also subject to friction and pressure drag. However, the flow about a two-dimensional body exhibits some peculiar properties that are not ordinarily found in the three-dimensional case of flow around a sphere.

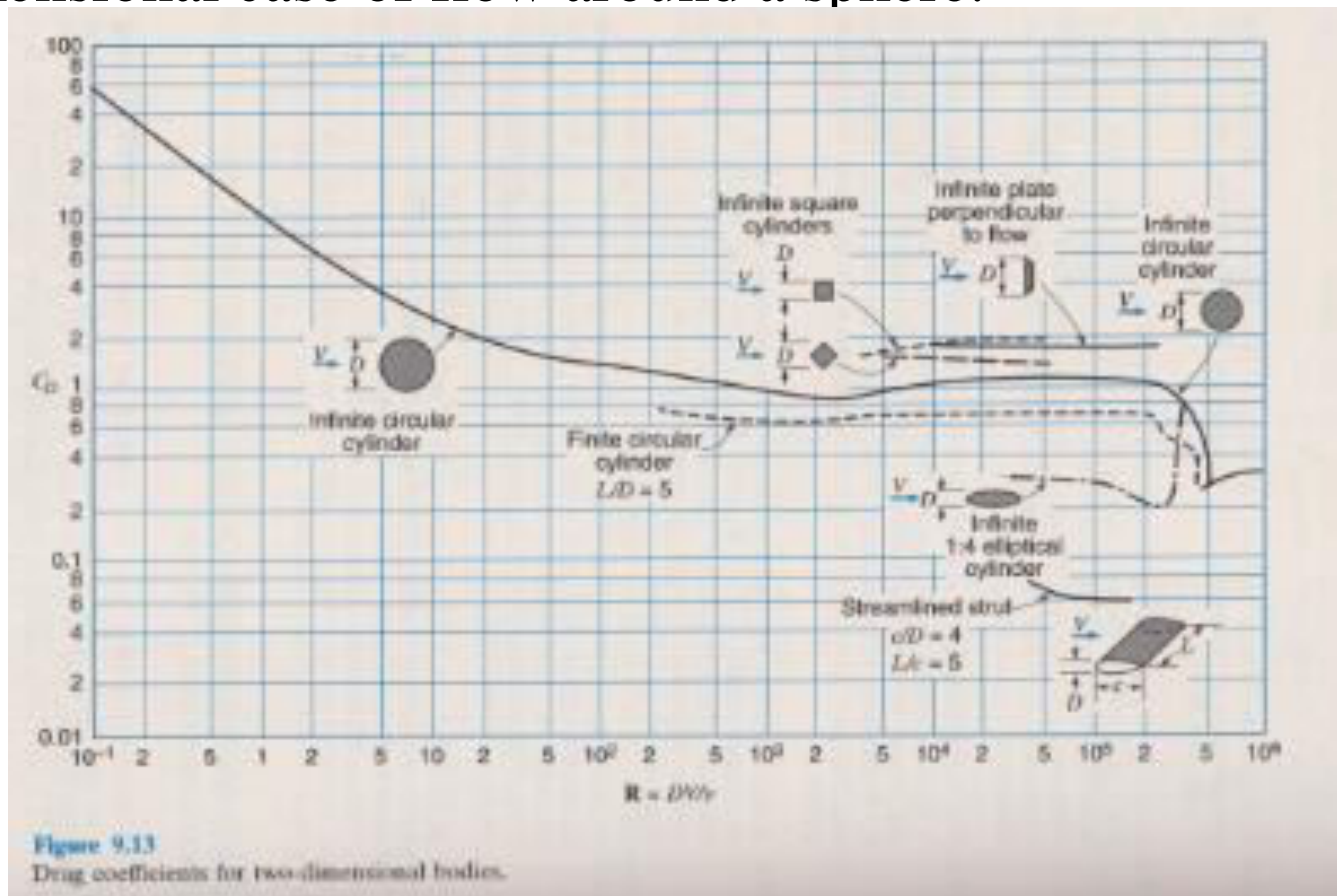


Figure 9.13
Drag coefficients for two-dimensional bodies.

Application to falling sphere (Stoke's law)

- The foregoing principles are vividly illustrated in the case of the flow around a sphere.
- For very low Reynolds numbers ($DV/\nu < 1$, in which D is the diameter of the sphere), the flow about the sphere is *completely viscous*, and the friction drag is given by Stokes' law,
$$F_D = 3\pi\mu VD$$
- Equating this equation to Eq. (9.32: $F_D = C_D\rho\frac{V^2}{2}A$), where A is defined as $\pi D^2/4$, the frontal area of the projected sphere, gives the result that $C_D = 24/R$.
- This regime of the flow about a sphere is shown as the straight line at the left of the log—log plot of C_D versus R in Fig. 9.10.

Application to falling sphere (Stoke's law)

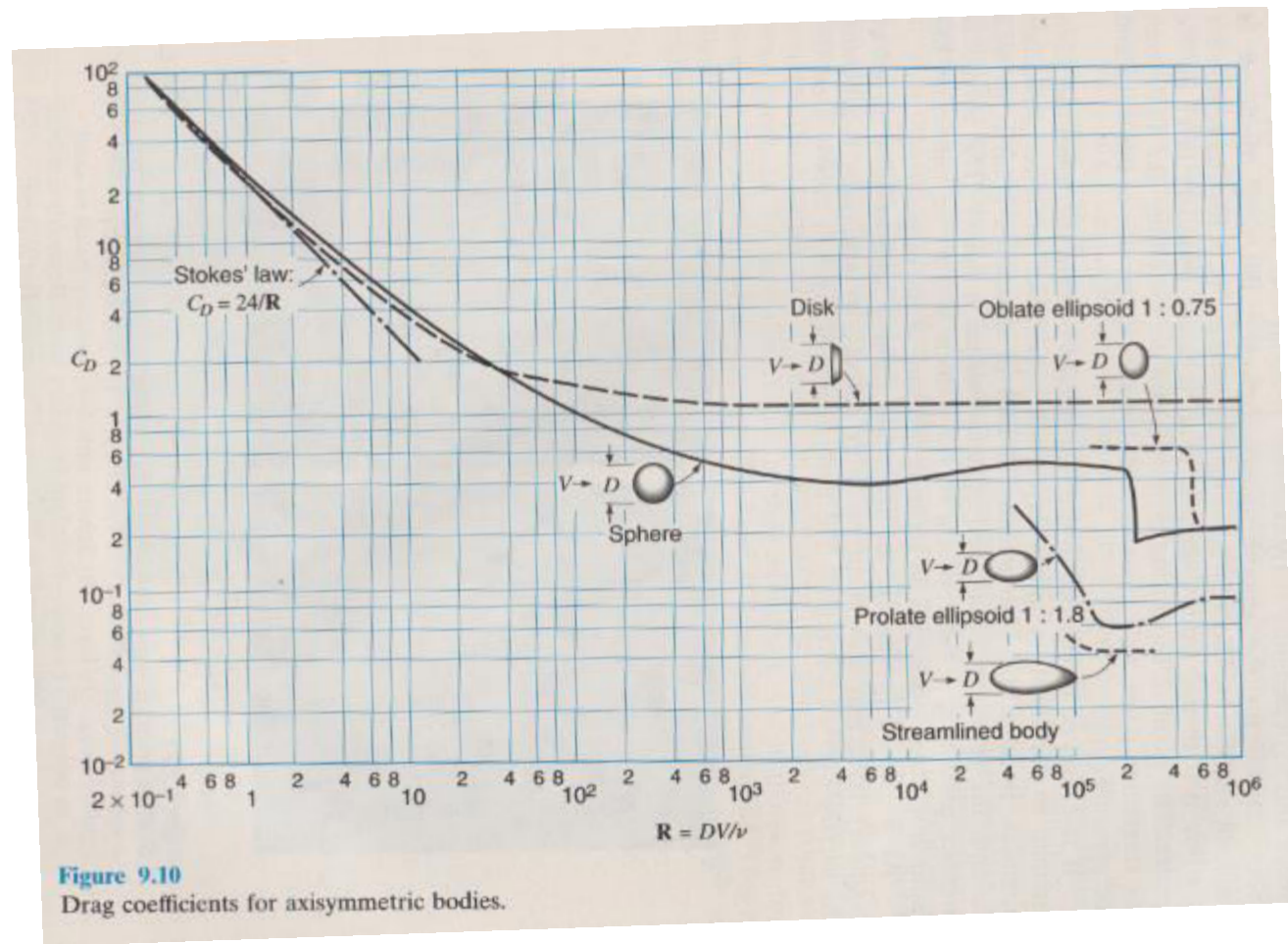


Fig. 9.10

This regime of the flow about a sphere is shown as the straight line at the left of the log—log plot of C_D versus R in Fig. 9.10.

Application to falling sphere (Stoke's law)

- The falling-sphere viscometer is used to measure viscosity, which is a fluid property (Lecture 1).
- In such a device the liquid is placed in a tall transparent cylinder and a sphere of known weight and diameter is dropped in it.
- If the sphere is small enough, Stokes' law will prevail and the fall velocity of the sphere will be approximately inversely proportional to the absolute viscosity of the liquid.
- That this is so may be seen by examining the free-body diagram of such a falling sphere (Fig. 11.2).

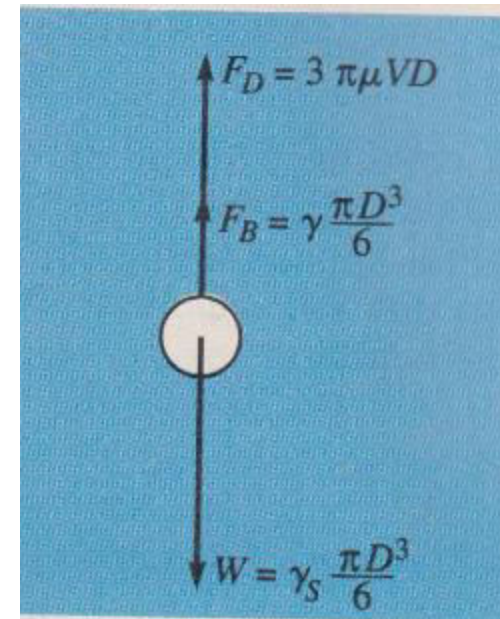


Figure 11.2

Free-body diagram of sphere falling at terminal velocity.

Application to falling sphere (Stoke's law)

- The forces acting include gravity, buoyancy, and drag. Stokes' law states that if $DV/\nu < 1$, the drag force on a sphere is given by $F_D = 3\pi\mu VD$, where V is the velocity of the sphere and D is its diameter.
- When the sphere is dropped in a liquid, it will quickly accelerate to terminal velocity, at which $\sum F_z = 0$. Then

$$W - F_B - F_D = \gamma_s \frac{\pi D^3}{6} - \gamma \frac{\pi D^3}{6} - 3\pi\mu VD = 0$$

where γ_s and γ represent the specific weight of the sphere and liquid, respectively. Hence

$$\mu = \frac{D^2(\gamma_s - \gamma)}{18V}$$

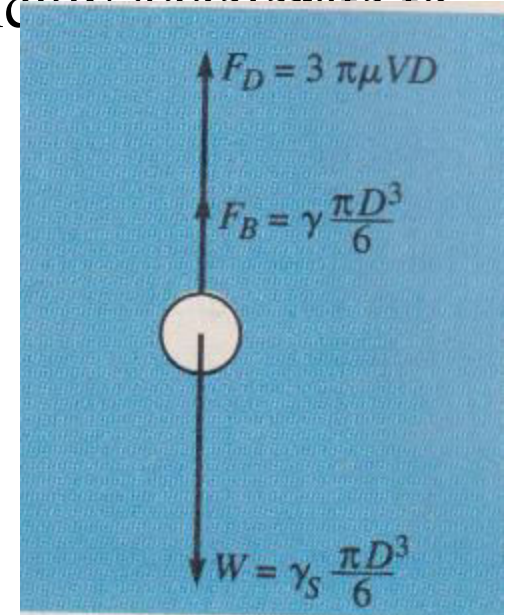


Figure 11.2

Free-body diagram of sphere falling at terminal velocity.

Example

EXAMPLE 8.1

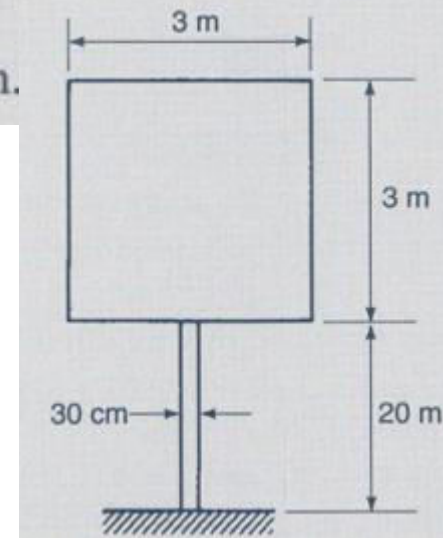
A square sign, $3\text{ m} \times 3\text{ m}$, is attached to the top of a 20-m-high pole that is 30 cm in diameter (Fig. E8.1). Approximate the maximum moment that must be resisted by the base for a wind speed of 30 m/s.

Solution: The maximum force F_1 acting on the sign occurs when the wind is normal to the sign; it is

$$F_D = C_D \rho \frac{V^2}{2} A \quad 9.32$$

$$\begin{aligned} F_1 &= C_D \times \frac{1}{2} \rho V^2 A \\ &= 1.1 \times \frac{1}{2} \times 1.20 \times 30^2 \times 3^2 = 5350\text{ N} \end{aligned}$$

where C_D is found in Table 8.2 and $\rho = 1.20\text{ kg/m}^3$ since it was not given.



Example

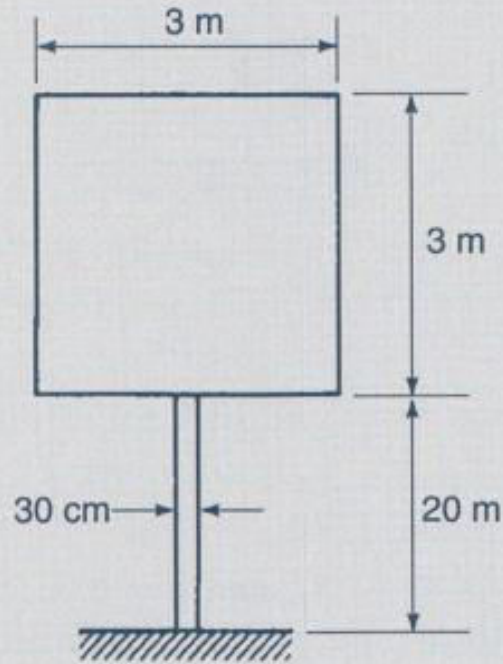


Figure E8.1

where C_D is found in Table 8.2 and $\rho = 1.20 \text{ kg/m}^3$ since it was not given. The force F_2 acting on the cylindrical pole is (using the projected area as $A = 20 \times 0.3 \text{ m}^2$)

$$\begin{aligned} F_2 &= C_D \times \frac{1}{2} \rho V^2 A \\ &= 0.8 \times \frac{1}{2} \times 1.20 \times 30^2 \times (20 \times 0.3) = 2600 \text{ N} \end{aligned}$$

where C_D is found from Fig. 8.8 with $\text{Re} = 30 \times 0.3 / 1.6 \times 10^{-5} = 5.63 \times 10^5$, assuming a high-intensity fluctuation level (i.e., a rough cylinder); since neither end is free, we do not use the multiplication factor of Table 8.1.

Example

TABLE 8.2 Drag Coefficients for Various Blunt Objects

<i>Object</i>	<i>L/w</i>	<i>Re</i>	<i>C_D</i>
Square cylinder $\rightarrow \square \begin{matrix} w \\ w \end{matrix}$	∞	$> 10^4$	2.0
Rectangular plates $\rightarrow \begin{matrix} w \\ w \end{matrix}$	∞	$> 10^3$	2.0
	20	$> 10^3$	1.5
	5	$> 10^3$	1.2
	1	$> 10^3$	1.1
Circular cylinder $\rightarrow \square \begin{matrix} L \\ w = D \end{matrix}$	0	$> 10^3$	1.10
	4	$> 10^3$	0.90
	7	$> 10^3$	1.0
Equilateral cylinders $\rightarrow \triangleleft$	2.0	$> 10^4$	2.0
$\rightarrow \triangleright$	1	$> 10^4$	1.4
Cone $\rightarrow \begin{matrix} 60^\circ \\ \diagdown \diagup \end{matrix}$		$> 10^4$	0.8
Parachute		$> 10^7$	1.4
Automobile			
1920	—	$> 10^5$	0.8
Modern	—	$> 10^5$	0.30
Van	—	$> 10^5$	0.42
Bicycle, upright rider			1.1
Semitruck			0.96
Semitruck with streamlined deflector			0.76