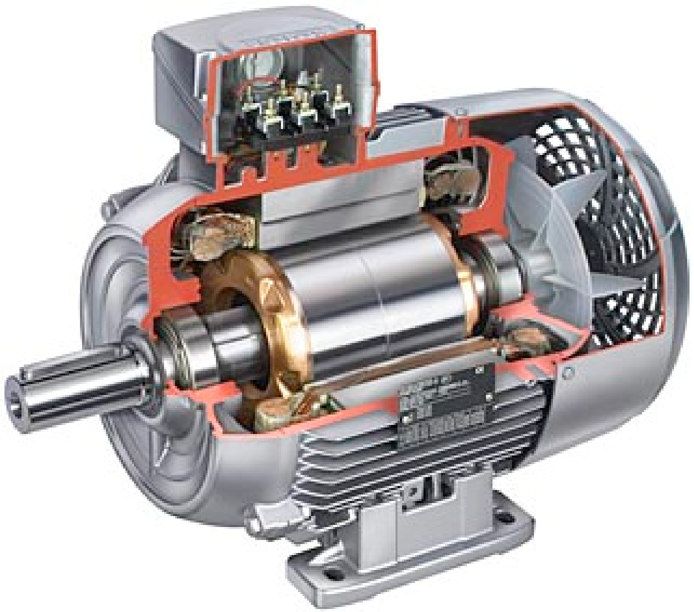
EEE 3352

Electromechanics & Electrical Machines



Lecture 2: Electromagnetic fields

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2.Electromagnetic fields

1. Introduction to electromagnetic fields
2. Electrical properties of matter and space
3. Lumped and field quantities
4. 0-dimensional fields
5. 1-dimensional fields
6. Stored energy and force in EM fields

- at the end of the lecture, students should be able to
  - associate EM quantities into groups of lumped and field terms
  - identify circuit parameters of RLC as properties of matter
  - organise material properties and spatial considerations onto lumped and field quantities
  - apply lumped and field approach to 0-D and 1-D EM problems
  - derive expressions for stored energy and mechanical force in EM fields
  - calculate values of stored energy and mechanical forces in 0-D EM field problems

2.1 Introduction to EM fields

1. Fields of
   1. electrostatics
   2. electrical conduction
   3. magnetostatics
2. Electrical properties of matter and space
   1. capacitance
   2. conductance / resistance
   3. permeance / reluctance

Flow and potential quantities

flow $\xrightarrow{\dfrac{d}{dA}}$ flow density

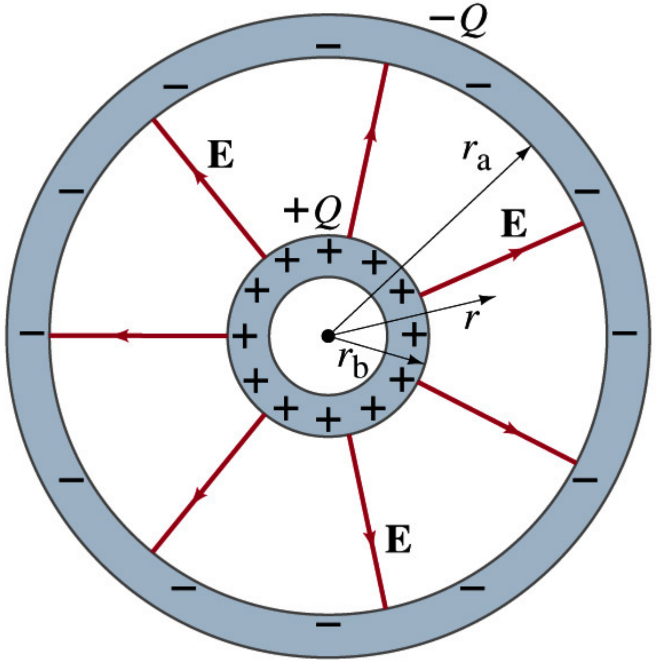
potential difference $\xrightarrow{\dfrac{d}{dl}}$ potential gradient

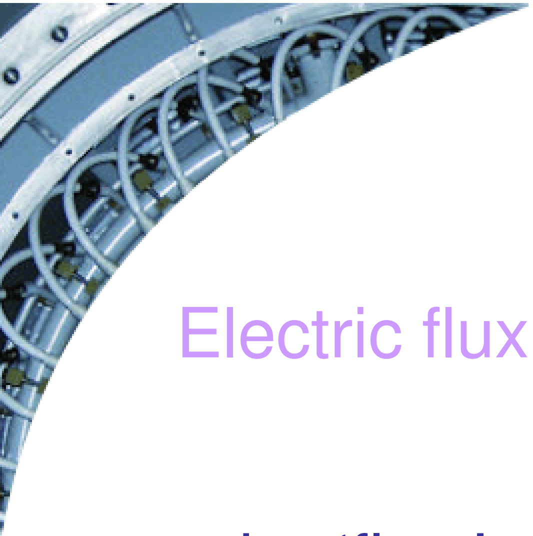
2.1.1 Electrostatics

- fields of electrostatics involve the quantities
  - q
  - ψ flow
  - D flow density
  - V potential difference
  - E potential gradient
- these quantities are defined as:

Electric charge, *q*

- an electrical property of the atomic particles of matter
- measured in coulombs [C]
- charge of an electron : -1.602×10^{-19} C
- charge is the source of electric field





Electric flux, ψ

- the ‘flow’ of electric effect
- flux of any field through a closed surface
  - tells how much the volume acts as a source of the field
- in electrostatics:

$$\psi = q$$

Electric flux density, *D*

$$D = \frac{d\psi}{dA} = \frac{\psi}{A}$$

Electric field strength, *E*

- defined as the force on a unit charge placed at that point
- is a vector quantity whose direction is that of the force on the unit charge
- applying Gauss's law:
  - at a distance *r*,
  - from a fixed point charge *q*,
  - the electric field strength in a radial direction from the point charge is

$$E = \frac{q}{4\pi\varepsilon r^2}$$

- ε is the permittivity of the medium enclosing the charge

$$\varepsilon = \varepsilon_o \varepsilon_r$$

  - ε_o = permittivity of free space,
  - ε_r = relative permittivity of the medium to that of vacuum

- knowing that

$$D = \frac{q}{A} = \frac{q}{4\pi r^2}$$

  - then

$$\boxed{D = \varepsilon E}$$

Electric potential, V

- the change in the potential of one coulomb of charge
  - if one joule of work is required to move it from point 1 to 2
- if a unit charge moves from point 1 to 2 in an electric field
  - changing its potential by V Volts,
  - work done is V Joules

- in general, work done is

$$W = qV$$

- when a unit charge is moved a distance Δl in a direction opposite to the field strength E, the work done is

$$W = E\Delta l$$

- if the associated change in potential of the unit charge is ΔV, then

$$\Delta V = E\Delta l$$

$$E = \frac{\Delta V}{\Delta l} = \frac{dV}{dl}$$

2.1.2 Electrical conduction

- fields of electrical conduction involve the quantities

• *I*	flow
• *J*	flow density
• *V*	potential difference
• *E*	potential gradient

- these quantities are defined as:

Current, *I*

- the flow of electric charge

Current density, *J*

- if current is distributed uniformly throughout the x-section of a wire,
    - the current density *J* is uniform,
    - *J* is given by the total current divided by the x-section area *A*

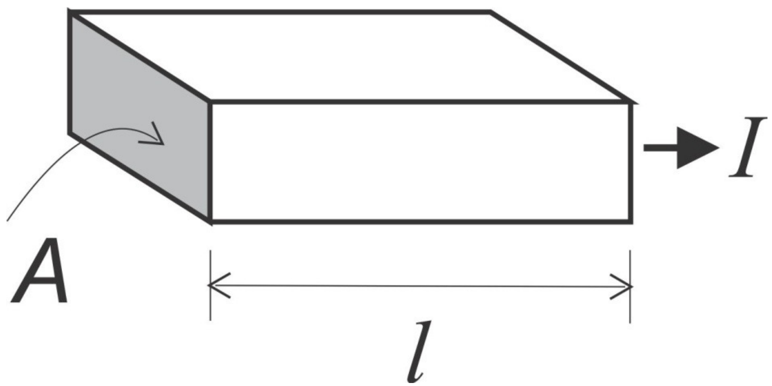
$$J = \frac{I}{A}$$

- if current density is not uniform, the local *J* is given by

$$J = \lim_{\Delta A \to 0} \frac{\Delta I}{\Delta A} = \frac{dI}{dA}$$

Electric Field Strength & Potential in conduction

- consider a small imaginary rectangle cell of
  - length l
  - x-section A,
- with J normal at a point of consideration in the cell;
- let V be the potential difference between the ends of the cell



- applying Ohm's law: $V = IR$

  - but $V = El$ and $I = JA$

  - so that

$$El = JAR \quad \text{or} \quad J = \frac{l}{AR}E = \sigma E$$

  - then

$$\boxed{J = \sigma E}$$

where $\sigma = \dfrac{l}{AR}$

- σ is the conductivity of the material
    - the reciprocal is resistivity, ρ

$$\rho = \frac{1}{\sigma} = \frac{AR}{l}$$

2.1.3 Magnetostatics

Magnetic field

- that region in which a charged particle in motion or a magnetic material is acted upon by a magnetic force

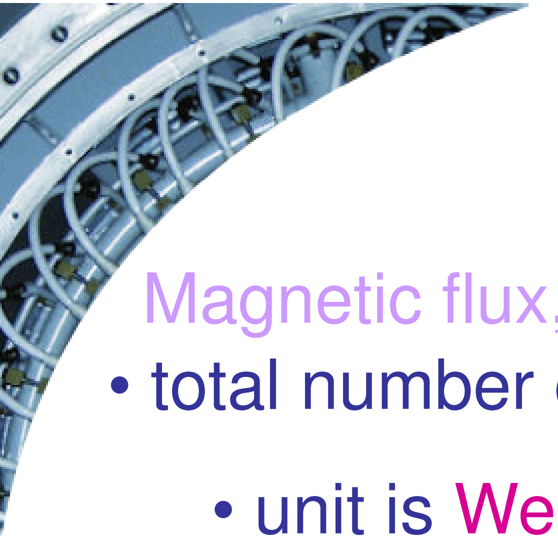
Magnetic lines of force

- have direction
- form complete loops
- represent a tension about their length which tends to make them as short as possible
- repel one another
- cannot intercept but must always form an individual closed loop

- fields of magnetostatics involve the quantities

  - ϕ | flow
  - B | flow density
  - F | potential difference
  - H | potential gradient

- these quantities are defined as:



Magnetic flux, ϕ

- total number of lines of force in a magnetic field,
    - unit is Weber, [Wb]
    - by definition,1 Wb = 10^8 lines of magnetic force

Magnetic flux density, (*B*)

- magnetic flux per unit area

$$B = \frac{d\phi}{dA}$$

- unit is [Wb/m^2] or Tesla [T]

- in a uniform field

$$B = \frac{\phi}{A}$$

Magnetomotive force, mmf, *F*

- magnetic flux is caused by a mmf
  - just as current in conductor is caused by an emf
- if a coil of *N* turns carrying a current *I* is the source of mmf, then

$$mmf = F = NI$$

Magnetic field strength or magnetic field intensity, *H*

- is defined as the mmf per unit length, and units are [A/m]

$$H = \frac{F}{l}$$

Permeability, μ

- in a uniform field magnetic flux density
  - B is related to magnetic filed intensity H by

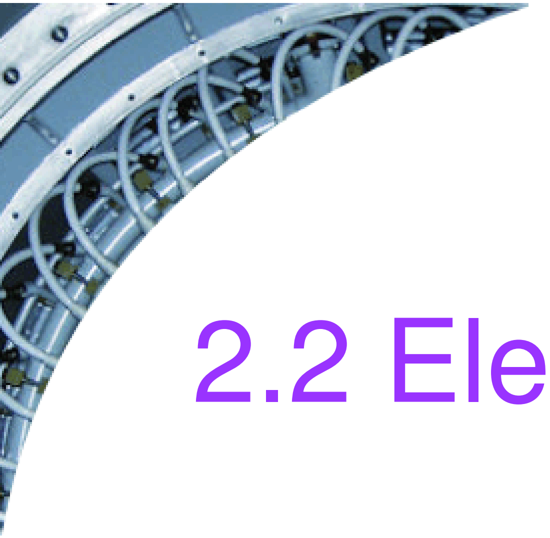
$$B = \mu H$$

where μ is the permeability of the medium

$$\mu = \mu_o \mu_r$$

μ_O = permeability of free space

μ_r = relative permeability



2.2 Electrical properties of matter and space

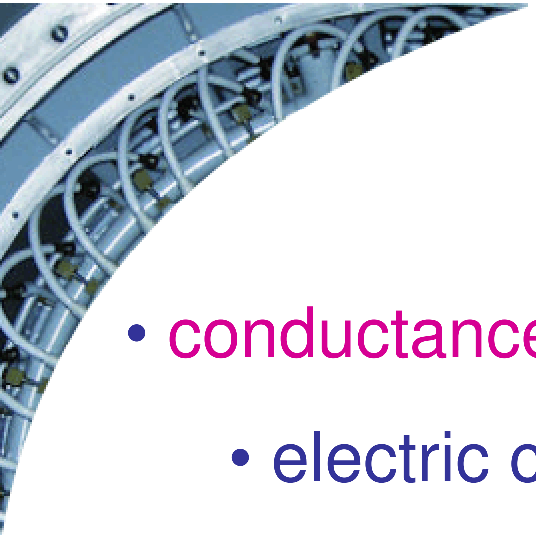
1. capacitance
2. Conductance / resistance
3. permeance / reluctance

2.2.1 Capacitance

- capacitance, *C*, is the property of matter and space whereby
  - electric charge *q* is accumulated when a p.d. *V* is applied to it
- if the applied p.d. is *V* and accumulated charge is *q*,
  - capacitance is defined as

$$C = \frac{q}{V}$$

- unit is Farad, [F]



2.2.2 Conductance

- conductance is the property of matter whereby
    - electric current flows when a p.d. is applied to it
- if the applied p.d. is *V* and the current that flows is *I*
    - conductance is defined as

$$G = \frac{I}{V}$$

- unit is Semiens, [S]
- reciprocal of conductance is Resistance, *R*

$$R = \frac{V}{I}$$

- unit is Ohm, [Ω]

2.2.3 Permeance

- permeance is the property of matter whereby
    - a magnetic flux is established when an mmf is applied to it
- if the applied mmf is F and the flux established is ϕ,
    - permeance is defined as

$$\Lambda = \frac{\phi}{F}$$

- unit is Henry, [H] or Weber per ampere [Wb/A]

- in special types of matter such as conductors,
  - inductance L is used to describe permeance
- L is the ability of a conductor
  - to have a voltage induced when the current changes
- in this case, if the total flux linkage, λ
  - causes a current I to be induced in a conductor then L

$$L = \frac{\lambda}{I}$$

- the reciprocal of *permeance* is *reluctance*, *S*
- unit of reluctance is Ampere per Weber [A/Wb]

2.3 Lumped and field quantities

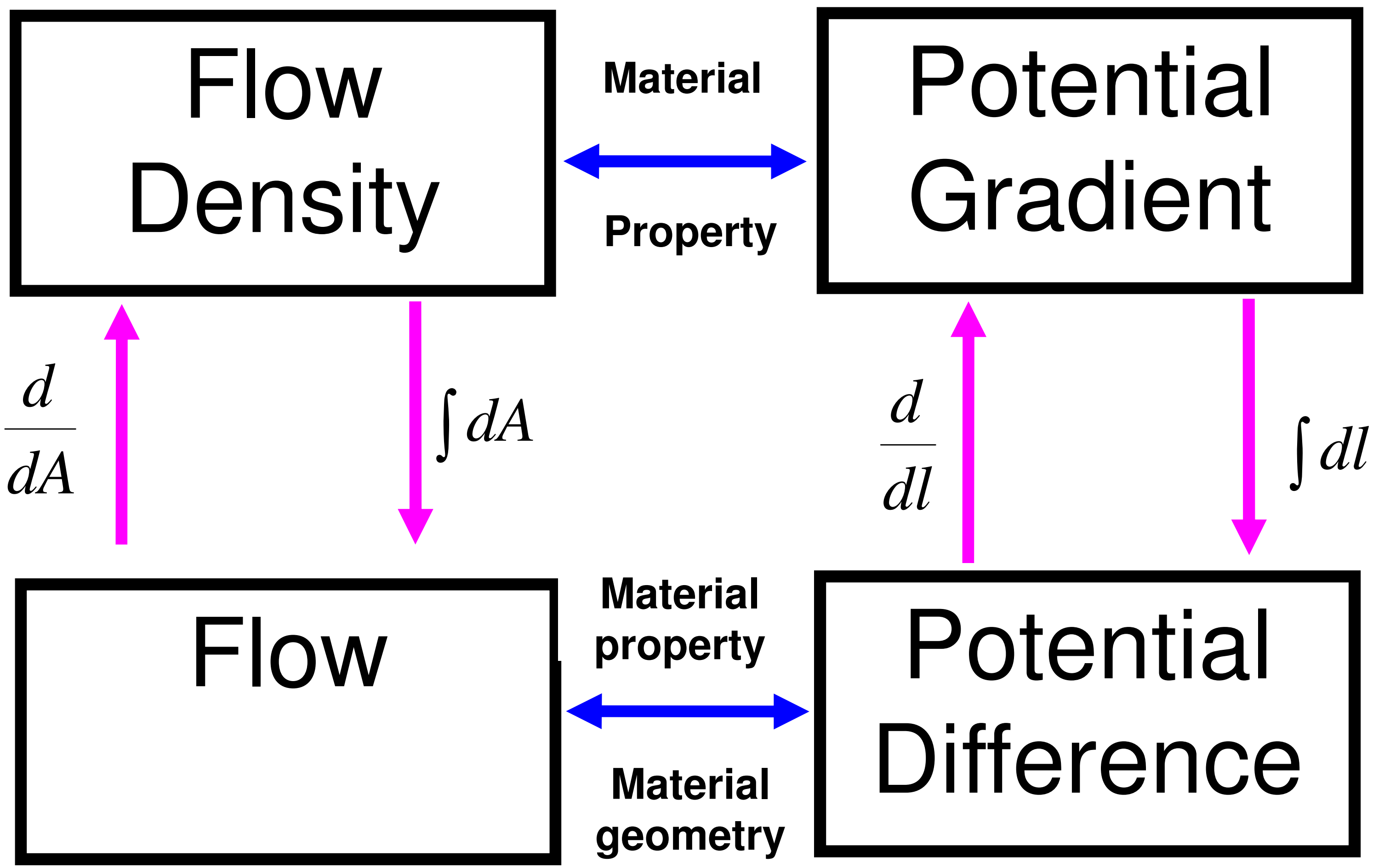
1. fields
2. relationship between field and lumped quantities

2.3.1 field

- field is used to indicate a region of space in which the effect is "felt"
- the region may be bounded or may extend to infinity
- field to be studied are electric and magnetic
- a field behaviour may be
  - 0-D [f = constant]
  - 1-D [f(x)]
  - 2-D [f(x,y)]
  - 3-D [f(x,y,z)]

2.3.2 Lumped and field quantities

- consider the relationships

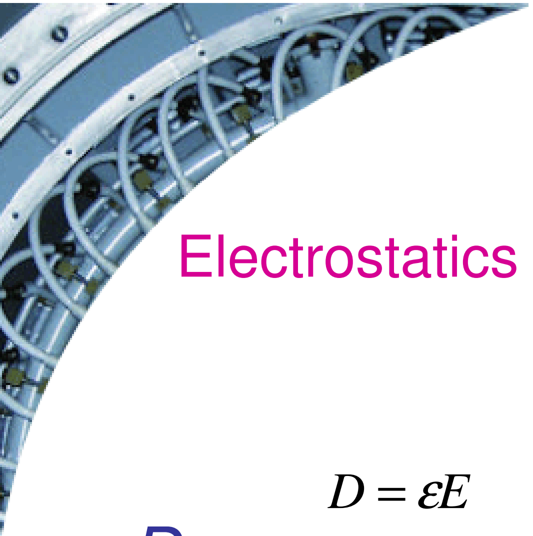


Field quantities:

- flow density & potential gradient

Lumped quantities:

- flow & potential difference



Electrostatics

D (C/m²) — E (V/m): $D = \varepsilon E$

D — Q: $D = \dfrac{dQ}{dA}$

E — V: $E = \dfrac{dV}{dl}$

Q (C) — V (V): $C = \dfrac{Q}{V}$

Current conduction

J (A/m²) — E (V/m): $J = \sigma E$

J — I: $J = \dfrac{dI}{dA}$

E — V: $E = \dfrac{dV}{dl}$

I (A) — V (V): $G = \dfrac{I}{V}$

Magnetostatics

B (Wb/m²) — H (A/m): $B = \mu H$

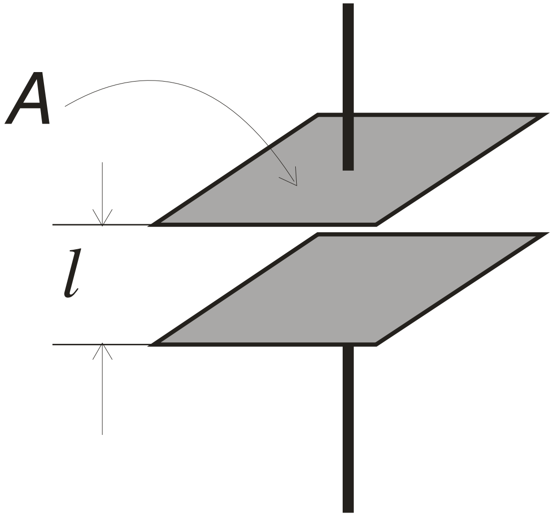
B — ϕ: $B = \dfrac{d\phi}{dA}$

H — F: $H = \dfrac{dF}{dl}$

ϕ (Wb) — F (A): $\Lambda = \dfrac{\phi}{F}$

2.4 0-D fields (Uniform fields)

Electrostatics

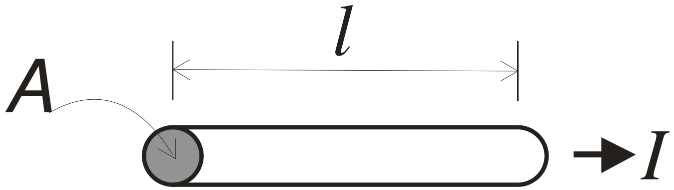


$$C = \frac{Q}{V}$$

$$= \frac{DA}{El}$$

$$\boxed{C = \varepsilon \frac{A}{l}}$$

Current conduction

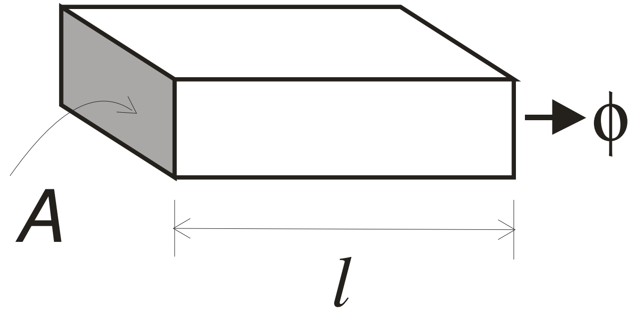


$$G = \frac{I}{V}$$

$$= \frac{JA}{El}$$

$$\boxed{G = \sigma \frac{A}{l}}$$

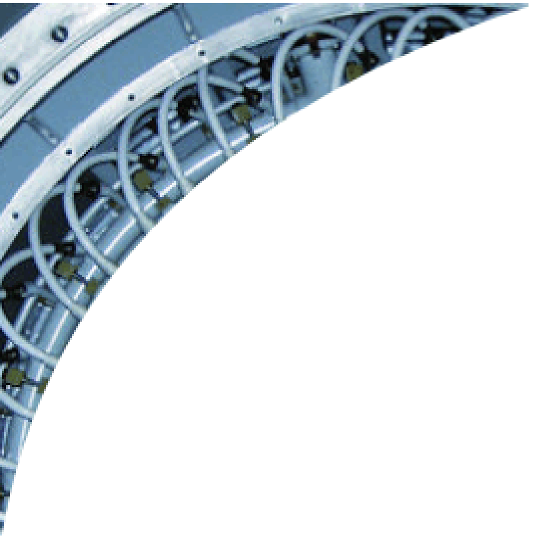
Magnetostatics



$$\Lambda = \frac{\phi}{F}$$

$$= \frac{BA}{Hl}$$

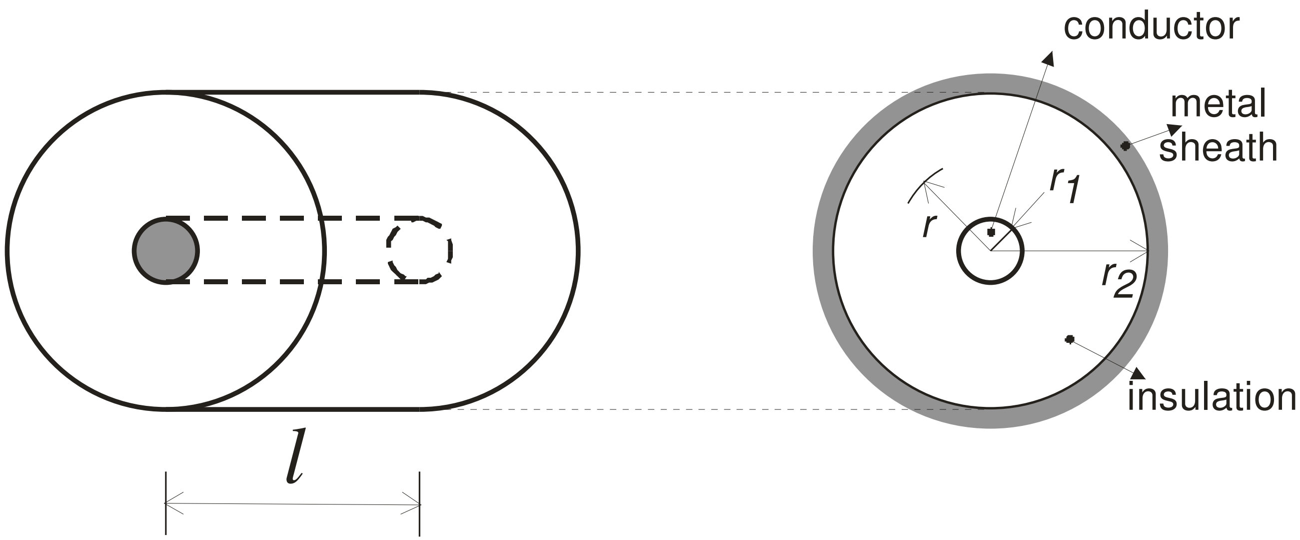
$$\boxed{\Lambda = \mu \frac{A}{l}}$$



2.5 1-D fields (Examples)

1. Concentric cable
2. Concentric sphere
3. Electrode boiler

2.5.1 Concentric cable (electrostatic field



- consider length *l* having charge *q*
- metal sheath is earthed, so is at zero potential

- total electric flux:

$$\psi = q$$

- at radius *r*

$$D = \frac{\psi}{A} = \frac{q}{2\pi r l}$$

$$E = \frac{D}{\varepsilon} = \frac{q}{2\pi r l \varepsilon}$$

$$V = \int_{r_1}^{r_2} E dr = \int_{r_1}^{r_2} \frac{q}{2\pi r l \varepsilon} dr = \frac{q}{2\pi l \varepsilon} \ln \frac{r_2}{r_1}$$

$$C = "\frac{Q}{V}" \longrightarrow \boxed{C = \frac{2\pi l \varepsilon}{\ln \frac{r_2}{r_1}}}$$

- consider *E*

$$E = \left(\frac{q}{2\pi l\varepsilon}\right)\frac{1}{r} = \left(\frac{V}{\ln\frac{r_2}{r_1}}\right)\frac{1}{r}$$

- for max *E* , *r* is minimum, i.e., $r = r_1$

$$\boxed{E_{\max} = \frac{V}{r_1 \ln\frac{r_2}{r_1}}}$$

- for given *V* and r_2, what value of r_1 gives minimum E_{max}?

$$\frac{dE_{\max}}{dr_1} = 0 = \frac{-V}{\left(r_1 \ln\frac{r_2}{r_1}\right)^2}\left[\ln\frac{r_2}{r_1} + r_1\left(\frac{r_1}{r_2}\right)\left(\frac{-r_2}{r_1^2}\right)\right]$$

$$\ln\frac{r_2}{r_1} - 1 = 0 \rightarrow \frac{r_2}{r_1} = e \qquad \longrightarrow \qquad \boxed{r_1 = \frac{r_2}{e}}$$

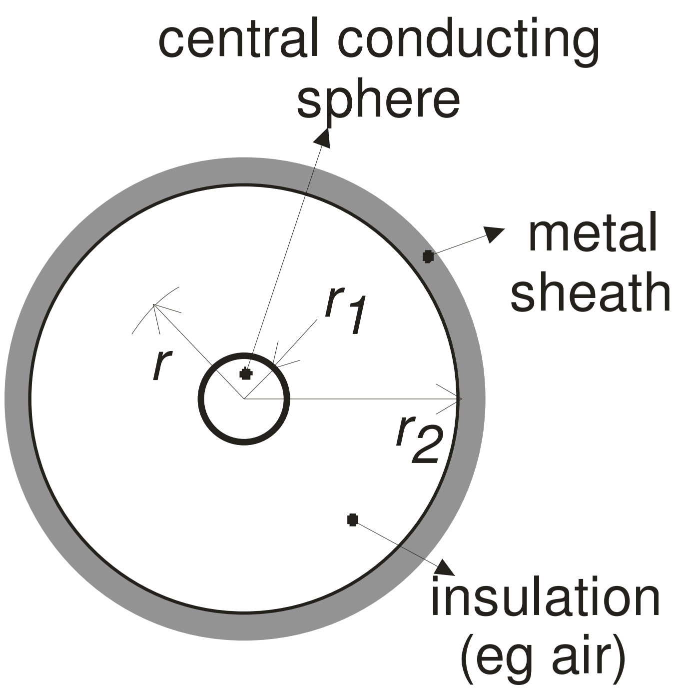
- on ac, capacitive current:

$$I = \frac{V}{Z} \qquad \text{with} \qquad Z = \frac{1}{j\omega C}$$

$$|I| = V\omega C = 2\pi f V C$$

- on ac, *V* is given as rms
  - use peak value to get ultimate stress E_{max}

2.5.2 Concentric sphere (electrostatic field)



$$D = \frac{\psi}{A} = \frac{q}{4\pi r^2}$$

$$E = \frac{D}{\varepsilon} = \frac{q}{4\pi r^2 \varepsilon}$$

$$V = \int E dr = \frac{q}{4\pi\varepsilon}\int_{r_1}^{r_2}\frac{dr}{r^2} = \frac{q}{4\pi\varepsilon}\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

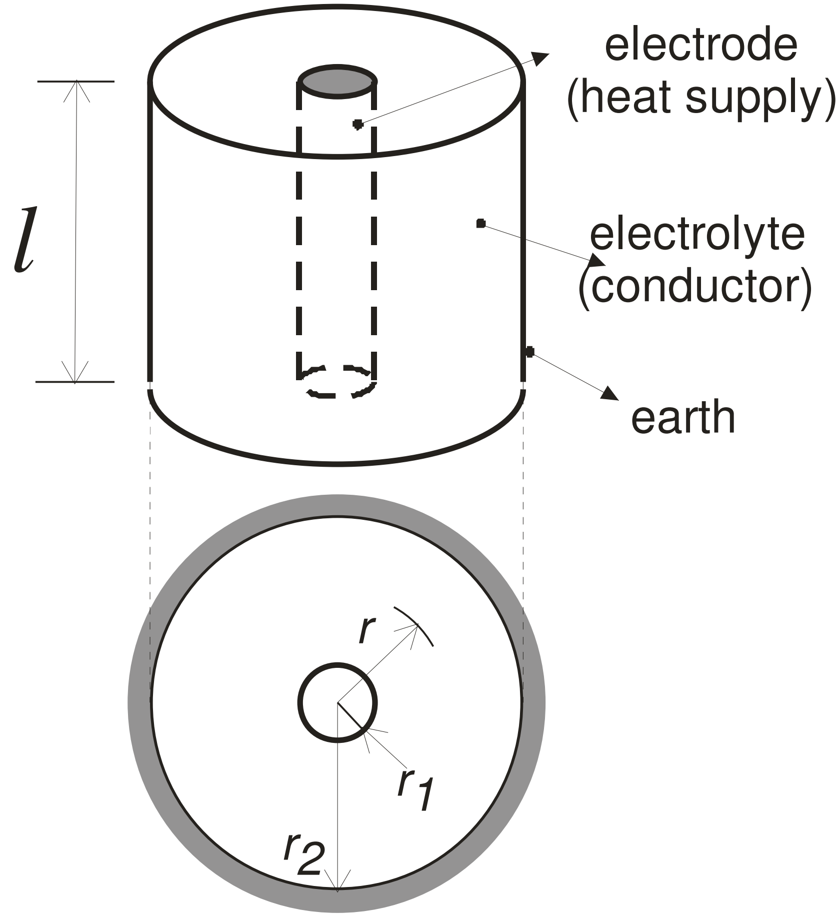
- capacitance

$$C = \frac{q}{V} = \frac{4\pi\varepsilon}{\left(\frac{1}{r_1}-\frac{1}{r_2}\right)}$$

- electric field stress

$$E = \frac{V}{\left(\frac{1}{r_1}-\frac{1}{r_2}\right)r^2} \longrightarrow \boxed{E_{\max} = \frac{V}{\left(\frac{1}{r_1}-\frac{1}{r_2}\right)r_1^2}}$$

2.5.3 Electrode boiler (conduction field)



- take total current I, depth of liquid l

$$J = \frac{I}{A} = \frac{I}{2\pi r l}$$

$$E = \frac{J}{\sigma} = \frac{I}{2\pi l \sigma}\frac{1}{r}$$

$$V = \int E dr = \frac{I}{2\pi l\sigma}\int_{r_1}^{r_2}\frac{dr}{r} = \frac{I}{2\pi l\sigma}\ln\frac{r_2}{r_1}$$

- conductance

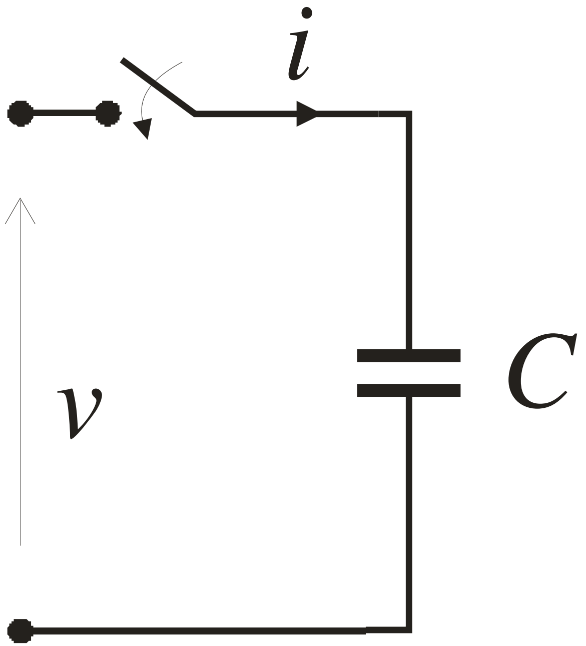
$$G = \frac{I}{V} = \frac{2\pi l\sigma}{\ln\frac{r_2}{r_1}}$$

- for max J, $r = r_1$

$$J_{\max} = \frac{I}{2\pi l r_1} = \frac{\sigma}{G}\frac{I}{r_1\ln\frac{r_2}{r_1}}$$

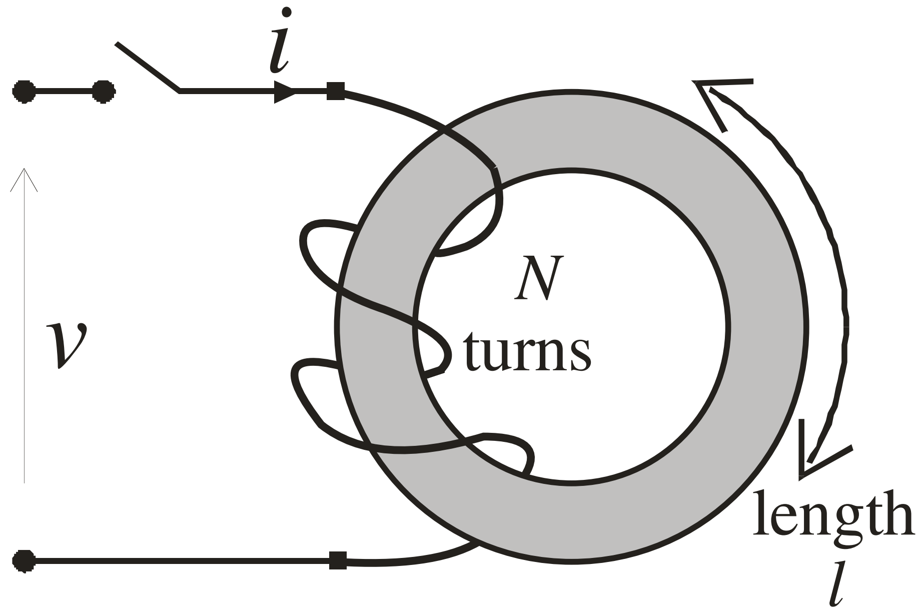
2.6 Stored energy in fields

2.6.1 Electrostatic field



• parallel plate capacitor, capacitance, *C*

2.6.2 Magnetostatic field



• toroidal coil, inductance *L*

	2.6.1 Electrostatic field	2.6.2 Magnetostatic field
	$q = CV$ $i = C\dfrac{dv}{dt}$	$Li = N\phi$ $v = L\dfrac{di}{dt}$
• power	$P = vi = Cv\dfrac{dv}{dt}$	$P = vi = Li\dfrac{di}{dt}$
• energy	$W = \int Pdt = \int_0^V cvdv$ $= C\dfrac{V^2}{2} = \dfrac{1}{2}qV$	$W = \int Pdt = \int_0^I Lidi$ $= L\dfrac{I^2}{2} = \dfrac{1}{2}N\phi I$
• energy/volume	$w = \dfrac{\frac{1}{2}qV}{Al} = \dfrac{1}{2}\dfrac{q}{A}\dfrac{V}{l}$ $\boxed{w = \dfrac{1}{2}DE}$ [J/m^3]	$w = \dfrac{\frac{1}{2}N\phi i}{Al} = \dfrac{1}{2}\dfrac{\phi}{A}\dfrac{NI}{l}$ $\boxed{w = \dfrac{1}{2}BH}$ [J/m^3]

2.6.1 Electrostatic field

$$w = \frac{1}{2} DE$$

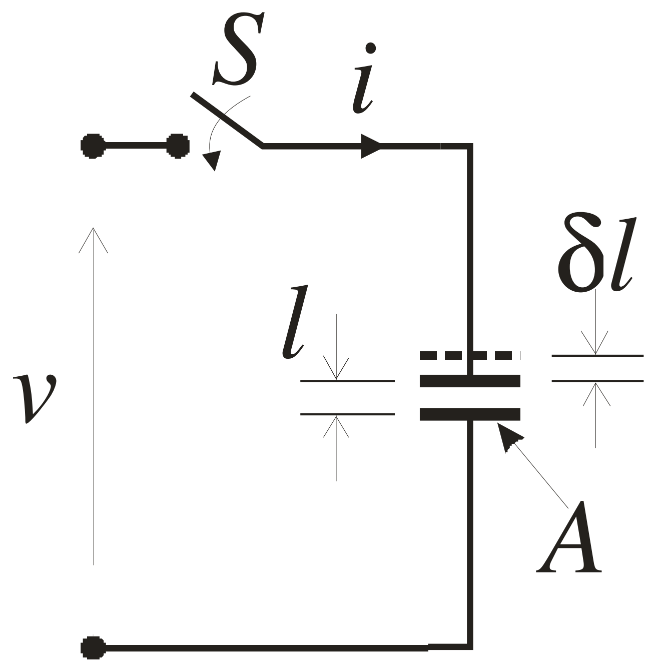
2.6.2 Magnetostatic field

$$w = \frac{1}{2} BH$$

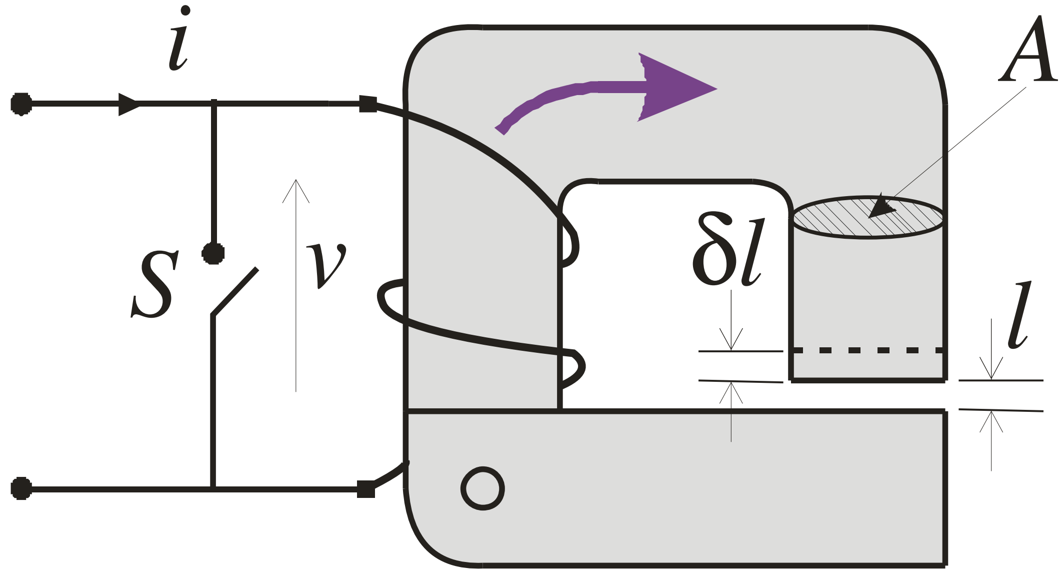
- the expression are derived using a uniform field
- as energy the expression apply at any point in the field
    - whether uniform or not

Force in fields

2.6.1 Electrostatic field



2.6.2 Magnetostatic field



- assume $\mu_{Fe} = \infty$, just consider airgap

2.6.1 Electrostatic field

1) close S, charge up C to voltage v, associated charge is

$$q = Cv$$

2) open S, charge q is trapped and is constant, ∴

$$q = \text{constant}$$

$$i = \frac{dq}{dt} = 0$$

$$\psi = \text{constant}$$

$$D = \text{constant}$$

$$E = \text{constant}$$

2.6.2 Magnetostatic field

1) open S, current i flows thru' coil, hence

$$\phi = \frac{Li}{N}$$

2) close S, making $v = 0$; since

$$v = N\frac{d\phi}{dt} = 0 \qquad \text{(Faraday's law)}$$

$$\phi = \text{constant} \qquad (\phi \text{ is 'trapped'})$$

$$B = \text{constant}$$

$$H = \text{constant}$$

2.6.1 Electrostatic field

3) gap changes from *l* to *l* + δ*l* ;
there are energy changes

a. (mech. energy ?) $= F_{\text{mech}}.\delta l$

b. (elect. energy ?) $= \int vidt = 0$

from $i = 0$

c. change of stored energy in electrostatic field:

$$= \delta\left(\tfrac{1}{2} DE \times \text{volume}\right)$$
$$= \tfrac{1}{2} DE(\delta l.A)$$

2.6.2 Magnetostatic field

3) gap changes from *l* to *l* + δ*l* ;
there are energy changes

a. (mech. energy ?) $= F_{\text{mech}}.\delta l$

b. (elect. energy ?) $= \int vidt = 0$

from $v = 0$

c. change of stored energy in magnetostatic field (airgap):

$$= \delta\left(\tfrac{1}{2} BH \times \text{volume}\right)$$
$$= \tfrac{1}{2} BH(\delta l.A)$$

4) applying law of conservation of energy, the two forms of energy changes relevant to motion δl can be equated

2.6.1 Electrostatic field

$$F_{\text{mech}}\delta l = \tfrac{1}{2} DE(\delta l . A)$$

$$F_{\text{mech}} = \tfrac{1}{2} DAE$$ [N]

Pressure = press

$$\text{press} = \frac{F_{\text{mech}}}{A}$$

$$\text{press} = \tfrac{1}{2} DE$$ [N/m²]

2.6.2 Magnetostatic field

$$F_{\text{mech}}\delta l = \tfrac{1}{2} BH(\delta l . A)$$

$$F_{\text{mech}} = \tfrac{1}{2} BAH$$ [N]

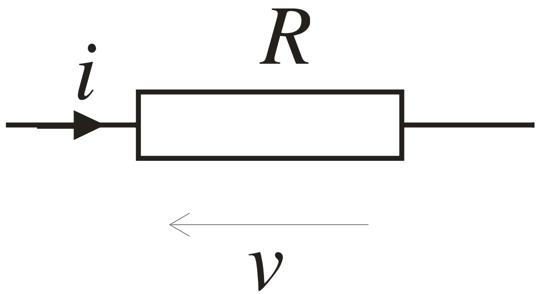
Pressure = press

$$\text{press} = \frac{F_{\text{mech}}}{A}$$

$$\text{press} = \tfrac{1}{2} BH$$ [N/m²]

2.6.3 Conduction field: energy dissipation

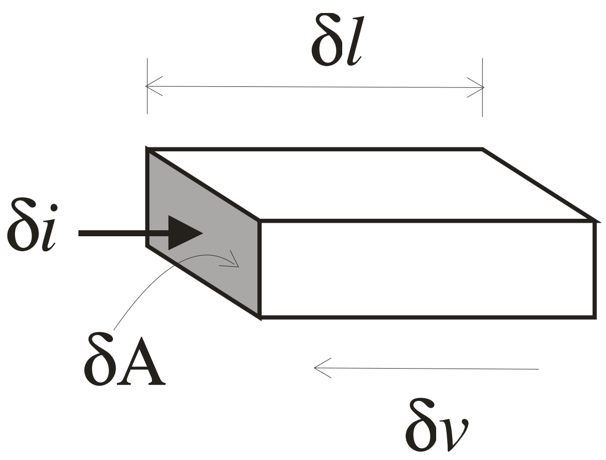
- for a resistance, the electrical energy goes into heat energy



$$P = vi = i^2 R = \frac{v^2}{R}$$

- no energy is stored

- for field study:



local: $J = \dfrac{\delta i}{\delta A}$

local: $E = \dfrac{\delta v}{\delta l}$

material property: $J = \sigma E$

- power dissipation in a small volume

$$\delta P = \delta v \delta i$$

- power dissipation per unit volume (power loss density)

$$= \frac{\delta v \delta i}{\delta l \delta A} = \frac{\delta v}{\delta l} \cdot \frac{\delta i}{\delta A} = JE = \sigma E^2 = \frac{J^2}{\sigma} \quad [\mathrm{J/m^3}]$$

Examples:

2.1

A 20-μF capacitor is charged at constant current of 5 μA for 10 minutes.

Calculate the final p.d. and the corresponding stored charge?

Examples:

2.2

An electrode boiler has an earthed metal cylinder of 0.8 m diameter with non conducting ends and a co-axial central electrode of 0.1 m diameter.

The boiler is designed to absorb 8 kW when connected to a 240-V supply, the depth of the water being 0.5 m in the axial direction.

To what specific conductivity should the water be treated in order to absorb this power?

Examples:

2.3

A coil wound uniformly round a wooden ring having a mean circumference of 600 mm and uniform cross-sectional area of 500 mm^2 produces an mmf of 0.8 kA.

Find in the ring,

a) the magnetic field strength;

b) the flux density;

c) the total flux.

.

Examples:

2.4

What force can one pole of an electromagnet exert on a movable steel object if

- there is a uniform air gap of 5 mm between them,
- the area of the pole and end face is 10^4 mm^2 , and
- the magnetic flux density is 0.5 T?

Assume that the flux path has negligible reluctance apart from the airgap.

- End of Lecture 2 -