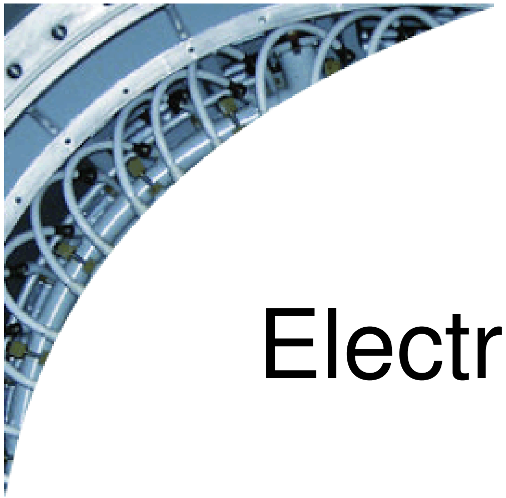
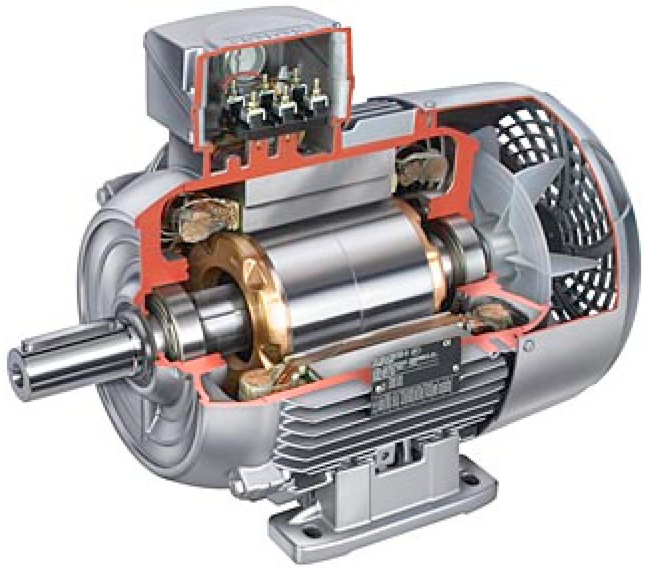


EEE 3352

Electromechanics & Electrical Machines



Lecture 3: Magnetic circuits

Dr A Zulu

3. Magnetic circuits

1. Introduction
2. Electric circuit analogy
3. Composite magnetic circuits
4. Inductance
    1. on dc
    2. on ac
5. Inductor (real)
    1. power loss
    2. equivalent circuit
    3. phasor diagram
6. Limitations of magnetic circuit model

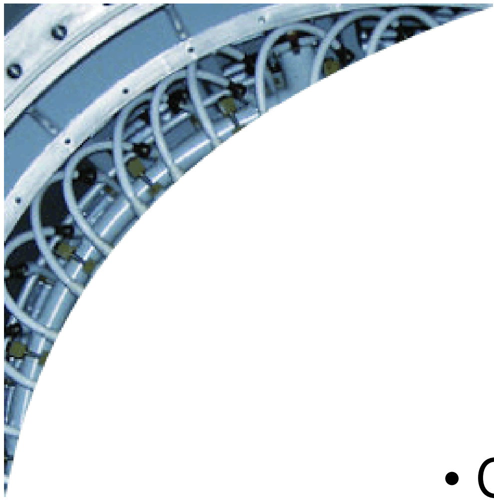
Lecture objectives

- at the end of the lecture, students should be able to
  - define a magnetic circuit
  - compare a magnetic circuit to an electric circuit
  - derive the expressions for reluctance and inductance of a uniform magnetic circuit
  - determine reluctances in composite arrangements
  - calculate reluctances for uniform magnetic circuits
  - determine the effects of inductance on application of ac voltage
  - derive equivalent circuit of a real inductor
  - develop phasor diagram of electrical quantities of an inductor

3.1 Introduction

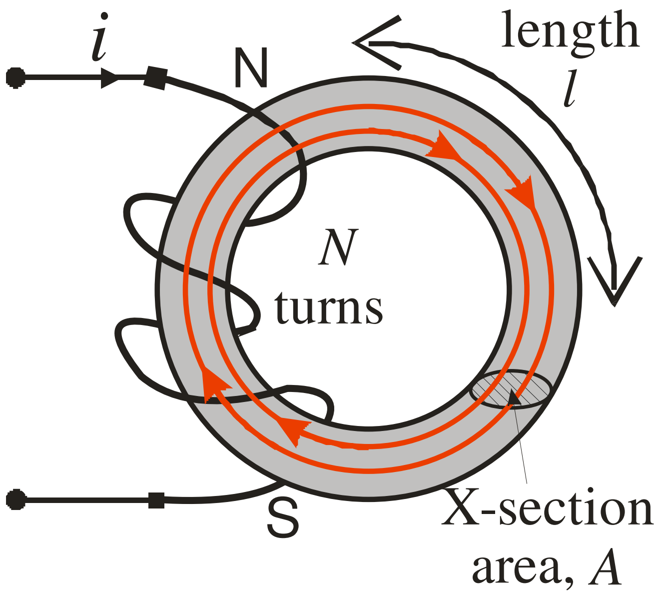
Definition

- characteristic of lines of magnetic flux:
    - each line is a closed path
- a complete closed path + any group of lines of magnetic flux
    - forms a magnetic circuit

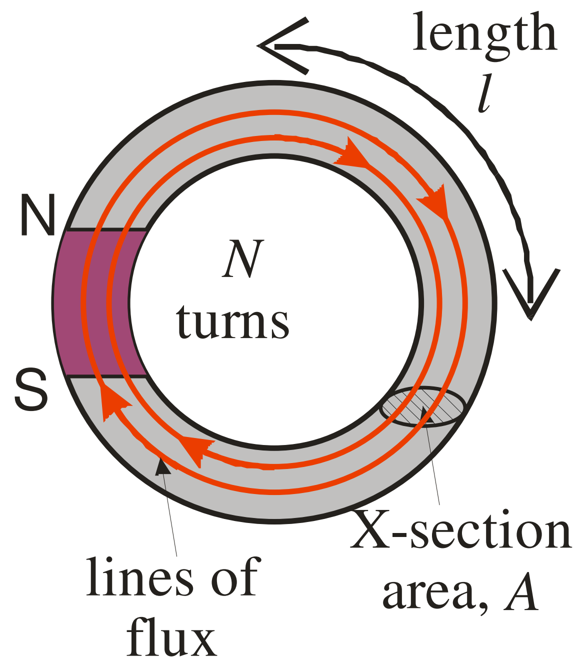


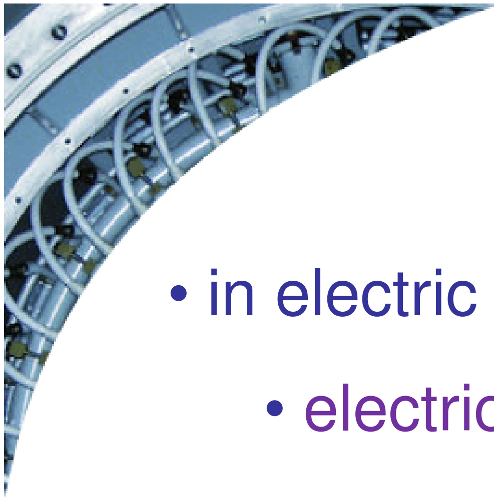
Sources of mmf, *F*

- Coil current



- Permanent magnet (PM)





3.2 Electric circuit analogy

- in electric circuits:
    - electric current is due to effect of emf, *V*
- in magnetic circuits:
    - magnetic flux is due to effect of mmf, *F*
- if a current-carrying coil is used as source of mmf
    - a current *i* flowing
    - coil of *N* turns
    - mmf *F* generated is

$$mmf = F = Ni$$

- mmf per unit length of the magnetic circuit
    - is called magnetic field strength, *H*
- if mean length of the magnetic circuit is l, then

$$F = Ni = Hl \qquad \rightarrow \qquad H = \frac{Ni}{l}$$

- *H* produces *B* wherever it exits:

$$B = \mu H$$

$$\mu = \mu_r \mu_o$$

- μ_o = permeability of free space; $\mu_o = 4\pi \times 10^{-7}$ H/m
- μ_r = relative permeability

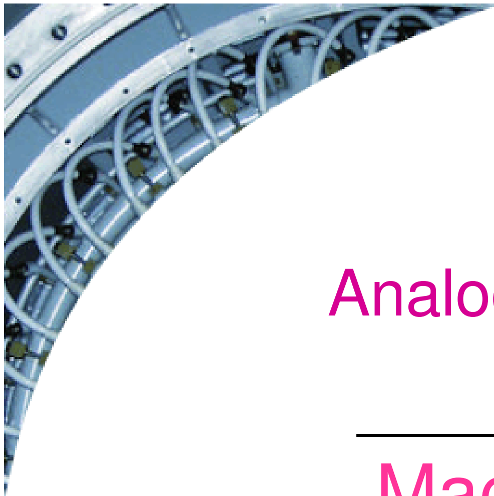
- for air, $u_r = 1$; for other magnetic materials u_r = 500-4000

- for uniform magnetic circuits:
  - X-section area, *A*;
  - length *l*,
  - permeability, μ

$$\Lambda = \frac{\phi}{F} = \frac{BA}{Hl} = \mu \frac{A}{l}$$

$$\phi = \frac{F}{S} = \frac{Ni}{l/\mu A}$$

$$S = \frac{1}{\Lambda} = \frac{l}{\mu A}$$



Analogy between magnet and electric circuits

Magnetic circuits	Electric circuits
ϕ	I
F	V
Λ	G
S	R

- in magnetic circuits:

$$F_{\text{source}} = Ni$$

$$F_{\text{used}} = F_{\text{drop}} = \phi S$$

- hence:

$$F = \phi S$$

$$S = \frac{l}{\mu A}$$

- analogous electric circuits:

$$emf_{\text{source}} = e$$

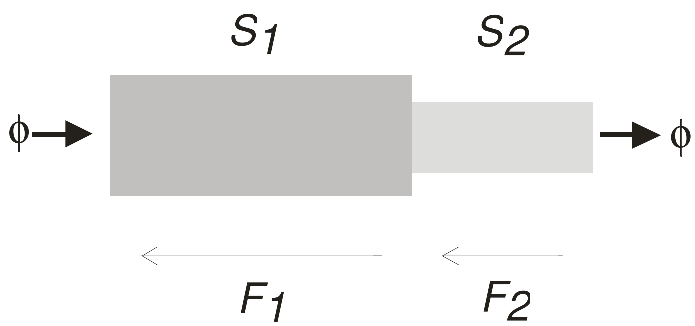
$$e_{\text{used}} = V_{\text{drop}} = IR$$

$$e = IR$$

$$R = \frac{l}{\sigma A}$$

3.3 Composite magnetic circuits

I. Series

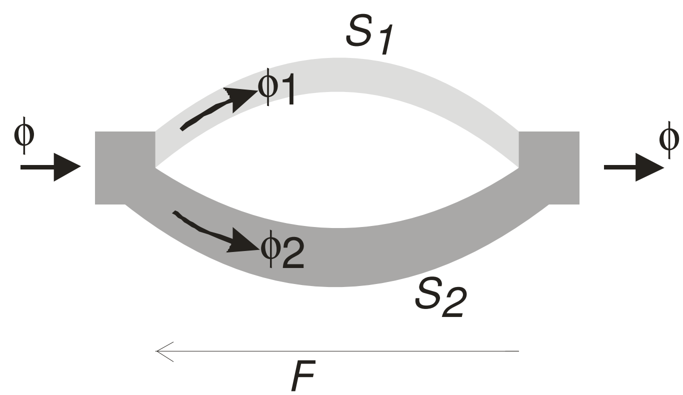


$$F = F_1 + F_2$$

$$\frac{F}{\phi} = \frac{F_1}{\phi} + \frac{F_2}{\phi}$$

$$S_{tot} = S_1 + S_1$$

II. Parallel



$$\phi = \phi_1 + \phi_2$$

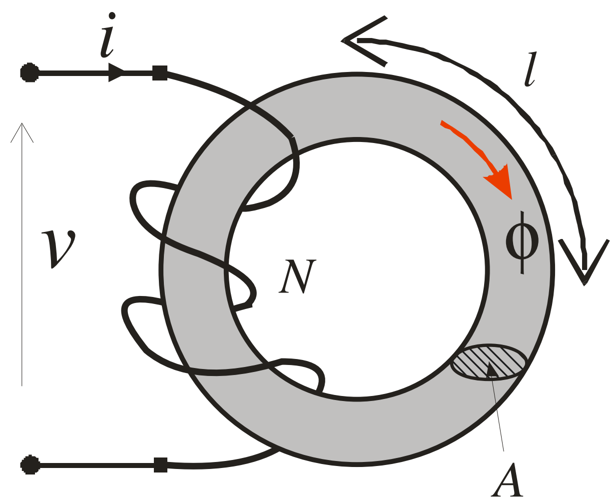
$$\frac{\phi}{F} = \frac{\phi_1}{F} + \frac{\phi_2}{F}$$

$$\frac{1}{S_{tot}} = \frac{1}{S_1} + \frac{1}{S_2}$$

- reluctances in series and parallel
  - combine together in the same way as resistances

3.4 Inductance

- inductance exists in any circuit in which
    - a change of current causes change of magnetic flux
    - or there is voltage due to change of magnetic flux



- definition (electrical circuit point of view) · · · [eq 1]:

$$v = L\frac{di}{dt}$$

- Faraday's law · · · [eq 2]:

$$v = N\frac{d\phi}{dt}$$

- equating [eq 1 & 2]:

$$L\frac{di}{dt} = N\frac{d\phi}{dt}$$

$$L = N\frac{d\phi}{di}$$

- for uniform case:

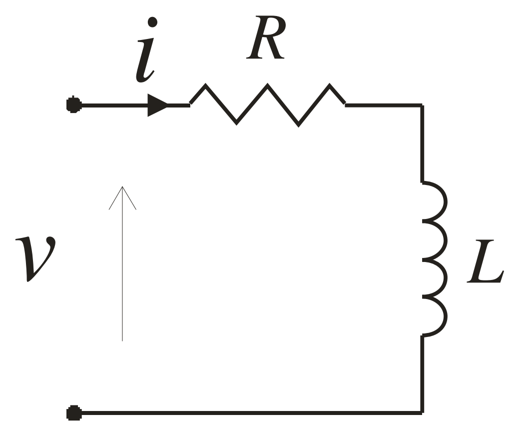
$$L = N\frac{\phi}{i}$$

- but

$$\phi = \frac{F}{S} = \frac{Ni}{S} \quad \Rightarrow \quad \boxed{L = \frac{N^2}{S}}$$

- thus, inductance *L* depends on:
  - geometry: *A* & *l*
  - material: μ
  - turns: *N*

3.4.1 Inductance on dc



- real coil, with resistance *R*

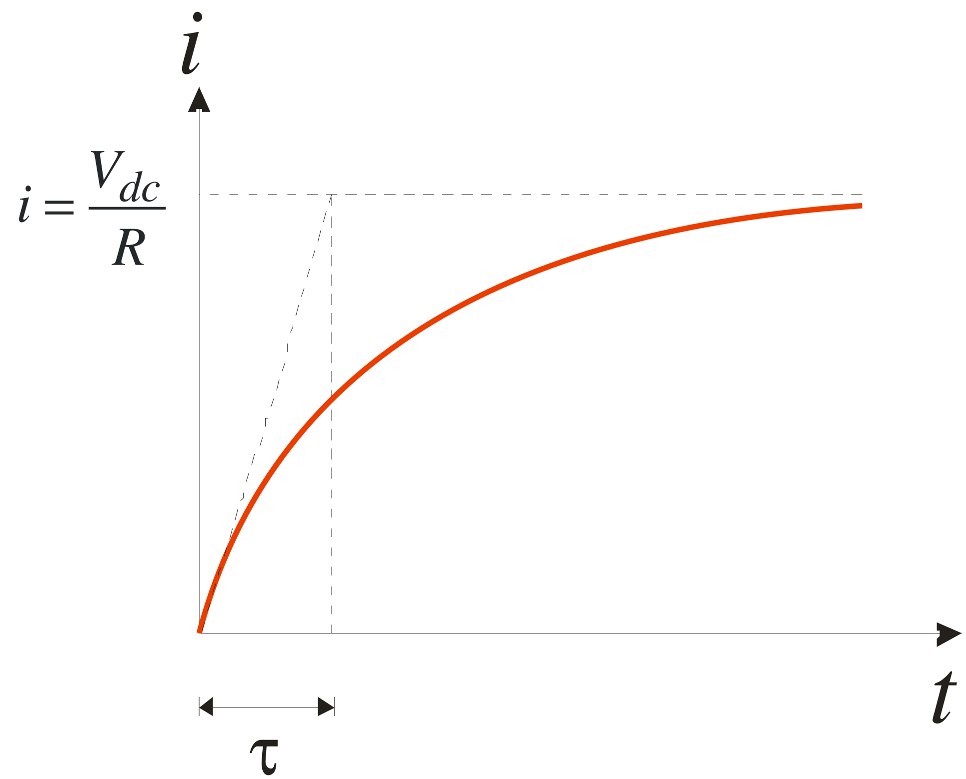
$$v = Ri + L\frac{di}{dt}$$

- thus, on dc

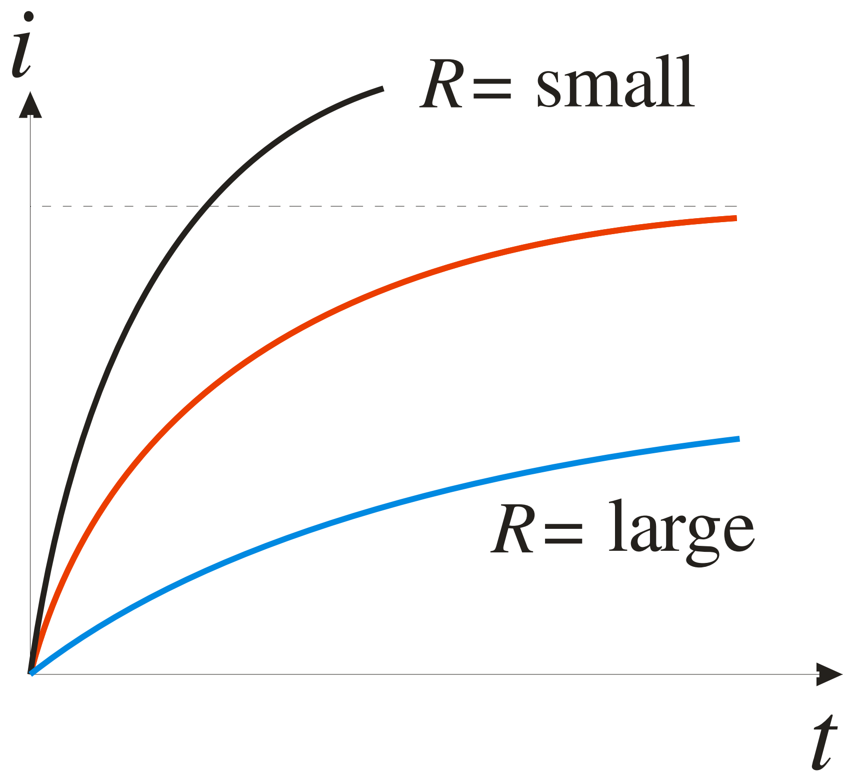
$$V_{dc} = Ri + L\frac{di}{dt}$$

$$i = \frac{V_{dc}}{R}\left(1 - e^{-t/\tau}\right)$$

$\tau = L/R$ = time constant

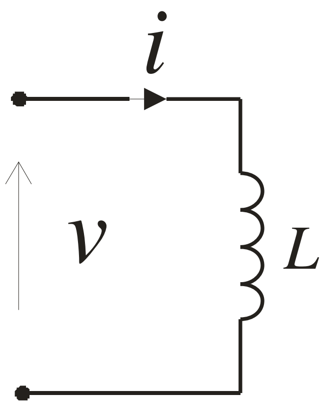


- given *L*, variable *R*



3.4.2 Inductance on ac

- consider pure *L*



$$v(t) = L\frac{di(t)}{dt}$$

- with sinusoidal supply, *v*

$$v(t) = V_m \sin \omega t$$

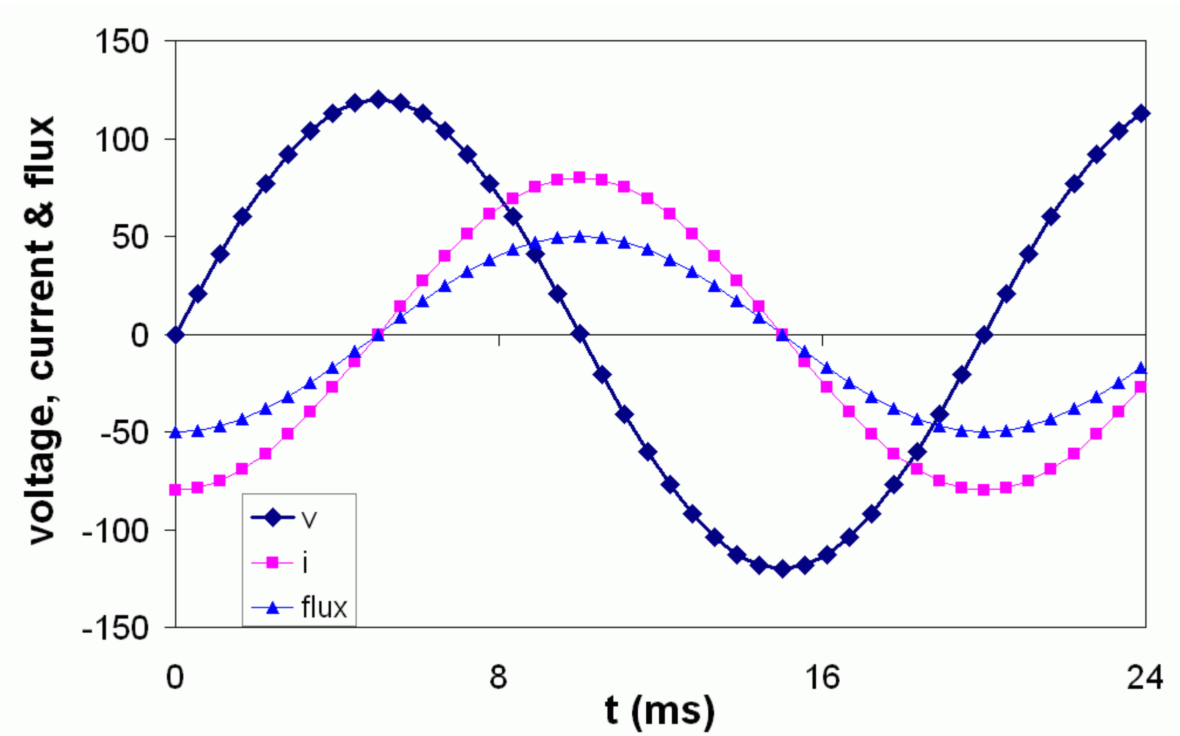
$$i(t) = \frac{1}{L}\int v dt = -\frac{V_m}{\omega L}\cos \omega t + \text{constant}$$

- after a long time, constant = 0

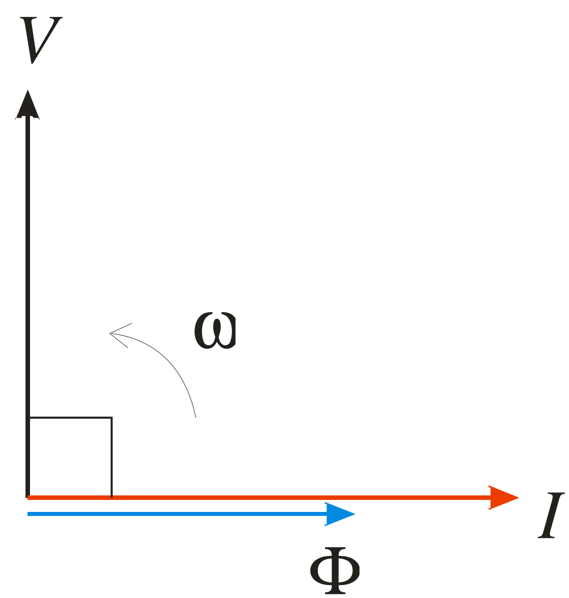
$$\phi = \frac{F}{S} = \frac{Ni}{S}$$

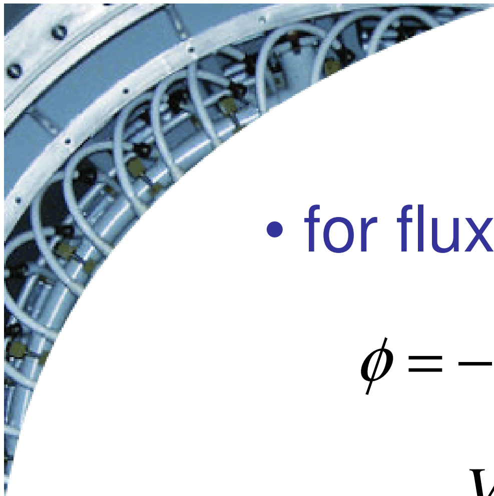
$$\phi(t) = -\frac{NV_m}{S\omega L}\cos\omega t$$

waveforms



Phasor diagram





- for flux:

$$\phi = -\frac{V_m}{\omega N}\cos\omega t$$

$$\Phi_m = \frac{V_m}{\omega N}$$

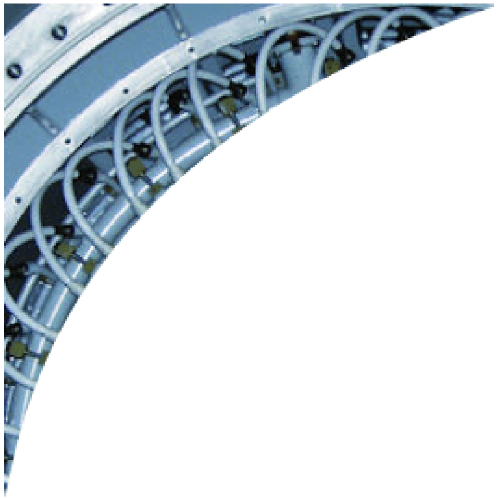
$$V_m = \omega N \Phi_m$$

$$\sqrt{2}V = 2\pi f N B_m A$$

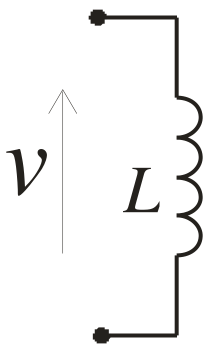
$$V = 4.44\, f N B_m A$$ $\rightarrow$ *the transformer equation*

rms value

peak value



5. Real inductor

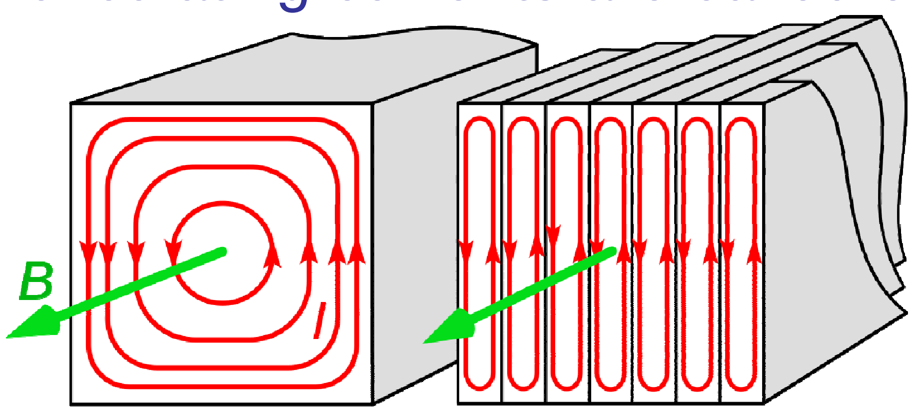


+ Power Losses:

- Copper loss (*R* of coil), P_{Cu}
- Iron loss, P_{Fe}
  - Eddy current, P_e
  - Hysteresis, P_h

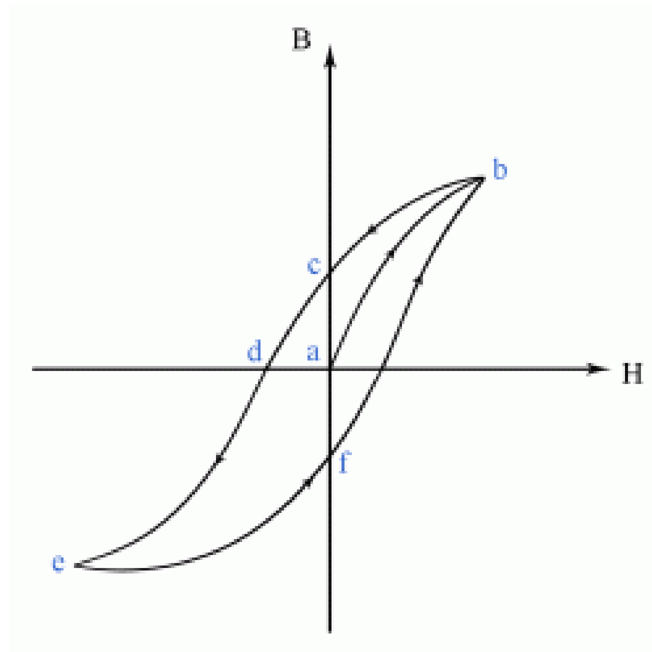
Eddy current:

- induced voltage in the core of the magnetic material
  - caused by the changing magnetic flux
  - (due to the ac current in coil)
- currents circulate in the core
  - due to induced voltage
- circulating currents are called eddy currents



Hysteresis:

- magnetic flux in the iron core
  - lags the increase or decrease of the magnetising force;
  - lagging creates a hysteresis loop



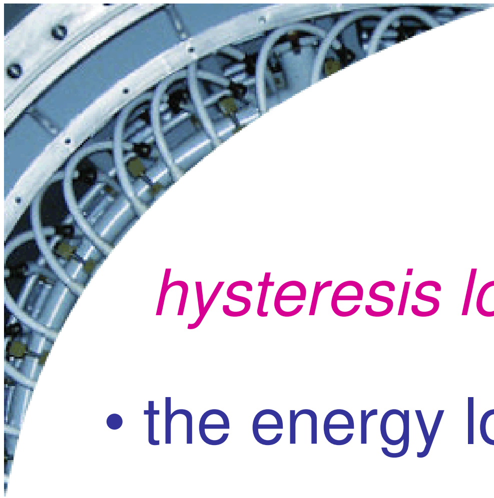
5.1 Iron loss

eddy current loss

- the power loss due to
  - eddy current
  - resistance of the iron core

$$P_e = k_e f^2 B_m^2 \qquad [\mathrm{W/m^3}]$$

- k_e = eddy constant
- f = frequency
- B_m = peak flux density



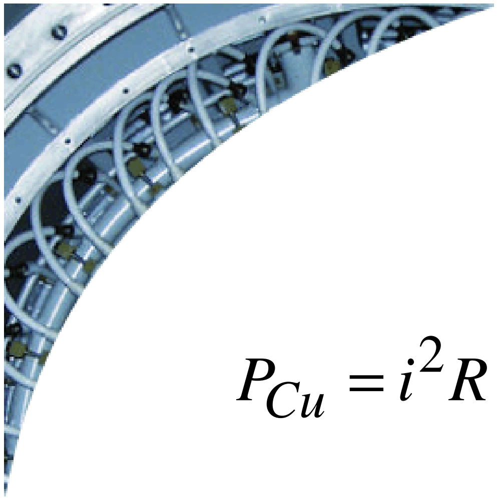
hysteresis loss

- the energy loss per cycle obtained by product of
- the area of the hysteresis loop and the volume of the core material
- 3 factors affect the shape & size of hysteresis loop:
  - material:
    - narrow - magnetises easily;
    - wide - does not easily magnetise
  - max value of B, $B_m \propto V_m$
  - initial magnetising state of iron piece

$$P_h = k_h f B_m^n \qquad [\mathrm{W/m^3}]$$

n = Steinmetz index (value of 1.6-2.5)

k_h = hysteresis constant



5.2 Inductor equivalent circuit

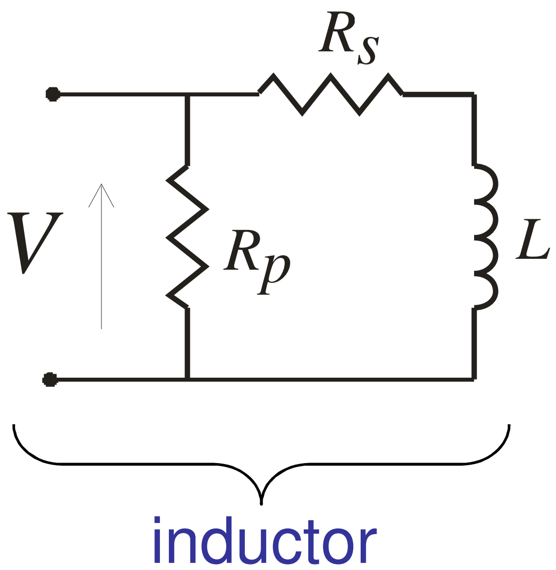
$$P_{Cu} = i^2 R$$

- represent by resistance, R_s = series resistance

$$P_{Fe} \propto \phi^2$$
$$\propto V^2$$
$$= kV^2$$

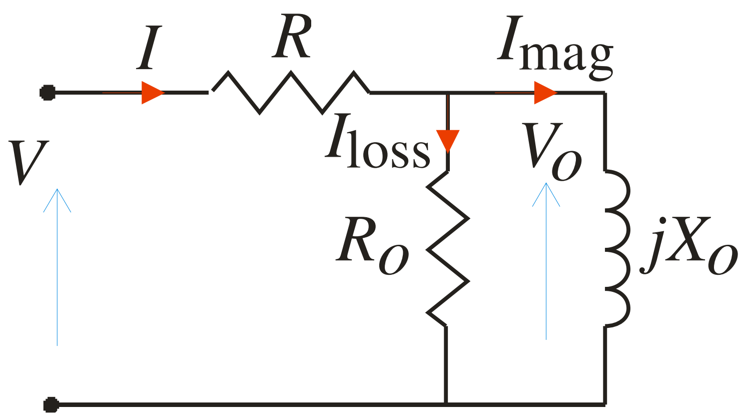
- represent by resistance, R_p such that

$$P_{Fe} = \frac{V^2}{R_p}$$
$$k = \frac{1}{R_p}$$



• inductance L + P_{Cu} + P_{Fe}

Equivalent circuit

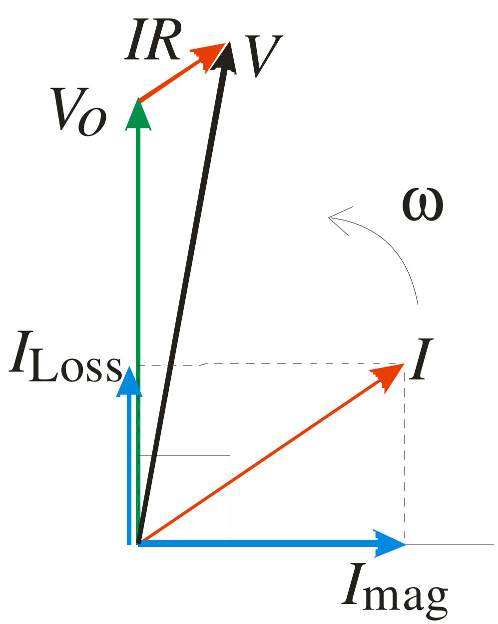


$$j\omega L = jX_o$$

$$I_{Loss} = \frac{V_o}{R_o}$$

$$I_{mag} = \frac{V_o}{jX_o}$$

Phasor diagram



$$\overline{I} = \overline{I}_{\mathrm{Loss}} + \overline{I}_{\mathrm{mag}}$$

$$\overline{V} = \overline{V}_{\mathrm{o}} + \overline{I}R$$

6. Limitations of magnetic circuit model

- requires homogeneous material
- requires isotropic material
- neglects non-linear characteristics
- neglects saturation
- neglects leakage and fringing flux

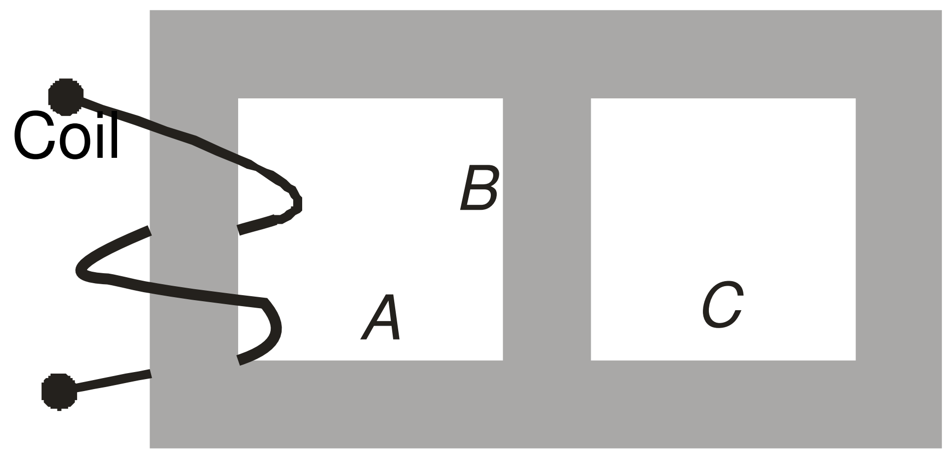
Examples:

The magnetic circuit shown in the figure has a cross-sectional area everywhere of 100 mm^2, and permeability of 10^{-3} H/m.

- If the coil has 100 turns and carries a steady current of 10 A, calculate the magnetic flux in the various portions *A*, *B* and *C*.

Assume that the effective length of each side of the squares is 100 mm, and neglect complications at the corners.

- What is the self-inductance of the coil?



- End of Lecture 3 -