EEE 3352

Electromechanics & Electrical Machines

Lecture 4: Single phase transformer

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4. Single phase transformer

1. introduction
2. ideal transformer
3. referred quantities
4. real transformer
5. determination of transformer parameters from tests
6. transformer rating
7. regulation
8. efficiency

- At the end of the lecture, students should be able to

1) describe features of an ideal and real transformer
2) derive relations for voltage ratio and current ratio
3) develop an equivalent circuit of the real transformer
4) express the operating parameters of the transformer on a phasor diagram
5) describe the tests for determining parameters of the transformer
6) determine equivalent circuit parameters of the transformer from tests

- At the end of the lecture, students should be able to

  7) determine rating of a transformer

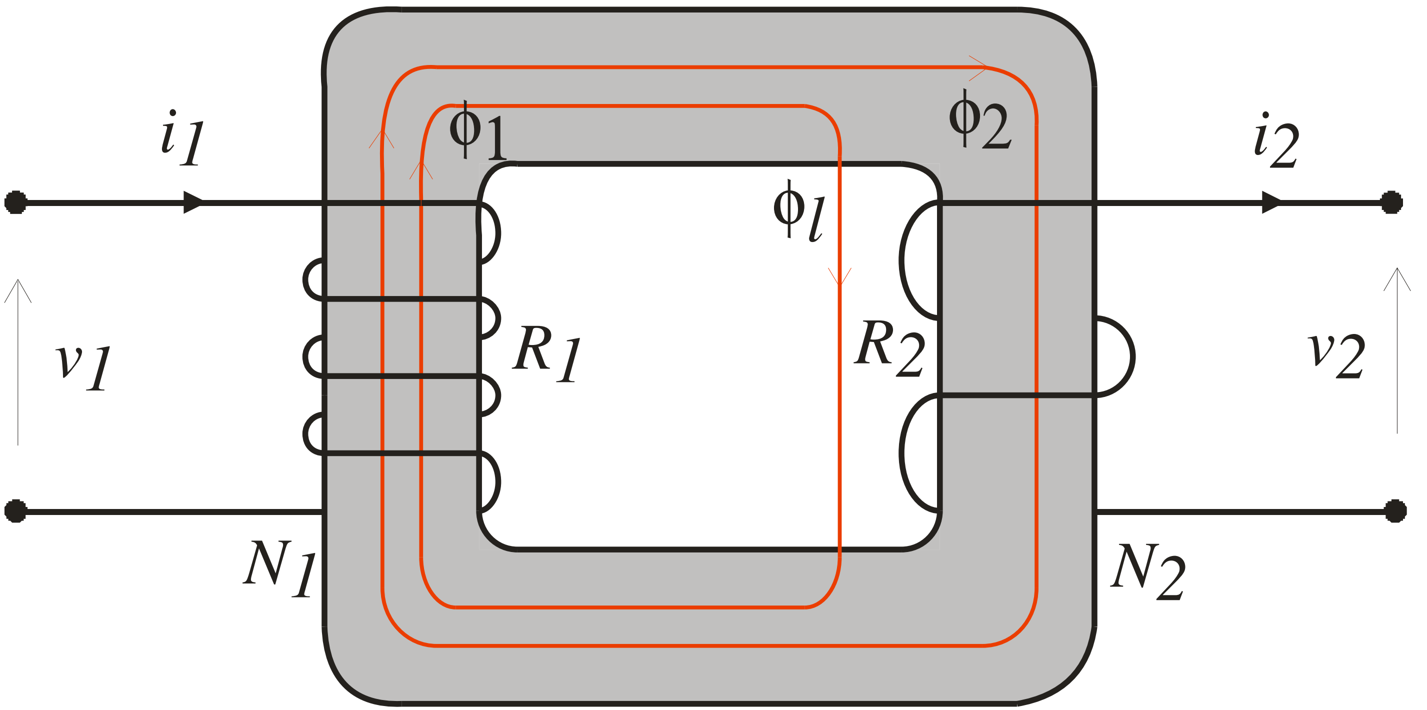
  8) derive expressions for performance of transformer: reg. and efficiency

  9) determine the performance of transformer: regulation and efficiency

4.1 Introduction

Demonstration transformer at KGRTC

Transformer schematic



- consists of ≥ 2 windings on a common magnetic circuit
- arranged so that all or nearly all flux linking 1st coil passes thru' 2nd coil
- achieved by well-defined low reluctance magnetic circuit linking the 2 coils

4.2 Ideal transformer

- ideal transformer applies 4 assumptions:

1) No leakage flux
   - so $\phi_1 = \phi_2$; or $\phi_l = 0$
2) No reluctance of magnetic circuit
   - $S = 0$; or $\mu_r = \infty$
3) No copper loss ($P_{cu} = 0$)
   - $R_1 = 0$, $R_2 = 0$
4) No iron loss ($P_{Fe} = 0$)
   - $R_p = \infty$

- Faraday's law

Coil 1: $v_1 = N_1 \dfrac{d\phi_1}{dt}$ | Coil 2: $v_2 = N_2 \dfrac{d\phi_2}{dt}$

- with $\phi_1 = \phi_2 = \phi$, for ideal trx

$$\frac{v_1}{v_2} = \frac{N_1}{N_2}$$

- using rms values

$$\frac{V_{1i}}{V_{2i}} = \frac{N_1}{N_2}$$

- subscript *i* denotes 'ideal'

- voltage ratio = turns ratio

- analogous KVL round magnetic circuit

$$F_1 - F_2 = \phi S$$

$$N_1 i_1 - N_2 i_2 = \phi S$$

- for ideal trx, $S = 0$

$$N_1 i_1 = N_2 i_2 \qquad \text{or} \qquad \frac{i_1}{i_2} = \frac{N_2}{N1}$$

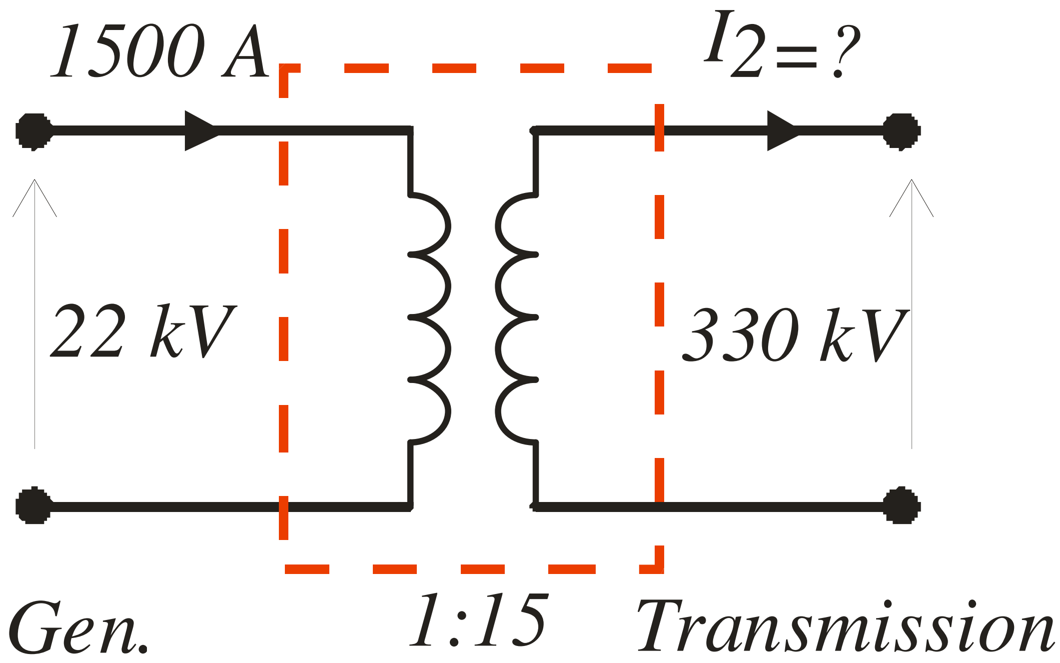
- using rms values

$$\frac{I_{1i}}{I_{2i}} = \frac{N_2}{N1}$$

- hence

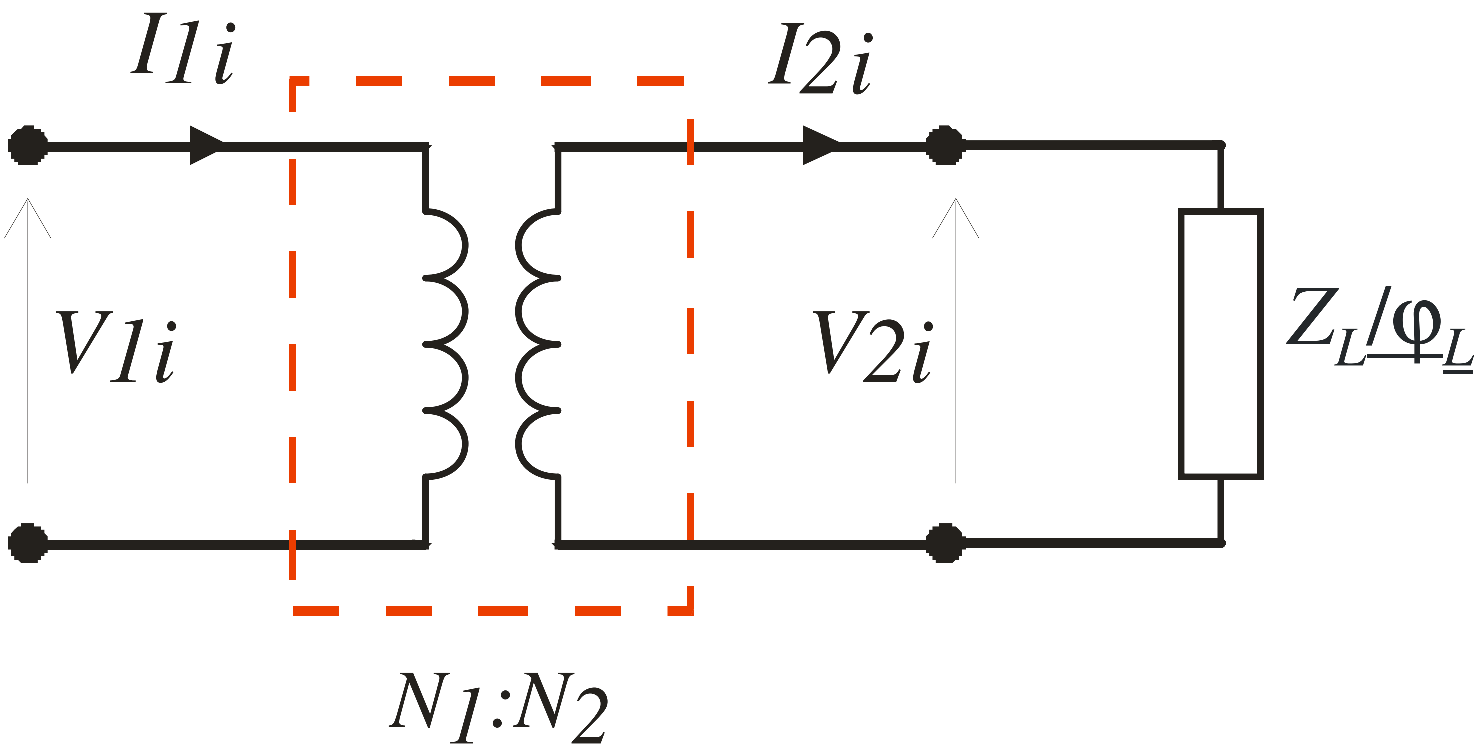
$$\boxed{\frac{V_{1i}}{V_{2i}} = \frac{N_1}{N_2} = \frac{I_{2i}}{I_{1i}}}$$

- example:

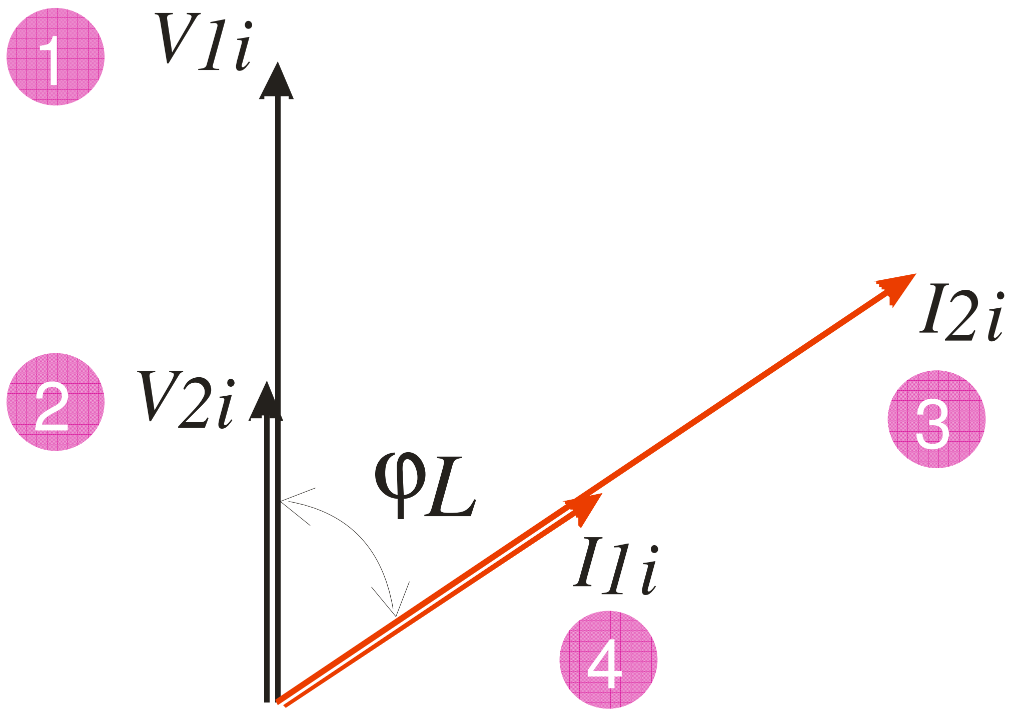


• on load:

Circuit representation



Phasor diagram



Step 2

$$e.g.\ \frac{N_1}{N_2} = 2$$

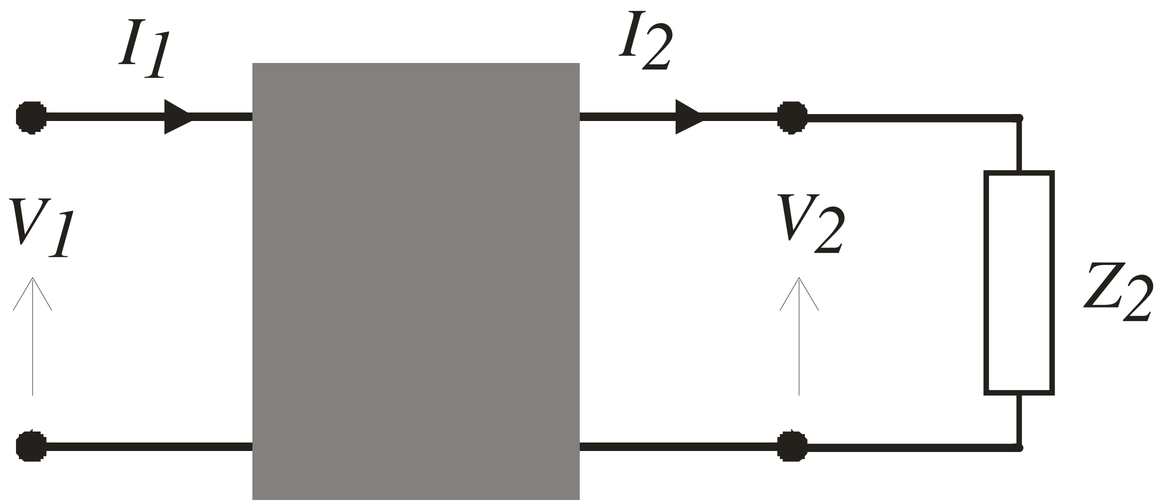
Step 3

$$I_{2i} = \frac{V_{2i}}{Z_L \angle \varphi_L}$$

- current & phase angle both on primary and secondary are determined
    - by the load
    - not by the supply or power station
- voltages are determined
    - by the supply or power station

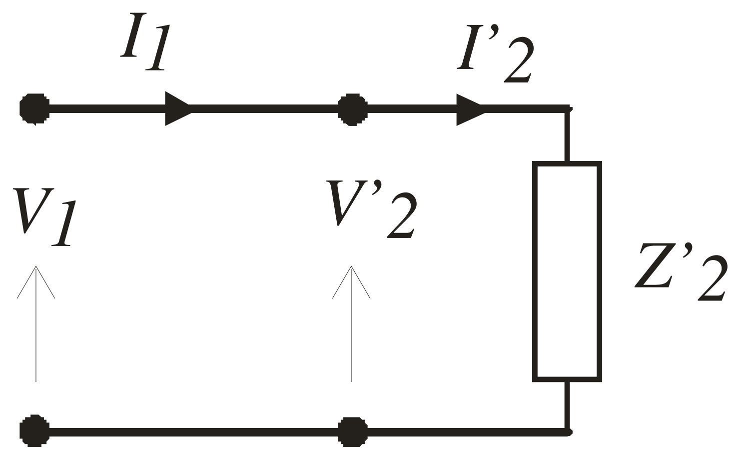
4.3 Referred quantities

Side 1 & 2 quantities



- referring *V*, *I* and *Z* from one side of trx to the other:

All quantities are seen from side 1



- the dash (′) is used to indicate a quantity has been referred from one side to the other

- what are V'_2, I'_2, and Z'_2 in terms of the original quantities?

$$V'_2 = V_1 = V_2 \frac{N_1}{N_2} \quad \longrightarrow \quad \boxed{V'_2 = V_2 \frac{N_1}{N_2}}$$

$$I'_2 = I_1 = I_2 \frac{N_2}{N_1} \quad \longrightarrow \quad \boxed{I'_2 = I_2 \frac{N_2}{N_1}}$$

$$Z'_2 = \frac{V'_2}{I'_2} = \frac{V_2}{I_2} \frac{N1/N2}{N2/N1} \quad \longrightarrow \quad \boxed{Z'_2 = Z_2 \left(\frac{N_1}{N_2} \right)^2}$$

4.4 Real transformer

- real (practical) transformer has:

1) leakage flux,

  - $\phi_l \neq 0$

2) finite reluctance of magnetic circuit

  - S = finite ;

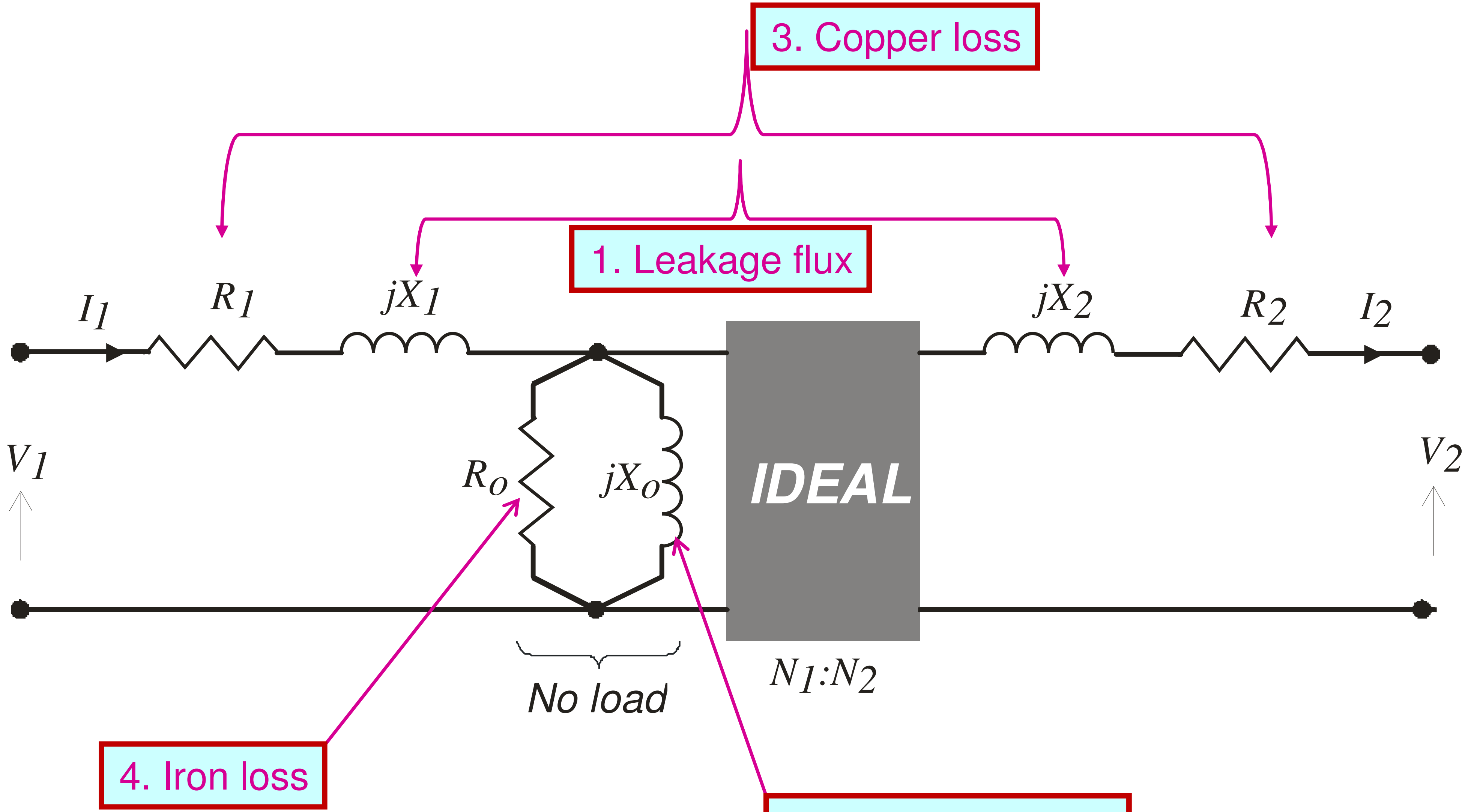
3) copper loss ($P_{cu} \neq 0$)

  - $R_1 \neq 0$, $R_2 \neq 0$

4) iron loss ($P_{Fe} \neq 0$)

  - R_p = finite

4.4.1 Equivalent circuit



- 1: leakage flux:
  - represented by two reactances in the primary and secondary X_1 and X_2;
  - they have small values compared to any other reactances

- 2: reluctance of magnetic circuit is not zero:
  - so there is a magnetising flux or mutual flux produced by magnetising current;
  - the effect is represented by a magnetising reactance X_o

$$I_1 N_1 - I_2 N_2 = \phi_m S$$

- ϕ_m is the mutual flux produced by magnetising current

$$\phi_m S = I_{mag} N_1$$

$$I_1 N_1 = I_2 N_2 + \phi_m S$$

- on no-load, $I_2 = 0$, and

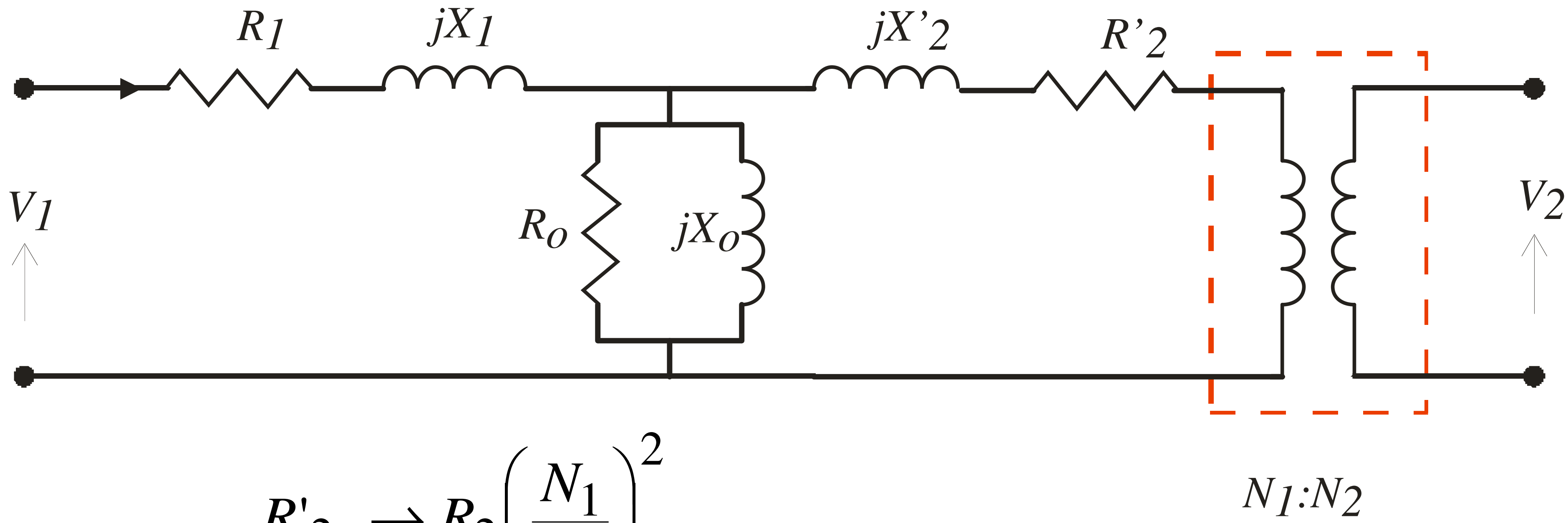
$$I_1 = I_{mag} = \frac{V_1}{jX_o} = \frac{V_1 S}{j\omega N_1^2}$$ since $$L_o = \frac{N_1^2}{S}$$

- 3: winding resistance, which incurs P_{cu},
  - is represented by the resistances R_1 and R_2,
  - for the primary and secondary, respectively

- 4: Iron loss, P_{Fe},
  - is represented by a resistance R_o
  - connected across the supply terminals

4.4.2 Simplifying equivalent circuit

1) Move R_2 and jX_2 from the secondary to primary side



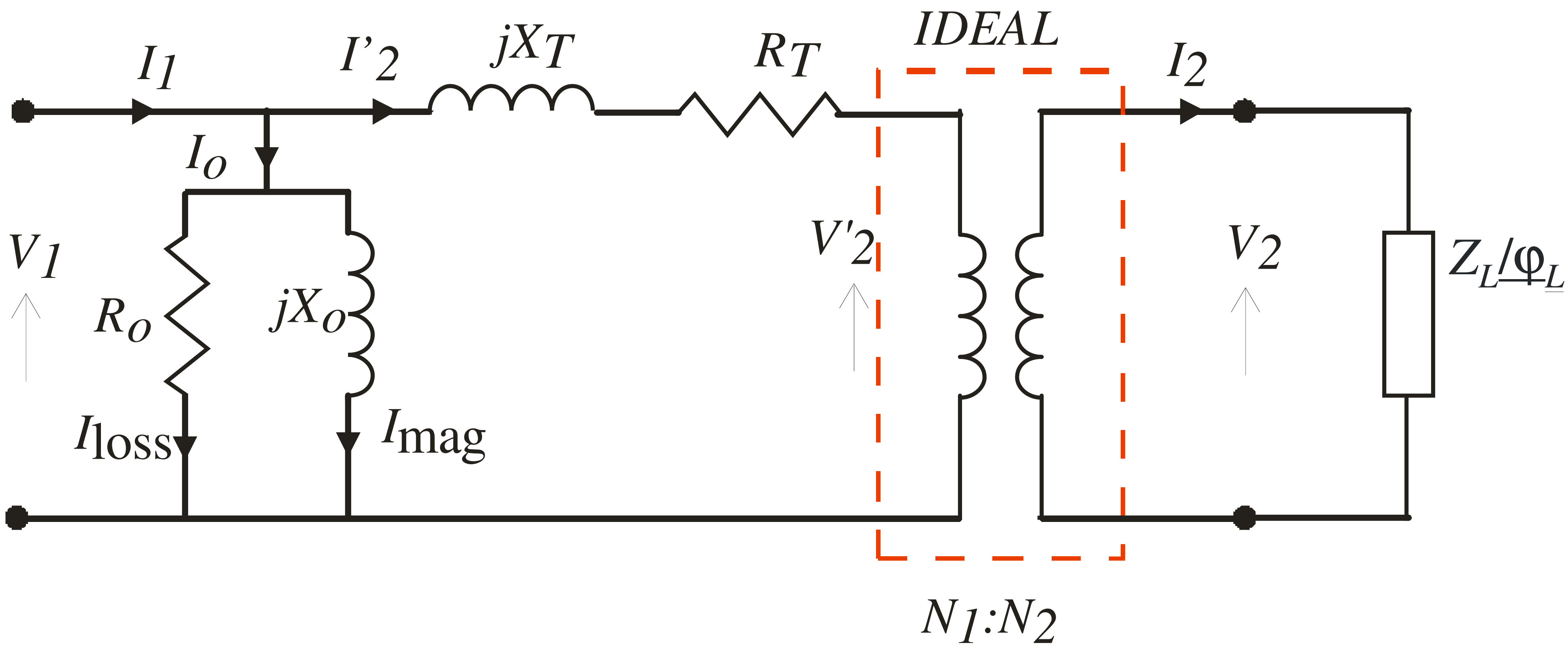
$$R'_2 \rightarrow R_2\left(\frac{N_1}{N_2}\right)^2$$

$$jX'_2 \rightarrow jX_2\left(\frac{N_1}{N_2}\right)^2$$

2) Change R_o and jX_o across the supply terminals

- strictly not legitimate
- series impedances in the circuit are much smaller than parallel components
- $\therefore$ the differences (errors) are insignificant

Final equivalent circuit



$$R_T = R_1 + R'_2$$

$$jX_T = j(X_1 + X'_2)$$

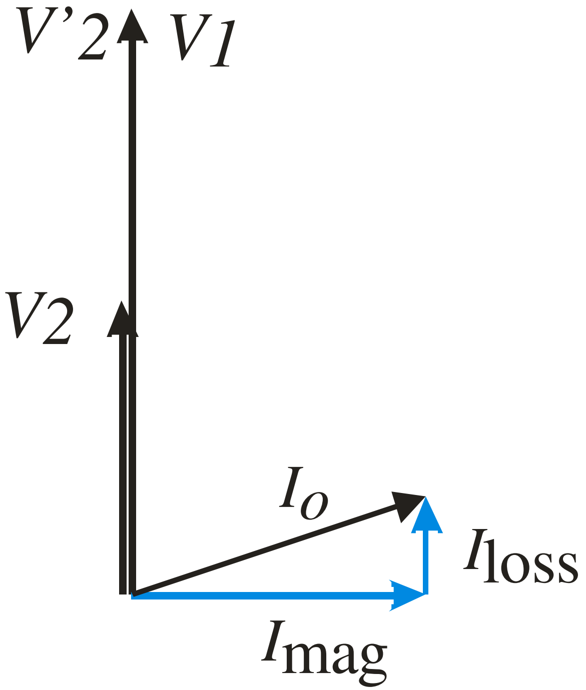
- In final equivalent circuit, the designer ensures
  - R_T and X_T are small compared to the load impendence Z'_L
  - I_o is small, so that R_o and X_o are large (>> Z'_L)
    - small P_{Fe} gives large R_o
    - large magnetising inductance L_m , or small S, gives large X_o

- Example: $N_1/N_2 = 2$, voltage equation is

$$\overline{V_1} = \overline{V'_2} + R_T \overline{I}'_2 + jX_T \overline{I}'_2$$

4.4.3 Phasor diagrams

(a) On no-load



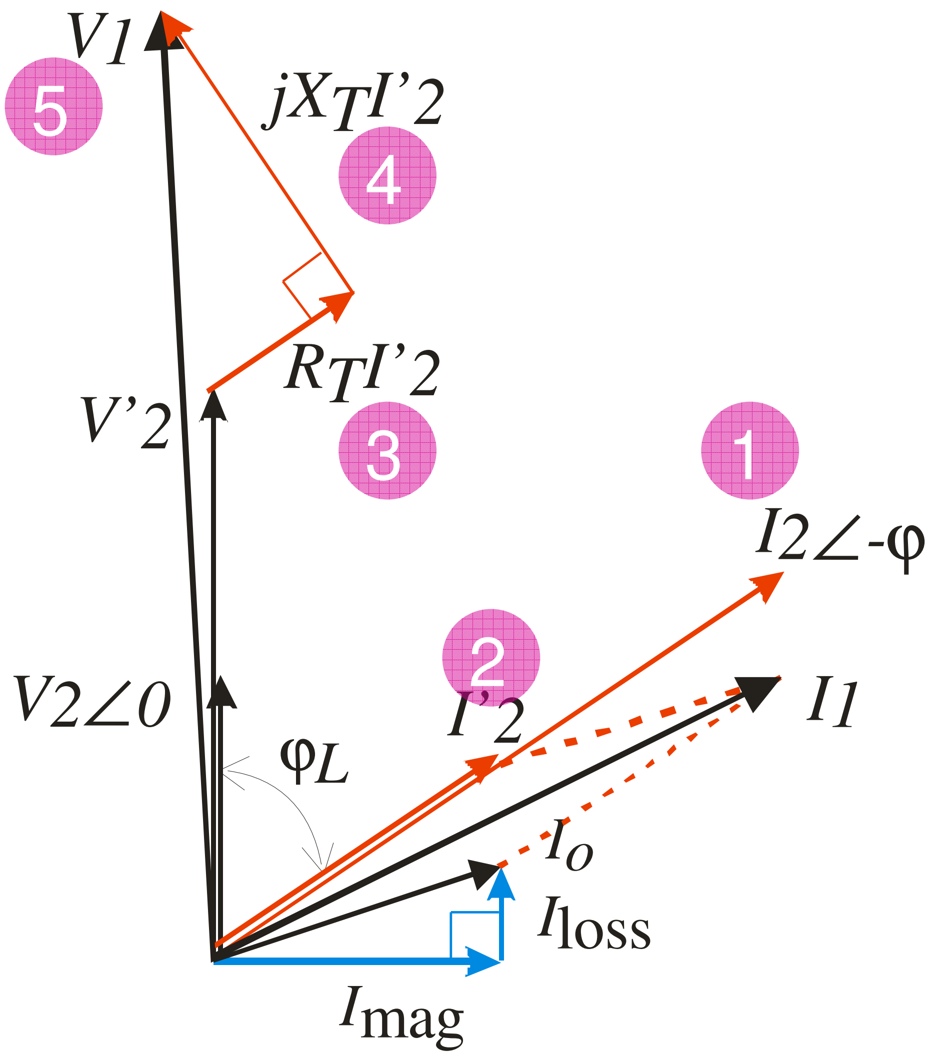
$$I_{Loss} = \frac{V_1}{R_o}$$

$$I_{mag} = \frac{V_1}{X_o}$$

$$I_o^2 = I_{Loss}^2 + I_{mag}^2$$

$$P_{Fe} = \frac{V_1^2}{R_o}$$

(b) On load



1 $$I_2 = \frac{V_2}{Z_L \angle \varphi_L}$$

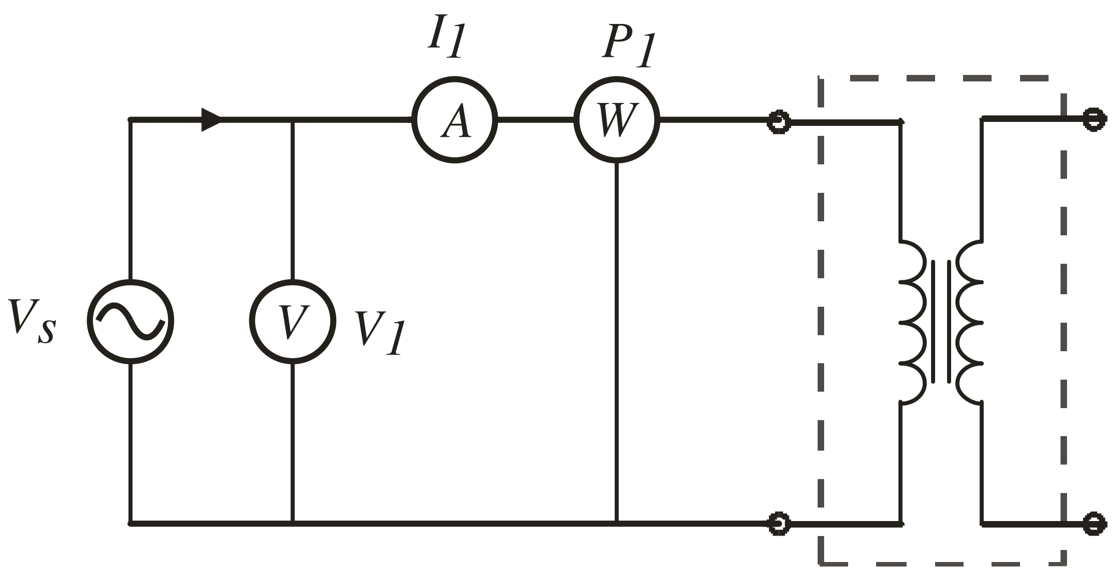
$$P_{Cu} = I'^2_2 R_T$$

$$P_{in} = V_1 I_1 \cos \varphi_1$$

$$P_{out} = V_2 I_2 \cos \varphi_L$$

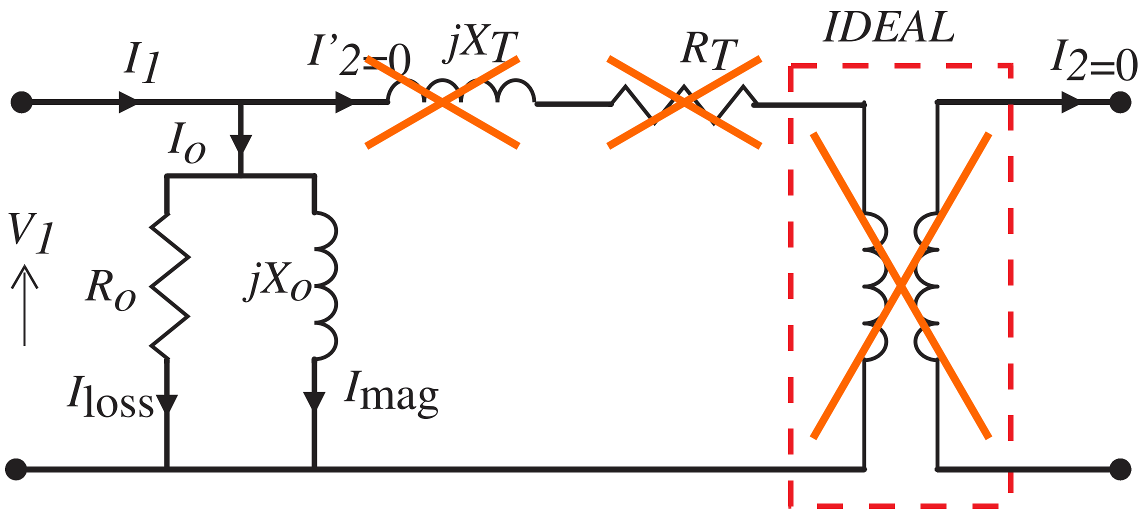
4.4 Determination of transformer parameters by tests

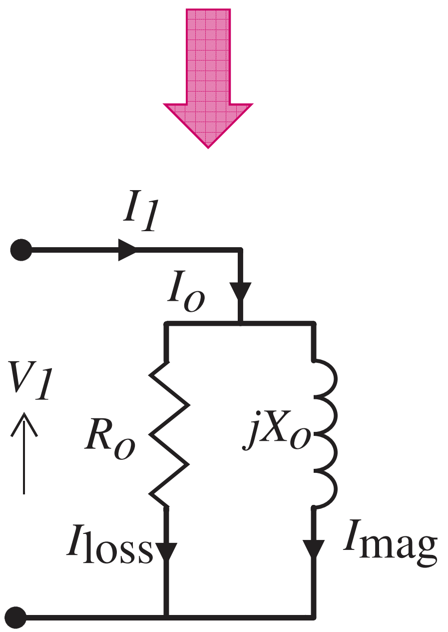
4.4.1 Open circuit test



1) Apply V_1 = rated voltage
2) Measure V_1, I_1, P_1

Equivalent circuit interpretation





1

$$P_1 = \frac{V_1^2}{R_o} \rightarrow R_o$$

2

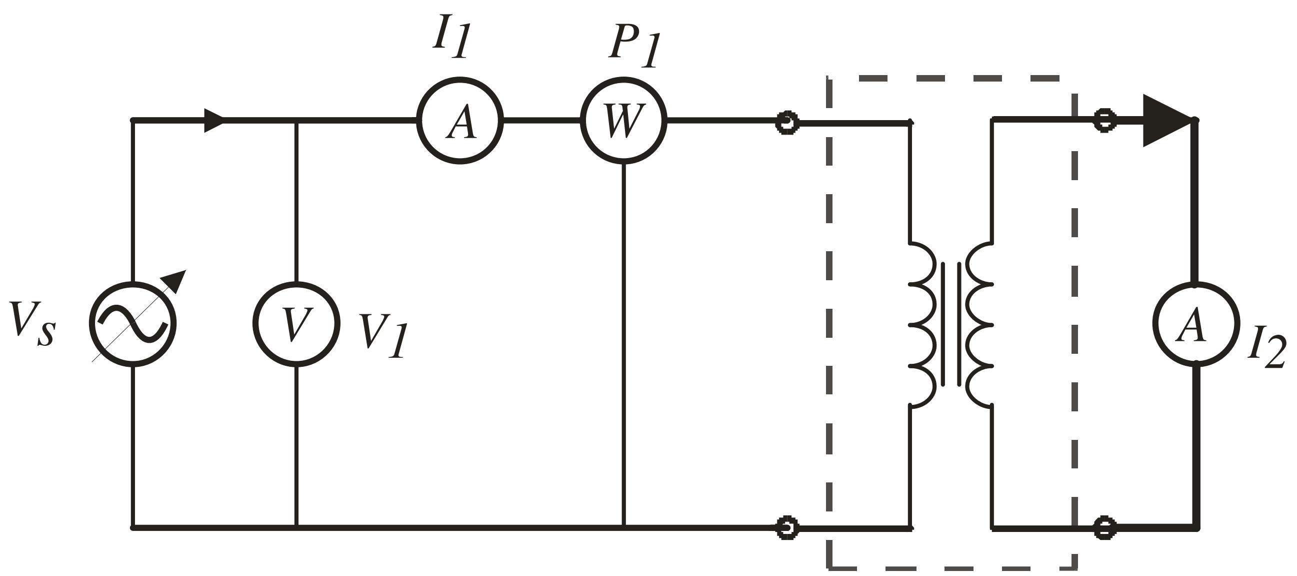
$$I_{Loss} = \frac{V_1}{R_o}$$

$$I_{mag}^2 + I_{Loss}^2 = I_o^2$$

$$X_o = \frac{V_1}{I_{mag}}$$

- gives R_o and X_o

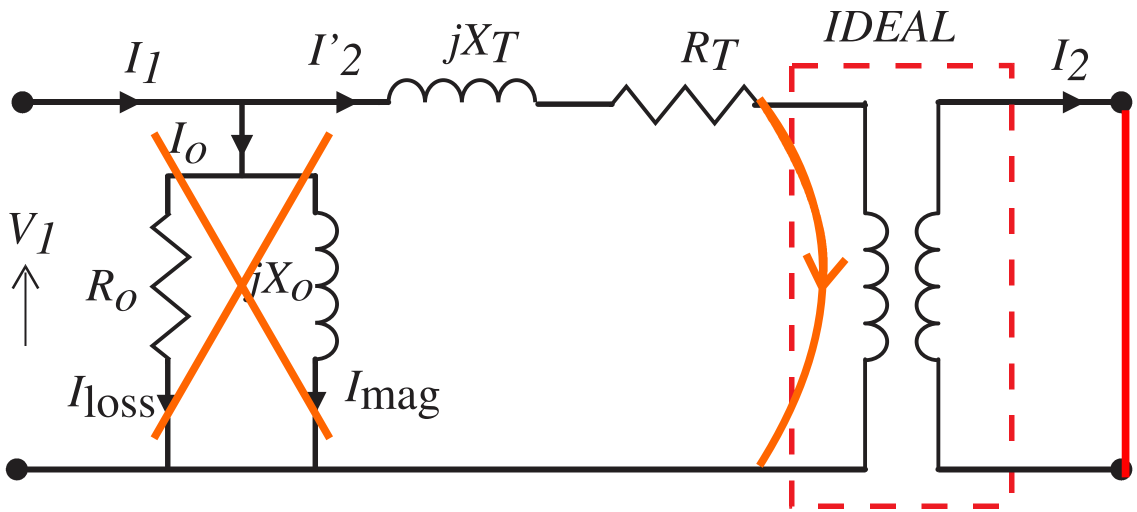
4.4.2 Short circuit test

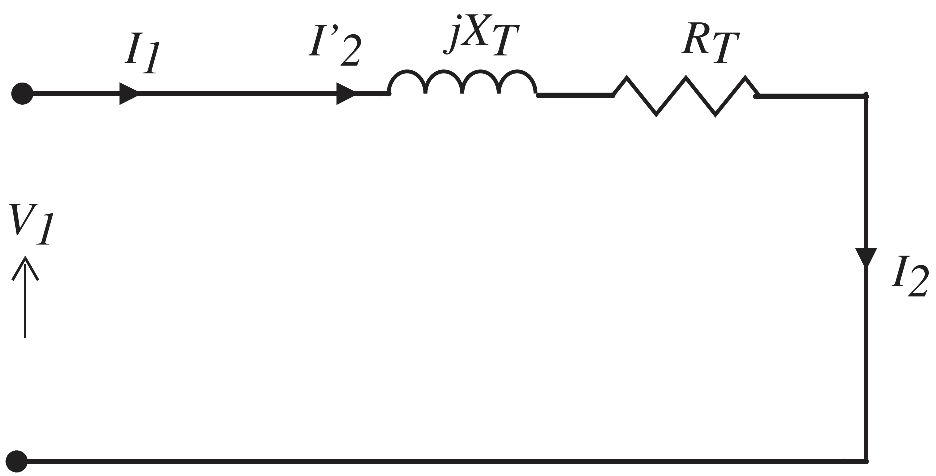


1) Apply low V_1 , gradually increasing until rated current flows

2) Measure V_1, I_1, P_1

Equivalent circuit interpretation





1

$$I_1^2 R_T = P_1 \rightarrow R_T$$

2

$$Z_T = \frac{V_1}{I_1}$$

$$Z_T^2 = R_T^2 + X_T^2$$

- gives R_T and X_T

4.5 Transformer rating

V_1	V_2	I_1	I_2	VA
$4.44N_1AfB_m$				$VA = S_r = V_1I_1 = V_2I_2$
1	2	3	4	5

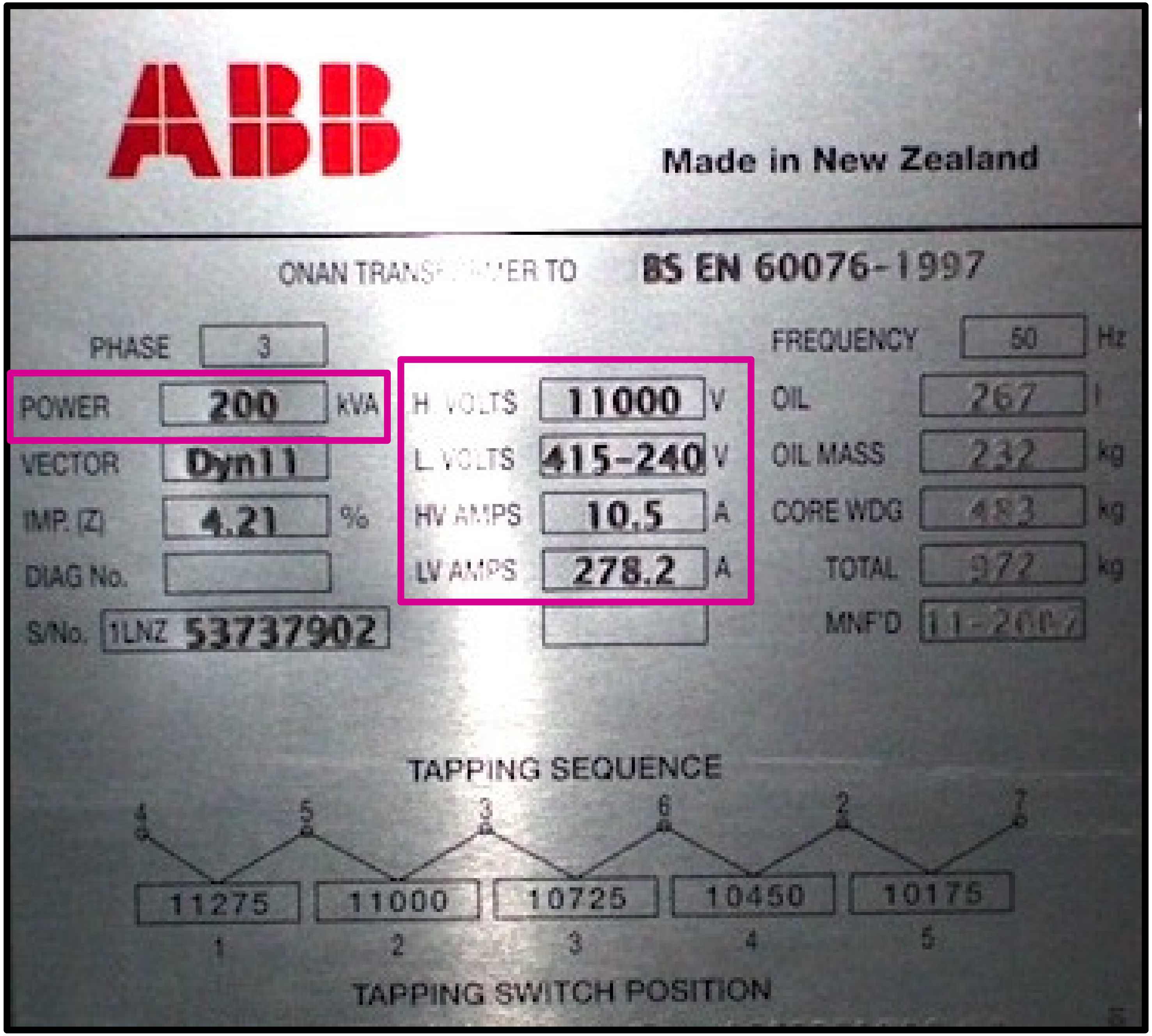
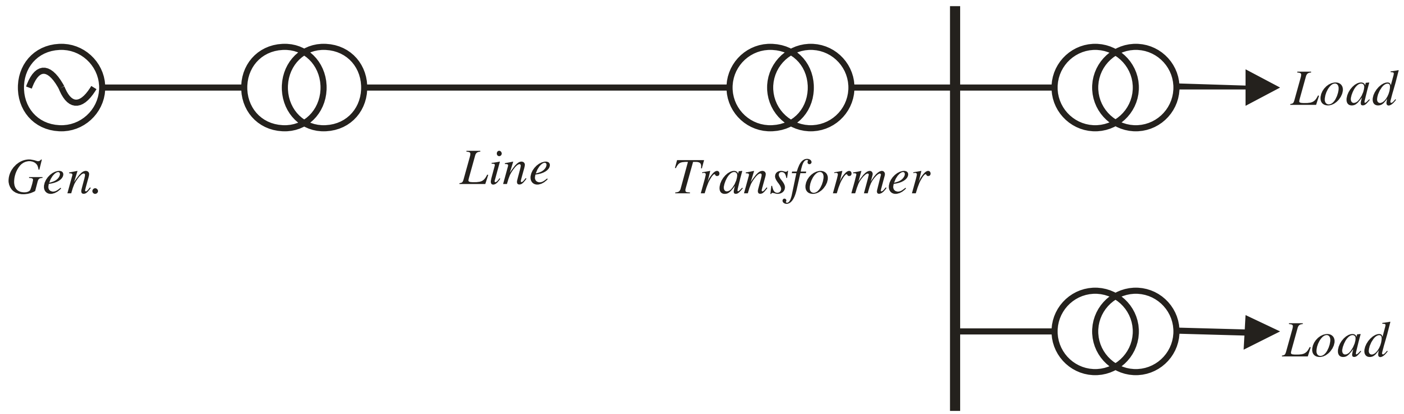


ABB
Made in New Zealand
ONAN TRANSFORMER TO BS EN 60076-1997
PHASE 3
FREQUENCY 50 Hz
POWER 200 kVA
H. VOLTS 11000 V
OIL 267 l
VECTOR Dyn11
L. VOLTS 415-240 V
OIL MASS 232 kg
IMP. (Z) 4.21 %
HV AMPS 10.5 A
CORE WDG 483 kg
DIAG No.
LV AMPS 278.2 A
TOTAL 972 kg
S/No. 1LNZ 53737902
MNF'D 11-2007
TAPPING SEQUENCE
11275 11000 10725 10450 10175
1 2 3 4 5
TAPPING SWITCH POSITION

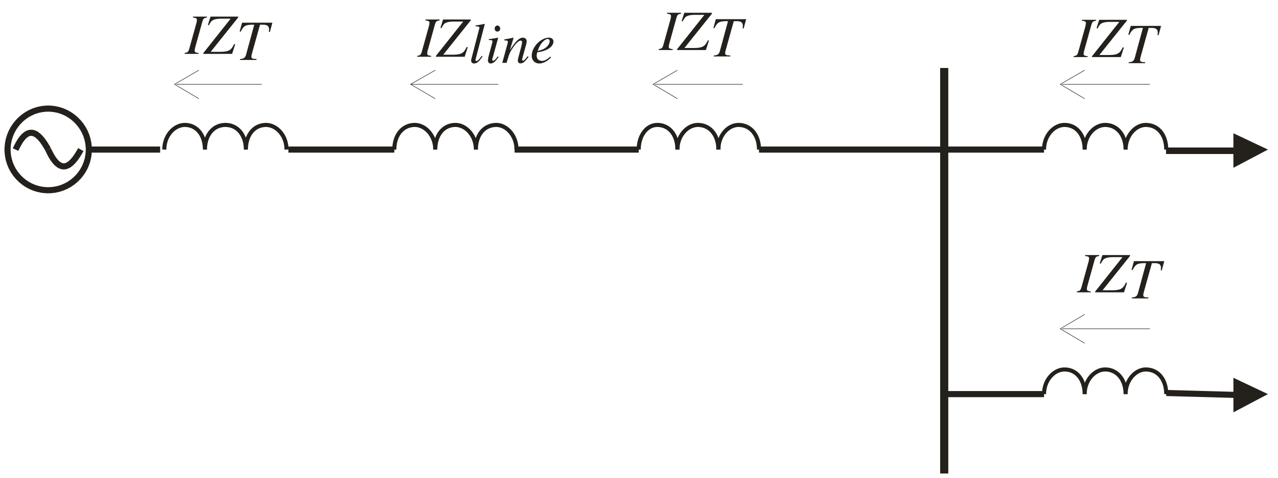
- Any 3 of the 5 quantities are essential to define trx rating,
  - the other 2 can be calculated
- *VA* rating for ideal trx is the same on primary as on secondary
- for real trx, the primary and secondary *VA* are slightly different, but so similar that the there is no distinction between the two

4.6 Voltage regulation

Power system



Equivalent circuit of power system



- Primary voltage of trx must be such that
  - delivers rated output power
  - maintains rated voltage at secondary
- Due to effect of transformer impendence
  - secondary voltage may vary from rated value
- Voltage variations are important to consumer
  - virtually all equipment is designed for some rated voltage

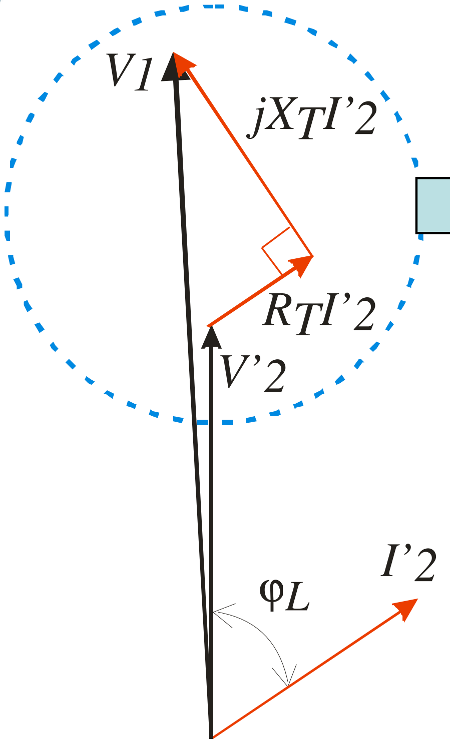
- Too high voltage:
  - overheating (shortening life of equipment)
  - efficiency may suffer
- Too low voltage
  - low power output (light, heat, mechanical power)
  - drop in efficiency
- In practice:
  - supply and consumer agree on tolerance of variation
    - e.g. $\pm$ 5%
  - variation governed by rules
    - eg Zambian regulations / standards, ZS 387/397

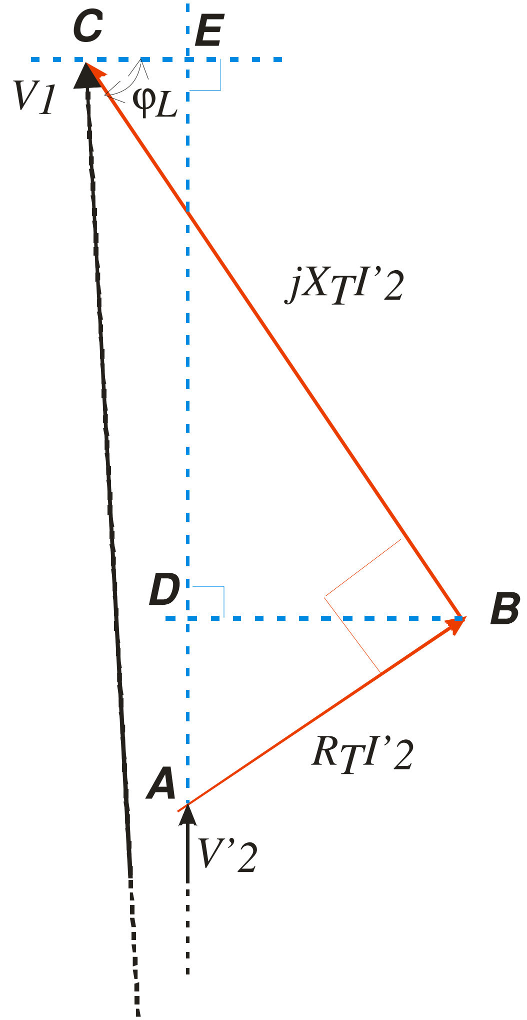
- Regulation definition:
  - fractional drop of voltage from no-load to some specified load, while keeping supply voltage constant

$$\mathrm{Reg} = \left.\frac{|V_{2NL}| - |V_{2L}|}{|V_{2NL}|}\right|_{V_1=\mathrm{const}}$$

$$= \left.\frac{|V'_{2NL}| - |V'_{2L}|}{|V'_{2NL}|}\right|_{V_1=\mathrm{const}}$$

$$= \left.\frac{|V_1| - |V'_{2L}|}{|V_1|}\right|_{V_1=\mathrm{const}}$$





$$|V_1| - |V'_{2L}| = OC - OA$$

$$OC \approx OE$$

$$|V_1| - |V'_{2L}| = OE - OA$$

$$= AD + DE$$

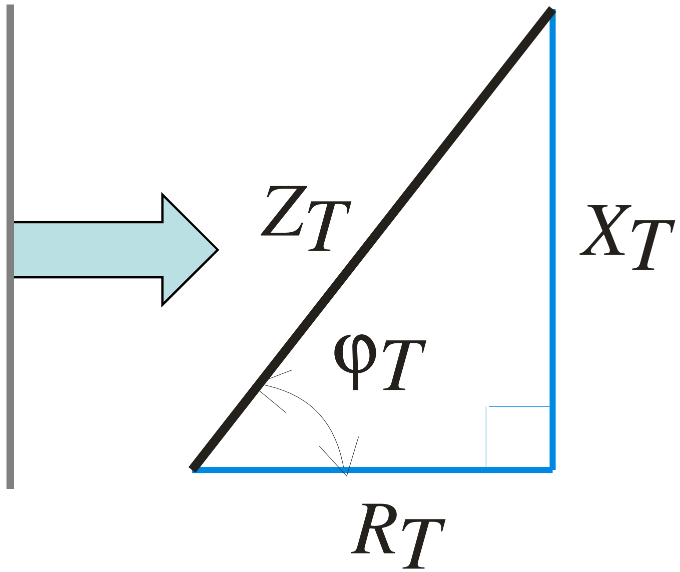
$$= I'_2\, R_T \cos\varphi_L + I'_2\, X_T \sin\varphi_L$$

$$\text{Reg} = \frac{|V_1| - |V'_{2L}|}{|V_1|}$$

$$= \frac{I'_2 R_T \cos\varphi_L + I'_2 X_T \sin\varphi_L}{V_1}$$

$$= \frac{I'_2}{V_1}(R_T \cos\varphi_L + X_T \sin\varphi_L)$$

- Define Z_T and φ_T, using the impedance triangle
  - (these are constant for a given trx)



$$\text{Reg} = \frac{I'_2}{V_1}\left(Z_T \cos\varphi_T \cos\varphi_L + Z_T \sin\varphi_T \sin\varphi_L\right)$$

$$\boxed{\text{Reg} = \frac{I'_2 \, Z_T}{V_1}\cos\left(\varphi_T - \varphi_L\right)}$$

- Reg depends on
  - trx parameters (R_T, X_T or Z_T, φ_T) and
  - load conditions (I_2, φ_L)

$$\text{Reg} = \frac{I'_2 Z_T}{V_1}\cos(\varphi_T - \varphi_L)$$

all primary

From primary side

$$\text{Reg} = \frac{I_2 Z_{T\sec}}{V_2}\cos(\phi_{T\sec} - \phi_L)$$

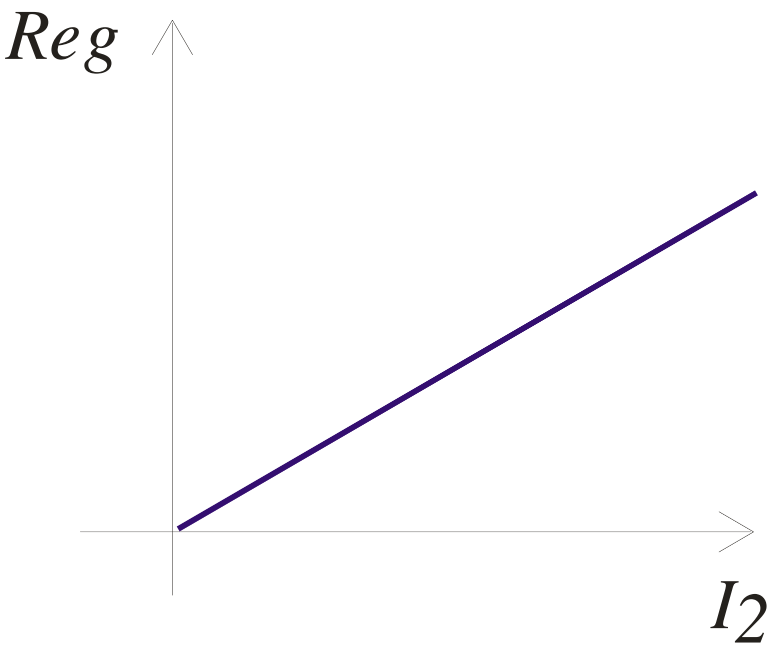
all secondary

From secondary side

Variation of Reg with load current and p.f.

$$\text{Reg} = \frac{I'_2 Z_T}{V_1} \cos(\varphi_T - \varphi_L)$$

- with p.f., const, Reg varies linearly with load current



- at specified current, Reg varies with p.f., as given by fn

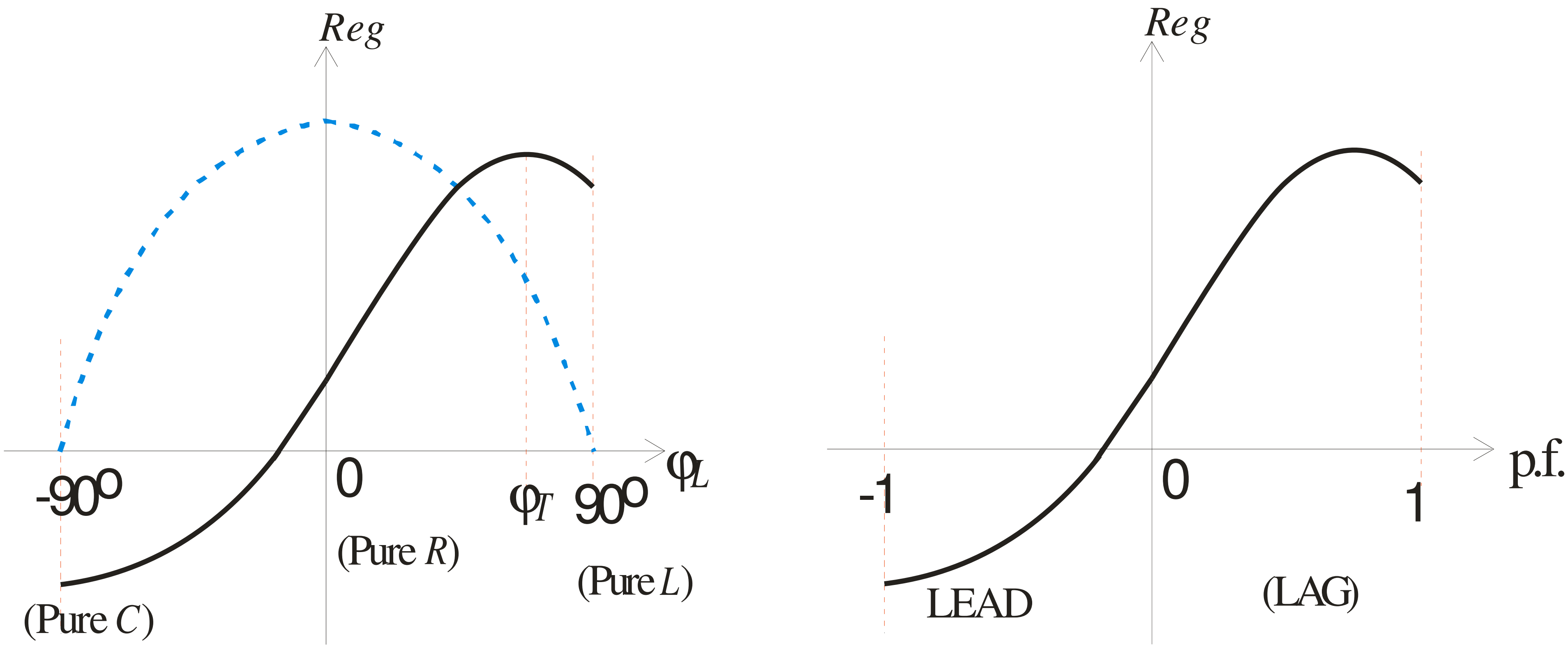
$$\text{Reg} = k_{reg}\cos(\varphi_T - \varphi_L)$$

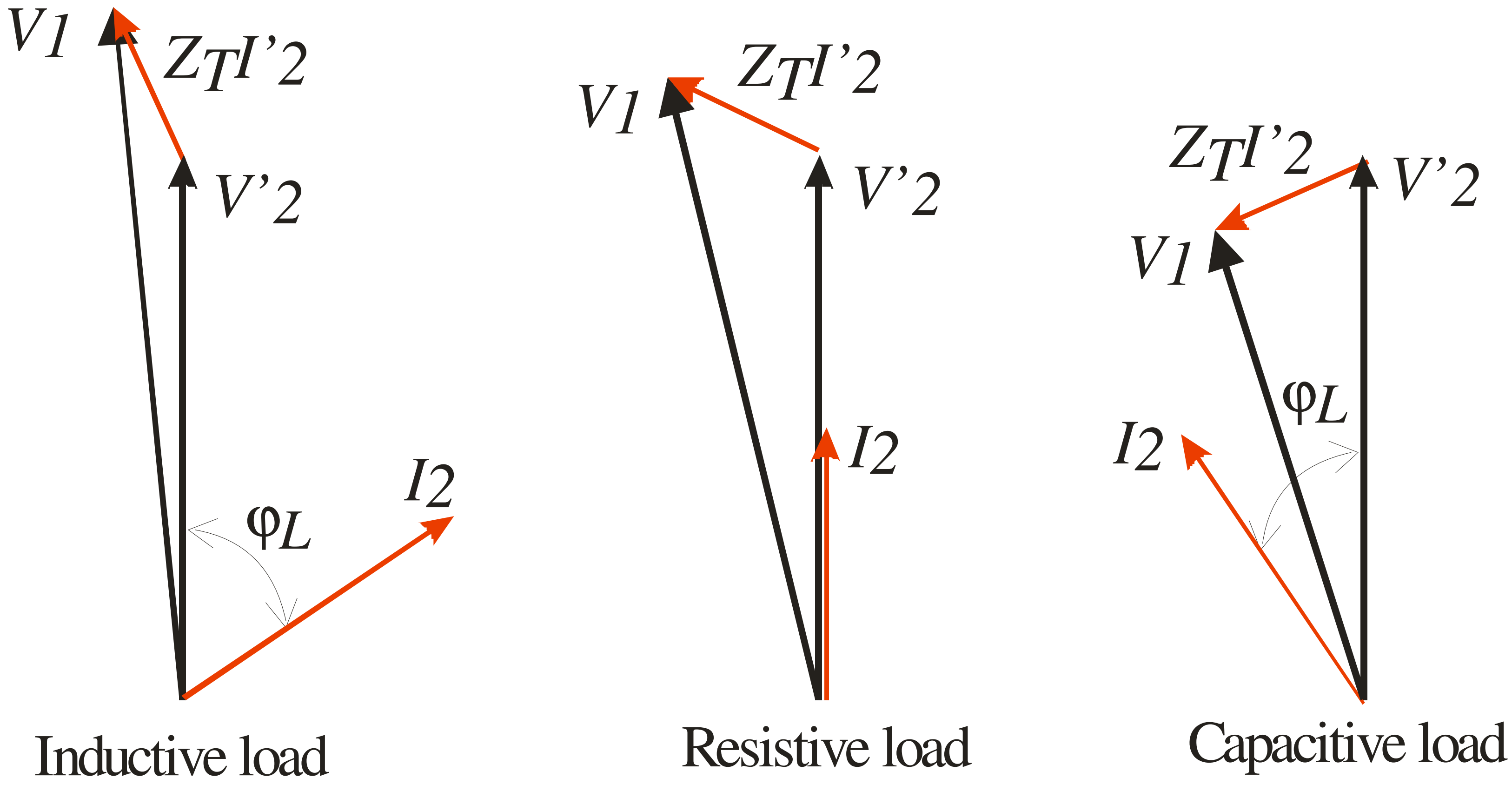
- Max. Reg occurs when $\varphi_L = \varphi_T$ and has value

$$\text{Reg} = \frac{I'_2 Z_T}{V_1}$$

- Zero Reg occurs when $\varphi_L - \varphi_T = \pm 90°$

- in terms of p.f., graph is:





- eg: $\phi_L = 60° \rightarrow$ p.f. = 0.5 (lag)

  $\phi_L = -60° \rightarrow$ p.f. = 0.5 (lead)

4.7 Efficiency

$$\text{power efficiency}, \eta = \frac{Outpt \;\; power, \;\; P_{out}}{Input \;\; power, \;\; P_{in}}$$

$$\eta = \frac{P_{out}}{P_{in}} = \frac{P_{out}}{P_{out} + Losses} = \frac{V_2 I_2 \cos\varphi_L}{V_2 I_2 \cos\varphi + P_{Fe} + I_2^2 R_{T\,\mathrm{sec}}}$$

$$\eta = f(I_2, \varphi_L)$$

- define ratio

$$x = \frac{I_2}{I_{2FL}} \qquad\qquad I_2 = xI_{2FL}$$

- x gives the function of load current

$$\eta = \frac{V_2(xI_{2FL})\cos\varphi_L}{V_2(xI_{2FL})\cos\varphi_L + P_{Fe} + (xI_{2FL})^2 R_{T\sec}}$$

$$\boxed{\eta = \frac{xS_r\cos\varphi_L}{xS_r\cos\varphi_L + P_{Fe} + x^2 P_{CuFL}}}$$

S_r = rated VA

Maximum efficiency

1) variable p.f., *current* constant:

$$\eta = \frac{xS_r}{xS_r + \dfrac{P_{Fe} + x^2 P_{CuFL}}{\cos\varphi_L}}$$

- denominator must be minimum
  - occurs when $\cos\varphi_L$ is max.,
  - i.e. $\cos\varphi_L = 1$, unity p.f.

- variable load current, x, p.f. const.

$$\eta = \frac{S_r \cos\varphi_L}{S_r \cos\varphi_L + \dfrac{P_{Fe}}{x} + xP_{CuFL}}$$

- denominator must be minimum

$$\frac{d}{dx} = \left(S_r \cos\varphi_L + \frac{P_{Fe}}{x} + xP_{CuFL} \right) = 0$$

$$0 - \frac{P_{Fe}}{x^2} + P_{CuFL} = 0$$

$$x^2 P_{CuFL} = P_{Fe}$$

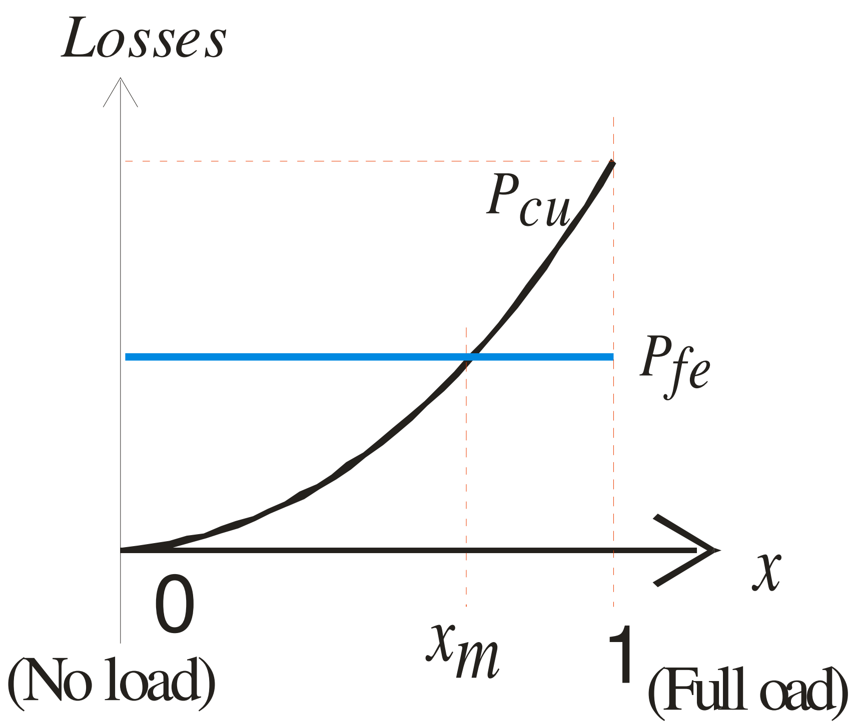
$$P_{Cu} = P_{Fe}$$

- max η occurs when variable P_{Cu} has the value of the const P_{Fe}
- max η occurs when

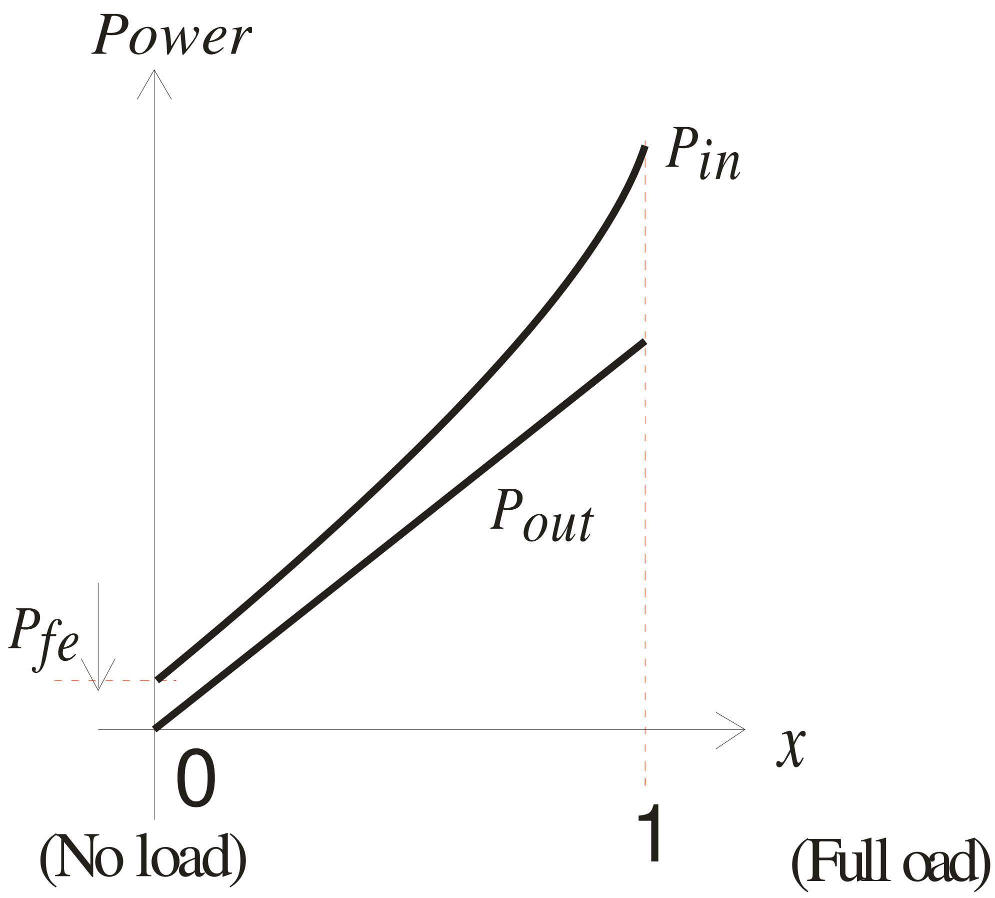
$$x^2 = \frac{P_{Fe}}{P_{CuFL}}$$

$$\boxed{x = \sqrt{\frac{P_{Fe}}{P_{CuFL}}}}$$

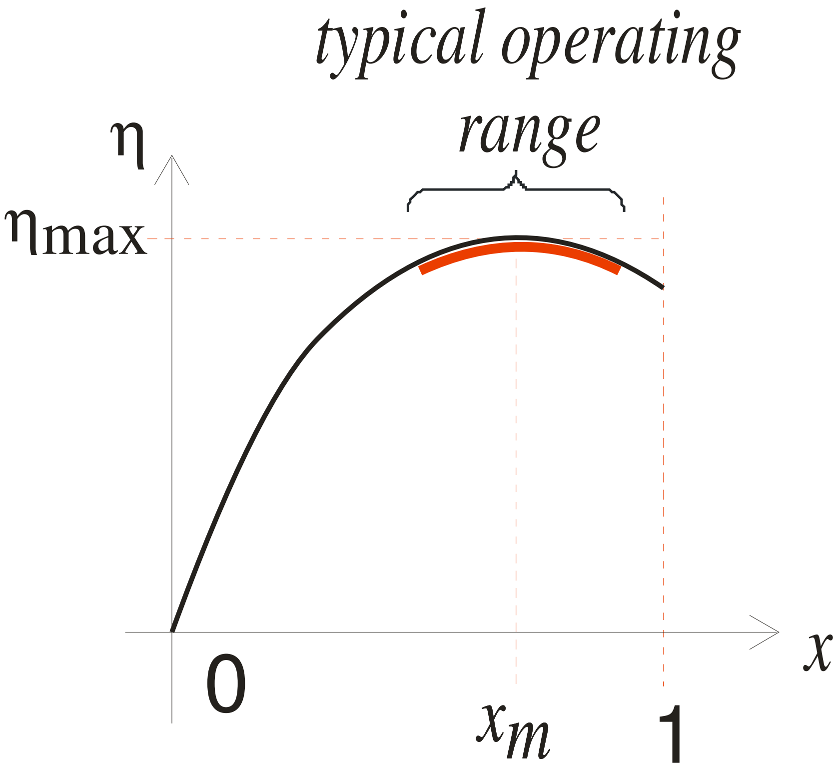
Variation of losses vs load



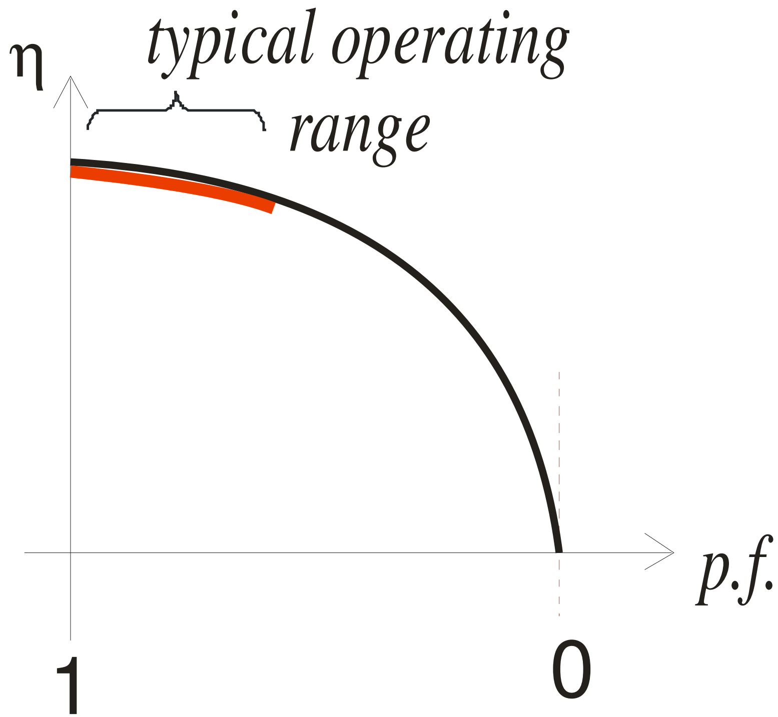
Variation of power vs load



η vs load



η vs p.f.



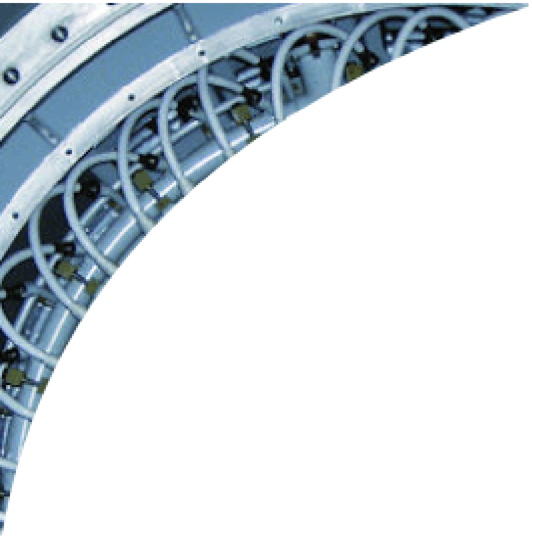
Examples:

1.

- A 220/100-V transformer has a total resistance of 2 Ω and a total leakage reactance of 3 Ω both referred to the primary. The secondary current is 10 A.
- Find the
  - regulation at zero power factor lag
  - regulation at unity power factor
  - regulation at zero power factor lead
  - power factor which gives maximum regulation
  - value of the maximum regulation
  - power factor which gives zero regulation

2.

- A single-phase transformer is rated at 10 kVA, 240/100 V
- When the secondary terminals are open-circuited and the primary winding is supplied at normal voltage, the input current is 2.6 A at a power factor of 0.3 lag
- When the secondary terminals are short-circuited, a voltage of 18 V applied to the primary causes the full-load current to flow in the secondary, the power input to the primary being 240 W.
- Calculate, at full load unity power factor, the:
  - regulation
  - efficiency



- End of Lecture 4 -