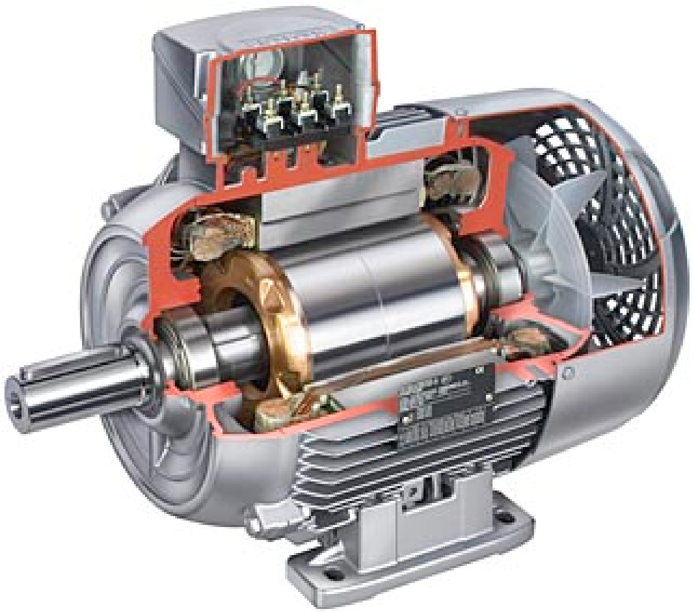
EEE 3352

Electromechanics & Electrical Machines



Lecture 2: Examples

Example 2.1

A parallel plate capacitor with waxed paper as the insulator has a

- capacitance of 3800 pF
- operating voltage of 600 V
- safety factor of 2.5

The waxed paper has

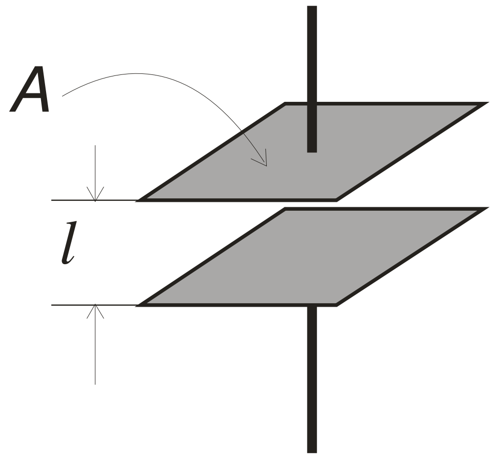
- a relative permittivity of 4.3 and
- breakdown voltage stress of 15 x 10^6 V/m

What is the physical size of the capacitor?

(i.e. find the spacing, *l,* between the two plates of the capacitor and the plate area, *A*.)

- Information:

  - C = 3800 pF
  - V = 600 V
  - SF = 2.5
  - ε_r = 4.3
  - ε_0 = 8.85×10^{-12} F/m
  - E_{bd} = 15×10^{6} V/m



- working voltage stress:

$$E_{\max} = \frac{E_{bd}}{SF} = \frac{15\times10^{6}}{2.5} = 6\times10^{6}\ \text{V/m}$$

- plates spacing, *l*:

$$E = \frac{V}{l} \rightarrow l = \frac{V}{E} = \frac{600}{6\times10^{6}} = 1\times10^{-4}\ \text{m} = \underline{\underline{0.1\ \text{mm}}}$$

- plate area, *A*:

$$C = \varepsilon\frac{A}{l} \rightarrow$$

$$A = \frac{Cl}{\varepsilon_r\varepsilon_0} = \frac{(3800\times10^{-12})\times(1\times10^{-4})}{4.3\times(8.85\times10^{-12})} = 10\times10^{-3}\ \text{m}^2 = \underline{\underline{100\ \text{cm}^2}}$$

Example 2.2

The diameter of the copper core of a single-core rubber-insulated cable is 4.5 mm.

The rubber insulation has a relative permittivity of 4 and breakdown field strength of 18×10^6 V/m.

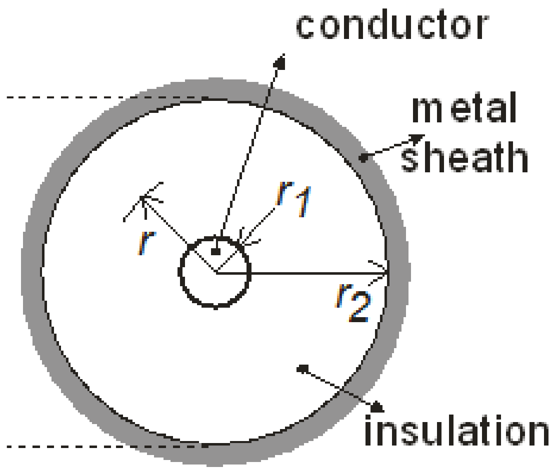
A voltage of 10 kV may be applied between the core and the lead sheath with a safety factor of 3.

Calculate the

- diameter of the lead sheath which covers the rubber insulation
- cable capacitance per metre

- Information:

  - d_1 = 4.5 mm
  - V = 10 kV
  - SF = 3
  - ε_r = 4
  - ε_0 = 8.85 x 10^{-12} F/m
  - E_{bd} = 18 x10^6 V/m



- working voltage stress:

$$E_{max} = \frac{E_{bd}}{SF} = \frac{18\times10^6}{3} = 6\times10^6 \text{ V/m}$$

- outer diameter, d_2:

$$E_{max} = \frac{V}{r_1 \ln\frac{r_2}{r_1}} \rightarrow r_2 = r_1 e^{\frac{V}{r_1 E_{max}}}$$

$$r_2 = \frac{4.5}{2}\times10^{-3}\times e^{\left(\frac{10\times10^3}{(\frac{4.5}{2}\times10^{-3})\times(6\times10^6)}\right)} = 4.72\times10^{-3}\text{ m}$$

$$d_2 = 2r_2 = 2\times(4.72\times10^{-3}) = 9.44\times10^{-3} = \underline{\underline{9.44 \text{ mm}}}$$

- capacitance per unit length:

$$C = \frac{2\pi l \varepsilon}{\ln \frac{r_2}{r_1}} \rightarrow \frac{C}{l} = \frac{2\pi\varepsilon}{\ln \frac{r_2}{r_1}}$$

$$\left[\frac{C}{l}\right] = \frac{2\pi \times 4 \times 8.85 \times 10^{-12}}{\ln \frac{9.44}{4.5}} = 3.0 \times 10^{-10} = \underline{\underline{300 \text{ pF/m}}}$$

Example 2.3

The voltage applied between empty plates separated by 1 cm is 20 kV.

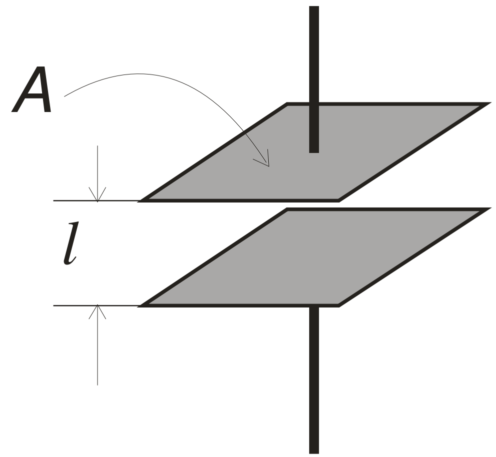
If a dielectric slab of relative permittivity 5 is introduced between its two plates, calculate for the parallel-plate capacitor the

- change in the stored energy
- stored energy, if plate area is 2 m^2

What is the pressure squeezing the plates when the slab is inserted?

- Information:

  - l = 1 cm
  - A = 2 m^2
  - V = 20 kV
  - ε_r = 5
  - ε_0 = 8.85 x 10^{-12} F/m



- change in stored energy:

$$w = \frac{1}{2}DE = \frac{1}{2}\varepsilon_r \varepsilon_0 E^2$$

- without slab: $w_1 = \frac{1}{2}\varepsilon_{r1}\varepsilon_0 E_1^2 \qquad E_1 = \frac{V}{l} = E$

- with slab: $w_2 = \frac{1}{2}\varepsilon_{r2}\varepsilon_0 E_2^2 \qquad E_2 = \frac{V}{l} = E$

$$\frac{w_2}{w1} = \frac{\frac{1}{2}\varepsilon_{r2}\varepsilon_0 E^2}{\frac{1}{2}\varepsilon_{r1}\varepsilon_0 E^2} = \frac{\varepsilon_{r2}}{\varepsilon_{r1}} = \frac{5}{1} = \underline{\underline{5 \text{ times}}}$$

- stored energy with slab:

$$volume = Al = 2\times(1\times10^{-2}) = 0.02\ \text{m}^3$$

$$W_2 = \frac{1}{2}\varepsilon_{r2}\varepsilon_0 E_2^2 \times volume$$

$$W_2 = \frac{1}{2}\times5\times8.85\times10^{-12}\times\left(\frac{20\times10^3}{1\times10^{-2}}\right)^2\times0.02 = 1.77\times10^{-12} = \underline{\underline{1.77\ \text{pJ}}}$$