

10 Determine

$$i) \int \frac{(2+3x)^2}{\sqrt{x}} dx = \int \frac{4+12x+9x^2}{\sqrt{x}} dx = \int \frac{4}{x^{1/2}} + \frac{12x}{x^{1/2}} + \frac{9x^2}{x^{1/2}} dx$$

$$= \int 4x^{-1/2} + 12x^{1/2} + 9x^{3/2} dx$$

$$= \frac{4x^{-1/2+1}}{-1/2+1} + \frac{12x^{1/2+1}}{1/2+1} + \frac{9x^{3/2+1}}{3/2+1}$$

$$= \frac{4x^{1/2}}{1/2} + \frac{12x^{3/2}}{3/2} + \frac{9x^{5/2}}{5/2}$$

$$= 8x^{1/2} + 8x^{3/2} + \frac{18}{5}x^{5/2} + C$$

$$ii) \int \frac{3}{4} e^{2x} dx = \frac{1}{2} \times \frac{3}{4} e^{2x} = \frac{3}{8} e^{2x} + C$$

$$iii) \int 6^{3x} dx = \int a^u du = \frac{a^u}{u \ln(a)} + C$$

$$= \frac{6^{3x}}{3 \ln(6)} + C$$

$$iv) \int 2 \csc^2 4\theta d\theta = \int \csc^2 ax dx = -\frac{1}{a} \cot ax + C$$

$$= -\frac{1}{4} \cot 4\theta = -\frac{1}{2} \cot 4\theta + C$$

$$v \int \frac{4}{3} \sec 4t \tan 4t dt \quad \text{Let } U = \tan 4t$$

$$\frac{dU}{dt} = 4 \sec^2 4t$$

$$\frac{4}{3} \int \sec 4t U \cdot \frac{dU}{4 \sec^2 4t}$$

$$\frac{4}{3} \int \frac{U}{4 \sec 4t} \cdot dU = \frac{1}{3} \int \frac{U}{\sqrt{1 + \tan^2 4t}} \cdot dU$$

$$= \frac{1}{3} \int \frac{U}{\sqrt{1 + U^2}} dU \quad \text{Let } m = 1 + U^2$$

$$\frac{dm}{dU} = 2U$$

$$= \frac{1}{3} \int \frac{U}{\sqrt{m}} \cdot \frac{dm}{2U} = \frac{1}{6} \int m^{-1/2} dm = \frac{1}{6} \left(\frac{m^{-1/2+1}}{-1/2+1} \right)$$

$$= \frac{1}{6} \frac{m^{1/2}}{1/2} = \frac{1}{3} m^{1/2} = \frac{1}{3} (1 + U^2)^{1/2} = \frac{1}{3} (1 + \tan^2 4t)^{1/2} + C$$

$$= \frac{1}{3} \sec 4t + C$$

$$vii \int \cot 2t \csc 2t dt \quad \text{Let } U = \cot 2t \quad dt = - \frac{dU}{2 \csc^2 2t}$$

$$\frac{dU}{dt} = -2 \csc^2 2t$$

$$\int \frac{U \csc 2t \cdot - \frac{dU}{2 \csc^2 2t}}{2 \csc^2 2t} = \int U \cdot - \frac{dU}{2 \csc 2t} = - \frac{1}{2} \int \frac{U}{\csc 2t} dU$$

$$= - \frac{1}{2} \int \frac{U}{\sqrt{\cot^2 2t + 1}} dU$$

$$\text{Let } m = U^2 + 1 \quad dU = \frac{dm}{2U}$$

$$= - \frac{1}{2} \int \frac{U}{\sqrt{U^2 + 1}} dU$$

$$\frac{dm}{dU} = 2U$$

$$-\frac{1}{2} \int \frac{u}{\sqrt{m}} \cdot \frac{dm}{2u} = -\frac{1}{4} \int m^{1/2} dm = -\frac{1}{4} \left(\frac{m^{3/2}}{3/2} \right)$$

$$= -\frac{1}{2} m^{1/2} = -\frac{1}{2} (u^2 + 1) = -\frac{1}{2} (\cot^2 2t + 1) + C$$

$$= -\frac{1}{2} \csc 2t + 1 + C$$

b) Evaluate

$$i) \int_1^3 \frac{2x^2 + 1}{x} dx = \int_1^3 2x + \frac{1}{x} dx = \left[x^2 + \ln x \right]_1^3$$

$$= (9 + \ln(3)) - (1 + \ln(1)) = 9 + \ln(3) - 1$$

$$= \underline{\underline{8 + \ln(3)}}$$

$$ii) \int_{-1}^2 \frac{2}{3e^{2x}} dx = \int_{-1}^2 \frac{2}{3} e^{-2x} dx = \int_{-1}^2 \frac{2}{3} \cdot -\frac{1}{2} e^{-2x}$$

$$\left[-\frac{1}{3} e^{-2x} \right]_{-1}^2 = \left[-\frac{1}{3} e^{-2(2)} \right] - \left[-\frac{1}{3} e^{-2(-1)} \right]$$

$$= -\frac{1}{3e^4} + \frac{e^2}{3} = 2.45691348$$

$$iii) \int_{\pi/4}^{\pi/2} (3 \sin 2x - 2 \cos 3x) dx = \left[-\frac{3}{2} \cos 2x - \frac{2}{3} \sin 3x \right]_{\pi/4}^{\pi/2}$$

$$\left(-\frac{3}{2} \cos 2\left(\frac{\pi}{2}\right) - \frac{2}{3} \sin 3\left(\frac{\pi}{2}\right) \right) - \left(-\frac{3}{2} \cos 2\left(\frac{\pi}{4}\right) - \frac{2}{3} \sin 3\left(\frac{\pi}{4}\right) \right)$$

$$= 2.638071187$$

Q 2) $\int \frac{x-2}{x^2-4x-2} dx$ ~~let~~ let $u = x^2 - 4x - 2$
 $\frac{du}{dx} = 2x - 4$ $dx = \frac{du}{2x-4}$

$$\int \frac{x-2}{u} \cdot \frac{du}{2x-4} = \frac{1}{2} \int \frac{x-2}{u} \cdot \frac{du}{x-2} = \frac{1}{2} \int \frac{1}{u} du$$

$$= \frac{1}{2} \ln u + C = \frac{1}{2} \ln(x^2 - 4x - 2) + C$$

ii) $\int \frac{4}{(5x-3)} dx = 4 \times \ln(5x-3) \times \frac{1}{5} = \frac{4}{5} \ln(5x-3) + C$

$$\int \frac{a}{bx \pm c} dx = a \times \ln(bx \pm c) \times \frac{1}{b} + C$$

let $u = bx \pm c$

iii) $\int \frac{2x}{\sqrt{4x^2-1}} dx$ let $u = 4x^2 - 1$ $dx = \frac{du}{8x}$
 $\frac{du}{dx} = 8x$

$$\int \frac{2x}{\sqrt{u}} \cdot \frac{du}{8x} = \frac{1}{4} \int u^{-1/2} du = \frac{1}{4} \left(\frac{u^{1/2}}{1/2} \right) = \frac{1}{2} u^{1/2}$$

$$= \frac{1}{2} (4x^2 - 1)^{1/2} + C$$

iv) $\int \frac{3 \ln t}{t} dt$

$$\int \frac{\ln ax}{x} dx = \frac{1}{2} \ln^2 ax + C$$

$$= \frac{3}{2} \ln^2 t + C$$

$$v \int 3\theta(4\theta^2-3)^5 d\theta \quad \text{let } u = 4\theta^2-3 \quad \frac{du}{d\theta} = 8\theta$$

$$\int 3\theta u^5 \frac{du}{8\theta} = \frac{3}{8} \int u^5 du = \frac{3}{8} \left(\frac{u^6}{6} \right)$$

$$\frac{1}{16} u^6 = \frac{1}{16} (4\theta^2-3)^6$$

$$vii) \int \frac{2e^t}{\sqrt{e^t+4}} dt \quad \text{let } u = e^t+4 \quad \frac{du}{dt} = e^t$$

$$\int \frac{2e^t}{\sqrt{u}} \frac{du}{e^t} = 2 \int u^{-1/2} du = 2 \left(\frac{u^{1/2}}{1/2} \right)$$

$$4\sqrt{u} = 4\sqrt{e^t+4} + C$$

b) Evaluate

$$i \int_0^1 2xe^{x^2+1} dx \quad \text{let } u = e^{x^2+1} \quad \frac{du}{dx} = 2xe^{x^2+1}$$

$$\frac{dx}{2xe^{x^2+1}} \quad \int_0^1 \frac{2xu}{2xu} du = \int_0^1 1 du$$

$$\frac{dx}{2xu} = \frac{du}{2xu} = \left[u \right]_0^1$$

$$\left[e^{x^2+1} \right]_0^1 = \left[e^{1^2+1} \right] - \left[e^{0^2+1} \right]$$

$$= e^2 - e^1$$

$$= 4.670774268$$

iv) $\int_0^{\frac{\pi}{6}} 24 \sin^3 \theta \cos \theta \, d\theta$ let $u = \cos \theta$ $du = -\sin \theta \, d\theta$

$$\int_0^{\frac{\pi}{6}} 24 \sin^3 \theta \cos \theta \, d\theta = \int_0^{\frac{\pi}{6}} -24 \sin^2 \theta \cos \theta \, d\theta$$

$$= -24 \int_0^{\frac{\pi}{6}} (1 - \cos^2 \theta) \cos \theta \, d\theta = -24 \int_0^{\frac{\pi}{6}} (1 - u^2) u \, du$$

$$= -24 \int_0^{\frac{\pi}{6}} (u - 2u^3 + u^5) \, du = -24 \left[\frac{u^2}{2} - 2u^4 + \frac{u^6}{6} \right]_0^{\frac{\pi}{6}}$$

$$= -24 \left[\frac{u^2}{2} - \frac{2u^4}{4} + \frac{u^6}{6} \right]_0^{\frac{\pi}{6}} = -24 \left[\frac{u^2}{2} - \frac{u^4}{2} + \frac{u^6}{6} \right]_0^{\frac{\pi}{6}}$$

$$= \left[-12 \cos^2 \theta + 12 (\cos \theta)^4 - 4 (\cos \theta)^6 \right]_0^{\frac{\pi}{6}}$$

$$= \left[-12 \left(\cos \frac{\pi}{6} \right)^2 + 12 \left(\cos \frac{\pi}{6} \right)^4 - 4 \left(\cos \frac{\pi}{6} \right)^6 \right] - \left[-12 (\cos 0)^2 + 12 (\cos 0)^4 - 4 (\cos 0)^6 \right]$$

$$= 0.0625$$

vi) $\int_1^3 5x \sqrt{(2x^2+7)} \, dx$ let $u = 2x^2+7$ $du = 4x \, dx$

$$\frac{du}{dx} = 4x$$

$$\int_1^3 5x \sqrt{u} \, dx = \frac{5}{4} \int_1^3 u^{\frac{1}{2}} \, du = \frac{5}{4} \left[\frac{u^{\frac{3}{2}}}{\frac{3}{2}} \right]_1^3$$

$$= \frac{5}{6} (2x^2+7)^{\frac{3}{2}} \Big|_1^3$$

$$\left(\frac{5}{6} (2(3)^2 + 7)^{3/2} \right) - \left(\frac{5}{6} (2(1)^2 + 7)^{3/2} \right)$$

$$= 81.6166067$$

$$\approx \underline{81.6}$$

$$\textcircled{c} \int \sin^2 2x \, dx$$

$$\sin^2 2x = \frac{1 - \cos 4x}{2}$$

$$\sin^2 2x = \frac{1 - \cos 4x}{2}$$

$$\int \frac{1 - \cos 4x}{2} \, dx = \frac{1}{2} \int (1 - \cos 4x) \, dx$$

$$= \frac{1}{2} \left(x - \frac{\sin 4x}{4} \right) = \frac{x}{2} - \frac{\sin 4x}{8} + C$$

$$\int 3 \cos^2 t \, dt = \int \cos^2 t = \frac{1 + \cos 2t}{2}$$

$$\int \frac{3}{2} \left(1 + \cos 2t \right) dt = \frac{3}{2} \left(t + \frac{\sin 2t}{2} \right)$$

$$= \frac{3t}{2} + \frac{3 \sin 2t}{4} + C$$

$$\text{iii) } \int 5 \tan^2 3\theta \, d\theta \quad \text{let } u = 3\theta \quad du = \frac{du}{3}$$

$$\int 5 \tan^2 u \, \frac{du}{3}$$

$$= \frac{5}{3} \int \tan^2 u \, du$$

$$\frac{5}{3} \int \tan^2 u \, du = \frac{5}{3} \int (\sec^2 u - 1) \, du$$

$$= \frac{5}{3} (\tan u - u) = \frac{5}{3} \tan u - \frac{5}{3} u$$

$$= \frac{5}{3} \tan 3\theta - \frac{5}{3} \times 3\theta = \frac{5}{3} \tan 3\theta - 5\theta + C$$

$$\text{iv) } \int 2 \cot^2 2t \, dt \quad \text{let } u = 2t \quad \frac{du}{dt} = 2$$

$$\int 2 \cot^2 u \cdot \frac{du}{2} = \int \cot^2 u \, du$$

$$= \int (\csc^2 u - 1) \, du = -\cot u + u$$

$$= -\cot 2t + 2t + C$$

d) Integrate

$$i) \int 2 \sin^3 t \cos^2 t \, dt \quad \text{let } u = \cos t \quad \frac{du}{dt} = -\sin t$$

$$= -2 \int \sin^2 t \cos^2 t \, dt = -2 \int \sin^2 t \, u^2 \, du$$

$$= -2 \int (1 - \cos^2 t) u^2 \, du = -2 \int (1 - u^2) u^2 \, du$$

$$= -2 \int (u^2 - u^4) \, du$$

$$-2 \left(\frac{u^3}{3} - \frac{u^5}{5} \right) = -\frac{2}{3} u^3 + \frac{2}{5} u^5 + C$$

$$= \frac{2}{5} \cos^5 t - \frac{2}{3} \cos^3 t + C$$

11) $\int \sin^2 t \cos^4 t \, dt$ let $u = \cos t$ $du = -\sin t \, dt$

$$\int \sin^2 t \cos^4 t \, dt = -\int \sin^2 t u^4 \, du$$

$$= -\int (1 - \cos^2 t) u^4 \, du = -\int (1 - u^2) u^4 \, du$$

$$= -\int u^4 - u^6 \, du = -\left(\frac{u^5}{5} - \frac{u^7}{7} \right)$$

$$= \frac{u^7}{7} - \frac{u^5}{5} = \frac{\cos^7 t}{7} - \frac{\cos^5 t}{5} + C$$

12) $\int 2 \sin^4 2t \, dt = 2 \int (\sin^2 2t)^2 \, dt = 2 \int \left(\frac{1 - \cos 4t}{2} \right)^2 \, dt$

$$= 2 \int \left(\frac{1 - \cos 4t}{2} \right) \left(\frac{1 - \cos 4t}{2} \right) \, dt$$

$$= 2 \int \frac{1 - \cos 4t}{2} + \frac{\cos^2 4t}{4} \, dt$$

$$= 2 \int \frac{1}{4} - \frac{\cos 4t}{2} + \frac{(1 + \cos 8t)}{8} \, dt$$

$$2 \int \left(\frac{3}{8} - \frac{\cos 4\theta}{2} + \frac{\cos 8\theta}{8} \right) d\theta$$

$$2 \left(\frac{3}{8} \theta - \frac{\sin 4\theta}{8} + \frac{\sin 8\theta}{64} \right)$$

$$\frac{3x}{4} - \frac{\sin 4\theta}{4} + \frac{\sin 8\theta}{32} + C$$

$$14 \int 4 \cos^3 \theta d\theta = 4 \int \cos^2 \theta \cos \theta d\theta$$

$$= 4 \int (1 - \sin^2 \theta) \cos \theta d\theta$$

$$= 4 \int \cos \theta d\theta - 4 \int \cos \theta \sin^2 \theta d\theta$$

$$= 4 \int \cos \theta d\theta - 4 \int \cos \theta \sin^2 \theta d\theta \quad \text{let } u = \sin \theta$$

$$du = \cos \theta d\theta$$

$$= 4 \times \frac{1}{2} \sin 2\theta - 4 \int \cos \theta d\theta$$

$$= 2 \sin 2\theta - 2 \int u^2 du = 2 \sin 2\theta - 2 \frac{u^3}{3}$$

$$= 2 \sin 2\theta - \frac{2 \sin^3 \theta}{3} + C$$

$$v) \int 2 \sin 3t \sin t \, dt$$

$$\left\{ \begin{array}{l} \sin A \sin B = \frac{1}{2} [\cos(A-B) - \cos(A+B)] \\ \text{Product Formula} \end{array} \right\}$$

$$= \int 2 \times \frac{1}{2} (\cos(3t-t) - \cos(3t+t)) \, dt$$

$$= \int \cos 2t - \cos 4t \, dt = \frac{\sin 2t}{2} - \frac{\sin 4t}{4} + C$$

$$vi) \int \cos 4t \cos 3t \, dt$$

$$\left\{ \begin{array}{l} \cos A \cos B = \frac{1}{2} [\cos(A+B) + \cos(A-B)] \\ \text{Product Formula} \end{array} \right\}$$

$$= \frac{1}{2} \int (\cos(4t+3t) + \cos(4t-3t)) \, dt = \frac{1}{2} \int \cos 7t + \cos t \, dt$$

$$= \frac{1}{2} \left(\frac{\sin 7t}{7} + \sin t \right) = \frac{\sin 7t}{14} + \frac{\sin t}{2} + C$$

$$vii) \int \frac{1}{2} \cos 4t \sin 2t \, dt$$

$$\left\{ \begin{array}{l} \cos A \sin B = \frac{1}{2} [\sin(A+B) - \sin(A-B)] \\ \text{Product Formula} \end{array} \right\}$$

$$= \int \frac{1}{2} (\sin(4t+2t) - \sin(4t-2t)) \, dt = \frac{1}{4} \int \sin 6t - \sin 2t$$

$$= \frac{1}{4} \left(-\frac{\cos 6t}{6} + \frac{\cos 2t}{2} \right) = \frac{\cos 2t}{8} - \frac{\cos 6t}{24} + C$$

$$Q 30) \int \sqrt{4-x^2} dx \quad x = a \sin \theta, \quad \sqrt{a^2-x^2}$$

$$x = a \sin \theta$$

$$dx = a \cos \theta d\theta$$

$$\int \sqrt{4-4\sin^2 \theta} \cdot a \cos \theta d\theta = \int \sqrt{4(1-\sin^2 \theta)} \cdot a \cos \theta d\theta$$

$$= 4 \int \cos^2 \theta d\theta = 4 \int \frac{1 + \cos 2\theta}{2} d\theta = 2 \int (1 + \cos 2\theta) d\theta$$

$$= 2 \left(\theta + \frac{\sin 2\theta}{2} \right) = 2\theta + \sin 2\theta$$

$$\theta = \arcsin \left(\frac{x}{2} \right)$$

$$= 2 \arcsin \left(\frac{x}{2} \right) + \sin 2 \left(\arcsin \left(\frac{x}{2} \right) \right) + C$$

$$ii) \int \sqrt{4-4x^2} dx = \int \sqrt{4 \left(\frac{1}{4} - x^2 \right)} dx = 2 \int \sqrt{\frac{1}{4} - x^2} dx$$

$$x = \frac{1}{2} \sin \theta \quad dx = \frac{1}{2} \cos \theta d\theta$$

$$\theta = \arcsin \left(\frac{2x}{1} \right)$$

$$= 2 \int \sqrt{\frac{1}{4} - \frac{1}{4} \sin^2 \theta} d\theta = 2 \int \sqrt{\frac{1}{4} (1 - \sin^2 \theta)} d\theta$$

$$= \frac{1}{2} \int \sqrt{1 - \sin^2 \theta} \cdot \frac{1}{2} \cos \theta d\theta = \frac{1}{4} \int \cos \theta \cdot \cos \theta d\theta$$

$$= \frac{1}{4} \int \cos^2 \theta d\theta = \frac{1}{4} \int \frac{1 + \cos 2\theta}{2} d\theta = \frac{1}{8} \int (1 + \cos 2\theta) d\theta$$

$$= \frac{1}{8} \left(\theta + \frac{\sin 2\theta}{2} \right) = \frac{1}{8} \theta + \frac{1}{16} \sin 2\theta$$

$$= \frac{1}{8} \arcsin \left(\frac{2x}{1} \right) + \frac{1}{16} \sin 2 \left(\arcsin \left(\frac{2x}{1} \right) \right) + C$$

$$(iii) \int \frac{5}{3+2x^2} dx = 5 \int \frac{1}{2(\frac{3}{2}+x^2)} dx \quad x = u \text{ then } dx = du$$

$$x = \frac{\sqrt{3}}{2} \tan \theta$$

$$= 5 \int \frac{1}{\frac{3}{2} + \frac{3}{2} \tan^2 \theta} \cdot \frac{\sqrt{3}}{2} \sec^2 \theta d\theta \quad dx = \frac{\sqrt{3}}{2} \sec^2 \theta d\theta$$

$$= \frac{5\sqrt{3}}{2} \int \frac{1}{\frac{3}{2}(1+\tan^2 \theta)} \cdot \sec^2 \theta d\theta \quad \theta = \tan^{-1} \left(\frac{\sqrt{3}x}{2} \right)$$

$$= \frac{5\sqrt{3}}{2} \int \frac{1}{\sec^2 \theta} \cdot \sec^2 \theta d\theta \quad \theta = \tan^{-1} \left(\frac{\sqrt{3}x}{2} \right)$$

$$= \frac{5\sqrt{3}}{2} (\theta) = \frac{5\sqrt{3}}{2} \tan^{-1} \left(\frac{\sqrt{3}x}{2} \right) + C$$

$$(iv) \int \frac{2}{x^2 \sqrt{1+x^2}} dx \quad x = u \tan \theta \quad dx = \sec^2 \theta d\theta$$

$$\theta = \tan^{-1} x$$

$$\int \frac{2}{\tan^2 \theta \sqrt{1+\tan^2 \theta}} \cdot \sec^2 \theta d\theta$$

$$\int \frac{2}{\tan^2 \theta \sec \theta} \cdot \sec^2 \theta d\theta = 2 \int \frac{\sec \theta}{\tan^2 \theta} d\theta$$

$$= 2 \int \frac{1}{\cos \theta} \cdot \frac{\cos \theta}{\sin^2 \theta} d\theta = 2 \int \frac{\cos \theta}{\sin^2 \theta} d\theta \quad \text{let } u = \sin \theta \quad du = \cos \theta d\theta$$

$$= 2 \int \frac{\cos \theta}{u^2} \cdot \frac{du}{\cos \theta} = 2 \int u^{-2} du = 2 \left(-u^{-1} \right)$$

$$= -2u^{-1} = -\frac{2}{u} = -\frac{2}{\sin \theta}$$

$$I = \int \frac{3}{\sqrt{4x^2-9}} dx \quad x = \frac{3}{2} \sec \theta \quad dx = \frac{3}{2} \tan \theta \sec \theta d\theta$$

$$= \int \frac{3}{\sqrt{4(x^2-\frac{9}{4})}} dx = \int \frac{3}{2\sqrt{x^2-\frac{9}{4}}} dx$$

$$= \frac{3}{2} \int \frac{1}{\sqrt{\sec^2 \theta - 1}} \cdot \frac{3}{2} \tan \theta \sec \theta d\theta$$

$$= \frac{3}{2} \int \frac{1}{\frac{3}{2} \sqrt{\sec^2 \theta - 1}} \cdot \frac{3}{2} \tan \theta \sec \theta d\theta$$

$$= \frac{3}{2} \int \frac{1}{\tan \theta \sec \theta} d\theta$$

$$= \frac{3}{2} \int \sec \theta d\theta = \frac{3}{2} \ln \left| \frac{2x}{3} + \tan \left(\arcsin \left(\frac{2x}{3} \right) \right) \right| + C$$

$$VI) \int \frac{\sqrt{a^2-x^2}}{x} dx \quad x = a \tan \theta \quad dx = a \sec^2 \theta d\theta$$

$$= \int \frac{\sqrt{a^2 - a^2 \tan^2 \theta}}{a \tan \theta} \cdot a \sec^2 \theta d\theta \quad \theta = \arctan \frac{x}{a}$$

$$= \int \frac{a \sqrt{1 - \tan^2 \theta}}{\tan \theta} \cdot a \sec^2 \theta d\theta = a \int \frac{\sec \theta \cdot \sec^2 \theta}{\tan \theta} d\theta$$

$$= a \int \frac{\sec \theta (1 + \tan^2 \theta)}{\tan \theta} d\theta$$

$$= a \int \frac{\sec \theta + \sec \theta \tan^2 \theta}{\tan \theta} d\theta$$

$$= a \int \frac{\sec \theta + \sec \theta \tan^2 \theta}{\tan \theta} d\theta$$

$$= a \int \frac{1}{\cancel{x} \cos \theta} + \sec \theta \tan \theta d\theta$$

$$= a \int \csc \theta + \sec \theta \tan \theta d\theta$$

$$= a \int \csc \theta d\theta + a \int \sec \theta \tan \theta d\theta$$

$$= -a \ln |\csc \theta + \cot \theta| + a \sec \theta$$

$$= -a \ln (\csc \theta \sec \theta) + \cot \theta \sec \theta + a \sec \theta \tan \theta$$

b. Determine

$$1) \int \frac{2x^2 + 1}{(x-1)(x-2)(x-3)} dx = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}$$

$$x=1, x=2, x=3$$

$$2x^2 + 1 = A(x-2)(x-3) + B(x-1)(x-3) + C(x-2)(x-1)$$

$$2) x=1$$

$$2(1)^2 + 1 = A(1-2)(1-3) + B(1-1)(1-3) + C(1-2)(1-1)$$

$$3 = 2A \quad A = \frac{3}{2}$$

$$3) x=2$$

$$2(2)^2 + 1 = A(2-2)(2-3) + B(2-1)(2-3) + C(2-2)(2-1)$$

$$9 = -B \quad B = -9$$

$$4) x=3$$

$$2(3)^2 + 1 = A(3-2)(3-3) + B(3-1)(3-3) + C(3-2)(3-1)$$

$$19 = 2C \quad C = \frac{19}{2}$$

$$\int \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3} dx \quad A = \frac{3}{2}$$

$$B = -9$$

$$\int \frac{3}{2x-2} - \frac{9}{x-2} + \frac{19}{2x-6} dx \quad C = \frac{19}{2}$$

$$3 \ln(2x-2) - 9 \ln(x-2) + 19 \ln(2x-6) + C$$

or

$$\frac{3}{2} \ln(x-1) - 9 \ln(x-2) + \frac{19}{2} \ln(x-3) + C$$

ii) $\int \frac{x}{x^4 - 13x^2 + 36} dx$ roots of $x^4 - 13x^2 + 36$

$$x = -2, x = -3, x = 3, x = 2$$

$$\frac{x}{(x+2)(x+3)(x-3)(x-2)} = \frac{A}{x+2} + \frac{B}{x+3} + \frac{C}{x-3} + \frac{D}{x-2}$$

$$x = A(x+3)(x-3)(x-2) + B(x+2)(x-3)(x-2) + C(x+2)(x+3)(x-2) + D(x+2)(x+3)(x-3)$$

i) $x = -2$

$$-2 = A(0+3)(0-3)(0-2) + B(2+2)(2-3)(2-2) + C(2+2)(2+3)(2-2) + D(2+2)(2+3)(2-3)$$

$$-2 = 20A \quad A = -\frac{1}{10}$$

ii) $x = -3$

$$-3 = A(-3+3)(-3-3)(-3-2) + B(-3+2)(-3-3)(-3-2) + C(-3+2)(-3+3)(-3-2) + D(-3+2)(-3+3)(-3-3)$$

$$-3 = 30B \quad B = -\frac{1}{10}$$

iii) $x = 3$

$$3 = C(3+2)(3+3)(3-2)$$

$$3 = 30C \quad C = \frac{1}{10}$$

14) $x=2$

$$2 = D(2+2)(2+3)(2-3)$$

$$2 = -10D \quad D = -\frac{1}{10}$$

$$\left(\frac{A}{x+2} + \frac{B}{x+3} + \frac{C}{x-3} + \frac{D}{x-2} \right) dx \quad A = -\frac{1}{10} \quad B = -\frac{1}{10}$$

$$C = \frac{1}{10} \quad D = -\frac{1}{10}$$

$$\int \left(\frac{1/10}{x+2} - \frac{1/10}{x+3} + \frac{1/10}{x-3} - \frac{1/10}{x-2} \right) dx$$

$$= \frac{1}{10} \ln(12x+21) - \frac{1}{10} \ln(12x+31) + \frac{1}{10} \ln(12x-31) - \frac{1}{10} \ln(12x-2) + C$$

15) $\int \frac{x^2}{(x^2-1)(x^2+4x+5)} dx$

$$\frac{x^2}{(x^2-1)(x^2+4x+5)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+4x+5}$$

$$x^2 = A(x^2+4x+5) + (Bx+C)(x-1)$$

when $x=1$

$$1^2 = A(1^2+4(1)+5) + (B(1)+C)(1-1)$$

$$1 = 10A \quad A = \frac{1}{10}$$

$$x^2 = Ax^2 + 4Ax + 5A + Bx^2 + 4Bx + 5C - Bx - C$$

$$1 = A + B$$

$$1 = \frac{1}{10} + B \quad C = \frac{1}{10}$$

$$B = \frac{9}{10}$$

$$\int \frac{x^2}{(x-1)(x^2+4x+5)} dx = \int \frac{\frac{1}{10}}{x-1} + \frac{\frac{9}{10}x + \frac{1}{2}}{x^2+4x+5} dx$$

$$= \frac{1}{10} \int \frac{1}{x-1} + \frac{9x+5}{x^2+4x+5} dx = \frac{1}{10} \int \frac{1}{x-1} dx + \frac{1}{10} \int \frac{9x+5}{x^2+4x+5} dx$$

$$9x+5 = \frac{9}{2}(2x+4) - 13$$

$$9x+5 = \frac{9(2x+4)}{2} - 26$$

KEY

$$= \frac{1}{10} \int \frac{1}{x-1} dx + \frac{1}{10} \int \frac{9(2x+4)}{2(2x^2+4x+5)} - 26 dx$$

$$= \frac{1}{10} \int \frac{1}{x-1} dx + \frac{1}{20} \int \frac{9(2x+4)}{x^2+4x+5} dx - \frac{1}{20} \int \frac{26}{x^2+4x+5} dx$$

For B let $u = x^2+4x+5$ $du = 2x+4 dx$

$$= \frac{1}{10} \int \frac{1}{x-1} dx + \frac{9}{20} \int \frac{(2x+4) \cdot \frac{du}{2}}{u} - \frac{13}{10} \int \frac{1}{x^2+4x+5} dx$$

$$= \frac{1}{10} \int \frac{1}{x-1} dx + \frac{9}{20} \int \frac{1}{u} du - \frac{13}{10} \int \frac{1}{x^2+4x+5} dx$$

$$= \frac{1}{10} \ln|x-1| + \frac{9}{20} \ln|u| - \frac{13}{10} \ln|x^2+4x+5|$$

$$= \frac{1}{10} \ln|x-1| + \frac{9}{20} \ln|x^2+4x+5| - \frac{13}{10} \ln|x^2+4x+5|$$

complete the square for $x^2 + 4x + 5$

$$= (x+2)^2 + 1$$

$$\frac{9}{20} \int \frac{1}{(x+2)^2 + 1} dx = \frac{13}{10} \int \frac{1}{(x+2)^2 + 1} dx$$

let $m = x+2$ $dm = dx$

$$\frac{9}{20} \int \frac{1}{m^2 + 1} dm = \frac{13}{10} \int \frac{1}{m^2 + 1} dm$$

let $m = \tan \theta$ $dm = \sec^2 \theta d\theta$

$$\frac{9}{20} \int \frac{1}{\tan^2 \theta + 1} \cdot \sec^2 \theta d\theta = \frac{13}{10} \int \frac{1}{\tan^2 \theta + 1} \cdot \sec^2 \theta d\theta$$

$$\frac{9}{20} \int 1 d\theta = \frac{13}{10} \int 1 d\theta = \frac{9}{20} \theta - \frac{13}{10} \theta$$

$\theta = \arctan x$

0

$$= \frac{1}{10} \ln(1x-1) + \frac{9}{20} \arctan x - \frac{13}{10} \arctan x$$

$$= \frac{21 \ln(1x-1) + 9 \arctan x - 26 \arctan x}{20}$$

$$= \frac{21 \ln(1x-1) - 17 \arctan x}{20} + C$$

$$10) \int \frac{x^4}{x^3 - 2x^2 - 7x - 4} dx$$

By long division $\frac{x^4}{x^3 - 2x^2 - 7x - 4} = x + 2 + \frac{11x^2 + 18x + 8}{x^3 - 2x^2 - 7x - 4}$

$$(x^3 - 2x^2 - 7x - 4) = (x-4)(x+1)^2$$

$$\frac{11x^2 + 18x + 8}{(x-4)(x+1)^2} = \frac{A}{x-4} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$$

$$A = \frac{256}{28} \quad B = \frac{19}{28} \quad C = -\frac{1}{5}$$

$$= \int x + 2 + \frac{256}{28(x-4)} + \frac{19}{28(x+1)} - \frac{1}{5(x+1)^2} dx$$

$$\int x dx + \int 2 dx + \frac{256}{28} \int \frac{1}{x-4} dx + \frac{19}{28} \int \frac{1}{x+1} dx - \frac{1}{5} \int \frac{1}{(x+1)^2} dx$$

Let $u = x+1$ Per E $du = dx$

$$\int x dx + \int 2 dx + \frac{256}{28} \int \frac{1}{x-4} dx + \frac{19}{28} \int \frac{1}{x+1} dx - \frac{1}{5} \int u^{-2} du$$

$$\frac{x^2}{2} + 2x + \frac{256}{28} \ln|x-4| + \frac{19}{28} \ln|x+1| - \frac{1}{5} \left(\frac{u^{-1}}{-1} \right)$$

$$\frac{x^2}{2} + 2x + \frac{256}{28} \ln|x-4| + \frac{19}{28} \ln|x+1| + \frac{1}{5} \ln|x+1| + C$$

$$\int \frac{1}{(x^2+1)(x+4)} dx$$

$$\frac{1}{(x^2+1)(x+4)} = \frac{A+Bx}{(x^2+1)} + \frac{C+Dx}{(x+4)}$$

$$A = -\frac{1}{3} \quad C = -\frac{1}{3} \quad B = 0 \quad D = 0$$

$$= \left(\frac{-\frac{1}{3}}{x^2+1} - \frac{\frac{1}{3}}{x+4} \right) dx$$

$$\frac{1}{3} \int \frac{1}{x^2+1} dx - \frac{1}{3} \int \frac{1}{x+4} dx \quad \begin{matrix} x = \tan \theta \\ dx = \sec^2 \theta d\theta \end{matrix}$$

$$\theta = \arctan(x) \quad \theta = \arctan\left(\frac{x}{2}\right) \quad dx = 2 \sec^2 \theta d\theta$$

$$\frac{1}{3} \int \frac{1}{\tan^2 \theta + 1} \cdot \sec^2 \theta d\theta - \frac{1}{3} \int \frac{1}{4(\tan^2 \theta + 1)} \cdot 2 \sec^2 \theta d\theta$$

$$\frac{1}{3} \theta - \frac{1}{6} \theta = \frac{1}{3} \arctan\left(\frac{x}{2}\right) - \frac{1}{6} \arctan\left(\frac{x}{2}\right) + C$$

11) $\int \frac{1}{1+e^x} dx = \int \frac{1+e^x - e^x}{1+e^x} dx$

$$= \int \frac{1+e^x}{1+e^x} - \frac{e^x}{1+e^x} dx$$

$$= \int 1 dx - \int \frac{e^x}{1+e^x} dx \quad \text{let } u = 1+e^x$$

$$du = e^x dx$$

$$= \int 1 dx - \int \frac{e^x}{u} \frac{du}{e^x}$$

$$= \int 1 dx - \int \frac{1}{u} du$$

$$= x - \ln(u)$$

$$= x - \ln(1+e^x) + C$$

C Find

1) $\int \frac{1}{\sin x + \cos x} dx$

Using Weierstrass Substitution

$$\sin x = \frac{2t}{1+t^2} \quad \cos x = \frac{1-t^2}{1+t^2}$$

$$dx = \frac{2}{1+t^2} dt$$

$$\int \frac{1}{\frac{2t}{1+t^2} + \frac{1-t^2}{1+t^2}} \cdot \frac{2}{1+t^2} dt$$

$$\int \frac{2}{2t+1-t^2} dt$$

$$\int \frac{2}{t^2 - 2t - 1} dt = \int \frac{2}{(t-1)^2 - 2} dt$$

Let $m = t - 1$ $dm = dt$

$$= \int \frac{2}{m^2 - 2} dt \quad m = \sqrt{2} \sec \theta \quad dm = \sqrt{2} \sec \theta \tan \theta d\theta$$

$$= \int \frac{2}{2 \sec^2 \theta - 2} \sqrt{2} \sec \theta \tan \theta d\theta$$

$$= \int \frac{2}{2(\sec^2 \theta - 1)} \sqrt{2} \sec \theta \tan \theta d\theta$$

$$= \int \frac{1}{\tan^2 \theta} \sqrt{2} \sec \theta \tan \theta d\theta$$

$$= \int \frac{\sqrt{2} \sec \theta}{\tan^2 \theta} d\theta = \sqrt{2} \int \frac{\sec \theta}{\tan^2 \theta} d\theta = \sqrt{2} \int \frac{1}{\cos \theta} \times \frac{\cos \theta}{\sin^2 \theta} d\theta$$

$$= \sqrt{2} \int \frac{1}{\sin \theta} d\theta = \sqrt{2} \int \csc \theta d\theta$$

$$= \sqrt{2} (\ln |\csc \theta + \cot \theta|) \quad \theta = \operatorname{arccsc} \left(\frac{m}{\sqrt{2}} \right)$$

$$= \sqrt{2} \ln (\csc \theta + \cot \theta) \quad \theta = \operatorname{arccsc} \left(\frac{t-1}{\sqrt{2}} \right)$$

$$\theta = \operatorname{arccsc} \left(\frac{\tan \left(\frac{t-1}{\sqrt{2}} \right) - 1}{\sqrt{2}} \right)$$

ii)

$$\int \frac{1}{2 + \cos \theta} d\theta$$

$$\cos \theta = \frac{1 - t^2}{1 + t^2}$$

$$d\theta = \frac{2}{1 + t^2} dt$$

$$\int \frac{1}{2 + 1 - t^2} \cdot \frac{2}{1 + t^2} dt$$

$$\int \frac{1}{2(1 + t^2) + 1 - t^2} \cdot \frac{2}{1 + t^2} dt$$

$$\int \frac{2}{2 + 2t^2 + 1 - t^2} dt = \int \frac{2}{t^2 + 3} dt$$

$$t = \sqrt{3} \tan \theta$$

$$dt = \sqrt{3} \sec^2 \theta d\theta$$

$$= \int \frac{2}{(\sqrt{3} \tan \theta)^2 + 3} \cdot \sqrt{3} \sec^2 \theta d\theta$$

$$= \int \frac{2}{3(\tan^2 \theta + 1)} \cdot \sqrt{3} \sec^2 \theta d\theta \quad \theta = \arctan \frac{t}{\sqrt{3}}$$

$$= \frac{2\sqrt{3}}{3} \theta \quad \theta = \arctan \left(\frac{\tan \left(\frac{2}{3} \right)}{\sqrt{3}} \right)$$

$$= \frac{2\sqrt{3}}{3} \arctan \left(\frac{\tan \frac{2}{3}}{\sqrt{3}} \right) + C$$

$$11) \int \frac{1}{1+2\sin x} dx$$

$$\sin x = \frac{2t}{1+t^2}$$

$$dx = \frac{2}{1+t^2} dt$$

$$\int \frac{1}{1+2\left[\frac{2t}{1+t^2}\right]} \cdot \frac{2}{1+t^2} dt$$

Let

$$m = t+2$$

$$dm = dt$$

$$\int \frac{1}{1 + \frac{4t}{1+t^2}} \cdot \frac{2}{1+t^2} dt$$

$$= \int \frac{2}{m^2-3} dm$$

$$\int \frac{1}{1+t^2+4t} \cdot \frac{2}{1+t^2} dt$$

$$m = \sqrt{3} \sec \theta$$

$$dm = \sqrt{3} \sec \theta \tan \theta d\theta$$

$$\int \frac{2}{t^2+4t+1} dt$$

$$= \int \frac{2}{3 \sec^2 \theta - 3} \cdot \sqrt{3} \sec \theta \tan \theta d\theta$$

$$\int \frac{2}{(t+2)^2-3} dt$$

$$= \int \frac{2}{3(\sec^2 \theta - 1)} \cdot \sqrt{3} \sec \theta \tan \theta d\theta$$

$$= \frac{2\sqrt{3}}{3} \int \sec \theta d\theta = \frac{2\sqrt{3}}{3} \int \frac{1}{\cos \theta} \times \frac{\cos \theta}{\sin \theta} d\theta$$

$$\theta = \arcsin\left(\frac{m}{\sqrt{3}}\right)$$

$$= \frac{2\sqrt{3}}{3} \ln(\sec \theta + \tan \theta)$$

$$\theta = \arcsin\left(\frac{t+2}{\sqrt{3}}\right)$$

$$\theta = \arcsin\left(\frac{\tan^2 \frac{x}{2} + 2}{\sqrt{3}}\right)$$

Integration by parts

$$i) \int 2 \ln(3x) dx$$

$$u = \ln(3x) \quad dv = 2$$

$$du = \frac{1}{x} dx \quad v = 2x$$

$$\int u dv = uv - \int v du$$

$$= 2x \ln(3x) - \int 2x \cdot \frac{1}{x} dx$$

$$= 2x \ln(3x) - 2x + C$$

$$ii) \int x \ln(x) dx$$

$$u = \ln(x) \quad dv = x$$

$$du = \frac{1}{x} dx \quad v = \frac{x^2}{2}$$

$$= \frac{x^2}{2} \ln(x) - \int \frac{x^2}{2} \cdot \frac{1}{x} dx$$

$$= \frac{1}{2} \ln(x) x^2 - \frac{1}{2} \int x dx$$

$$= \frac{1}{2} \ln(x) x^2 - \frac{1}{4} x^2 + C$$

$$iii) \int x^2 \ln(3x) dx$$

$$u = x^2 \quad dv = \ln(3x)$$

$$du = 2x dx \quad v = -\frac{1}{3} \cos(3x)$$

$$= -\frac{x^2}{3} \cos(3x) - \int -2x \cdot \frac{1}{3} \cos(3x) dx$$

$$= -\frac{x^2}{3} \cos(3x) + \frac{2}{3} \int x \cos(3x) dx$$

$$2x \ln(3x) - 2x + C$$

$$\begin{aligned} x \ln(3x) &= \ln(3x) \\ x &= 1 \end{aligned}$$

$$x \ln(3x) = \ln(3x) + x \ln(3x) = \ln(3x) + x \ln(3x)$$

$$x \ln(3x) = \ln(3x) + x \ln(3x) = \ln(3x) + x \ln(3x)$$

$$x \ln(3x) = \ln(3x) + x \ln(3x) = \ln(3x) + x \ln(3x)$$

$$x \ln(3x) = \ln(3x) + x \ln(3x) = \ln(3x) + x \ln(3x)$$

$$x \ln(3x) = \ln(3x) + x \ln(3x) = \ln(3x) + x \ln(3x)$$

$$\begin{aligned} x \ln(3x) &= \ln(3x) \\ x &= 1 \end{aligned}$$

$$x \ln(3x) = \ln(3x) + x \ln(3x) = \ln(3x) + x \ln(3x)$$

$$x \ln(3x) = \ln(3x) + x \ln(3x) = \ln(3x) + x \ln(3x)$$

$$x \ln(3x) = \ln(3x) + x \ln(3x) = \ln(3x) + x \ln(3x)$$

$$x \ln(3x) = \ln(3x) + x \ln(3x) = \ln(3x) + x \ln(3x)$$

Reduction Formulas

$$i) \int \ln^n x \, dx \quad u = \ln^n x \quad dv = dx \\ du = \frac{n \ln^{n-1} x}{x} dx \quad v = x$$

$$= x \ln^n x - \int x \cdot \frac{n \ln^{n-1} x}{x} dx$$

$$= x \ln^n x - n \int \ln^{n-1} x \, dx$$

$$\boxed{\int \ln^n x \, dx = x \ln^n x - n \int \ln^{n-1} x \, dx}$$

$$ii) \int x^m (\ln x)^n \, dx$$

$$u = (\ln x)^n \quad dv = x^m \\ du = \frac{n (\ln x)^{n-1}}{x} dx \quad v = \frac{x^{m+1}}{m+1}$$

$$\int x^m (\ln x)^n \, dx = \frac{x^{m+1}}{m+1} (\ln x)^n - \int \frac{x^{m+1}}{m+1} \cdot \frac{n (\ln x)^{n-1}}{x} dx$$

$$\boxed{\int x^m (\ln x)^n \, dx = \frac{x^{m+1}}{m+1} (\ln x)^n - \frac{n}{m+1} \int x^m (\ln x)^{n-1} \, dx}$$

$$iii) \int x^n e^{ax} \, dx$$

$$u = x^n \quad dv = e^{ax}$$

$$du = nx^{n-1} dx \quad v = \frac{e^{ax}}{a}$$

$$\frac{x^n e^{ax}}{a} - \int \frac{nx^{n-1} e^{ax}}{a} dx$$

$$\frac{x^n e^{ax}}{a} - \frac{n}{a} \int x^{n-1} e^{ax} dx$$

$$\boxed{\int x^n e^{ax} \, dx = \frac{x^n e^{ax}}{a} - \frac{n}{a} \int x^{n-1} e^{ax} \, dx}$$

$$iv) \int \sec^n x \, dx = \int \sec^{n-2} x \sec^2 x \, dx$$

$$u = \sec^{n-2} x \quad dv = \sec^2 x$$

$$du = (n-2) \tan x \sec x \sec^{n-3} x \, dx \quad v = \tan x$$

$$du = (n-2) \tan x \sec^{n-2} x \, dx$$

$$= \tan x \sec^{n-2} x - \int (n-2) \tan^2 x \sec^{n-2} x \, dx$$

$$= \tan x \sec^{n-2} x - (n-2) \int (\sec^2 x - 1) \sec^{n-2} x \, dx$$

$$= \tan x \sec^{n-2} x - (n-2) \int \sec^n x - \sec^{n-2} x \, dx$$

$$= \tan x \sec^{n-2} x - (n-2) \int \sec^n x + (n-2) \int \sec^{n-2} x \, dx$$

$$I_n = \int \sec^n x \, dx \quad \rightarrow -(n-2) I_n$$

$$I_n = \frac{\tan x \sec^{n-2} x + (n-2) \int \sec^{n-2} x \, dx}{n-1}$$

$$\boxed{\int \sec^n x \, dx = \frac{\tan x \sec^{n-2} x + (n-2) \int \sec^{n-2} x \, dx}{n-1}}$$

$$v) \int \sin^n x \, dx = \int \sin^{n-1} x \sin x \, dx$$

$$u = \sin^{n-1} x \quad du = \sin x$$

$$du = (n-1) \sin^{n-2} x \cos x \quad v = -\cos x$$

$$= -\cos x \sin^{n-1} x - \int (n-1) \cos^2 x \sin^{n-2} x \, dx$$

$$= -\cos x \sin^{n-1} x + (n-1) \int (1 - \sin^2 x) \sin^{n-2} x \, dx$$

$$= -\cos x \sin^{n-1} x + (n-1) \int \sin^{n-2} x - \sin^n x \, dx$$

$$= -\cos x \sin^{n-1} x + (n-1) \int \sin^{n-2} x \, dx - (n-1) \int \sin^n x \, dx$$

$$= -\frac{\cos x \sin^{n-1} x}{n} + \frac{(n-1) \int \sin^{n-2} x \, dx}{n}$$

$$\boxed{\int \sin^n x \, dx = \frac{(n-1)}{n} \int \sin^{n-2} x \, dx - \frac{\cos x \sin^{n-1} x}{n}}$$

$$vi) \int \cos^n x \, dx = \int \cos^{n-1} x \cos x \, dx$$

$$u = \cos^{n-1} x \quad du = -\cos x$$

$$du = -(n-1) (\sin x) \cos^{n-2} x \, dx \quad v = \sin x$$

$$= \sin x \cos^{n-1} x + \int (n-1) \sin^2 x \cos^{n-2} x \, dx$$

$$= \sin x \cos^{n-1} x + (n-1) \int (1 - \cos^2 x) \cos^{n-2} x \, dx$$

$$= \sin x \cos^{n-1} x + (n-1) \int \cos^{n-2} x - \cos^n x \, dx$$

$$= \sin x \cos^{n-1} x + (n-1) \int \cos^{n-2} x \, dx - (n-1) \int \cos^n x \, dx$$

$$\boxed{\int \cos^n x \, dx = \frac{\sin x \cos^{n-1} x + (n-1) \int \cos^{n-2} x \, dx}{n}}$$

$$\begin{aligned}
 \text{ii)} \quad & \int x^5 \cos x \, dx \\
 & \int x^n \cos x \, dx = x^n \sin x + n x^{n-1} \cos x - n(n-1) \int x^{n-2} \cos x \, dx \\
 & = x^5 \sin x + 5x^4 \cos x - 20 \int x^3 \cos x \, dx \\
 & = x^5 \sin x + 5x^4 \cos x - 20 \left(x^3 \sin x + 3x^2 \cos x - 6 \int x \cos x \, dx \right) \\
 & = x^5 \sin x + 5x^4 \cos x - 20x^3 \sin x - 60x^2 \cos x + 120 \int x \cos x \, dx \\
 & = x^5 \sin x + 5x^4 \cos x - 20x^3 \sin x - 60x^2 \cos x + 120(x \sin x + \cos x - 0) \\
 & = x^5 \sin x + 5x^4 \cos x - 20x^3 \sin x - 60x^2 \cos x + 120x \sin x + 120 \cos x + C
 \end{aligned}$$

$$\text{iii)} \quad \int x^5 (\ln x)^2 \, dx$$

$$\int x^m (\ln x)^n \, dx = \frac{x^{m+1}}{m+1} (\ln x)^n - \frac{n}{m+1} \int x^m (\ln x)^{n-1} \, dx$$

$$= \frac{x^6}{6} (\ln x)^2 - \frac{2}{6} \int x^5 (\ln x) \, dx$$

$$= \frac{x^6}{6} (\ln x)^2 - \frac{2}{6} \left(\frac{x^6}{6} (\ln x) - \frac{1}{6} \int x^5 \, dx \right)$$

$$= \frac{x^6}{6} (\ln x)^2 - \frac{2}{6} \left(\frac{x^6}{6} (\ln x) - \frac{x^6}{36} \right)$$

$$= \frac{x^6}{6} (\ln x)^2 - \frac{x^6 (\ln x)}{18} + \frac{x^6}{108} + C$$

$$iii) \int x^5 e^{5x} dx$$

$$\int x^n e^{ax} dx = \frac{x^n e^{ax}}{a} - \frac{n}{a} \int x^{n-1} e^{ax} dx$$

$$= \frac{x^5 e^{5x}}{5} - \int x^4 e^{5x} dx = \frac{x^5 e^{5x}}{5} - \left(\frac{x^4 e^{5x}}{5} - \frac{4}{5} \int x^3 e^{5x} dx \right)$$

$$= \frac{x^5 e^{5x}}{5} - \frac{x^4 e^{5x}}{5} + \frac{4}{5} \left(\frac{x^3 e^{5x}}{5} - \frac{3}{5} \int x^2 e^{5x} dx \right)$$

$$= \frac{x^5 e^{5x}}{5} - \frac{x^4 e^{5x}}{5} + \frac{4}{25} x^3 e^{5x} - \frac{12}{25} \left(\frac{x^2 e^{5x}}{5} - \frac{2}{5} \int x e^{5x} dx \right)$$

$$= \frac{x^5 e^{5x}}{5} - \frac{x^4 e^{5x}}{5} + \frac{4}{25} x^3 e^{5x} - \frac{12}{125} x^2 e^{5x} + \frac{24}{125} \left(\frac{x e^{5x}}{5} - \frac{1}{25} e^{5x} \right)$$

$$= \frac{x^5 e^{5x}}{5} - \frac{x^4 e^{5x}}{5} + \frac{4}{25} x^3 e^{5x} - \frac{12}{125} x^2 e^{5x} + \frac{24}{625} x e^{5x} - \frac{24}{3125} e^{5x} + C$$

$$iv) \int \sin^7 x dx$$

$$\int \sin^n x dx = \frac{(n-1)}{n} \int \sin^{n-2} x dx - \frac{\cos x \sin^{n-1} x}{n}$$

$$= -\frac{\cos x \sin^6 x}{7} + \frac{6}{7} \int \sin^5 x dx$$

$$= -\frac{\cos x \sin^6 x}{7} + \frac{6}{7} \left(-\frac{\cos x \sin^4 x}{5} + \frac{4}{5} \int \sin^3 x dx \right)$$

$$= -\frac{\cos x \sin^6 x}{7} - \frac{6}{35} \cos x \sin^4 x + \frac{24}{35} \int \sin^3 x dx$$

$$= -\frac{\cos x \sin^6 x}{7} - \frac{6 \cos x \sin^4 x}{35} + \frac{24}{35} \left(\frac{-\cos x \sin^3 x}{3} + \frac{2}{3} \int \sin x dx \right)$$

$$= -\frac{\cos x \sin^6 x}{7} - \frac{6 \cos x \sin^4 x}{35} - \frac{24 \cos x \sin^3 x}{105} + \frac{48 \cos x}{105} + C$$

V $\int \cos^6 x dx$

$$\int \cos^n x dx = \frac{\sin x \cos^{n-1} x}{n} + (n-1) \int \cos^{n-2} x dx$$

$$= \frac{\sin x \cos^5 x}{6} + \frac{5}{6} \int \cos^4 x dx$$

$$= \frac{\sin x \cos^5 x}{6} + \frac{5}{6} \left(\frac{\sin x \cos^3 x}{4} + \frac{3}{4} \int \cos^2 x dx \right)$$

$$= \frac{\sin x \cos^5 x}{6} + \frac{5}{24} \sin x \cos^3 x + \frac{5}{8} \int \cos^2 x dx$$

$$= \frac{\sin x \cos^5 x}{6} + \frac{5}{24} \sin x \cos^3 x + \frac{5}{8} \left(\frac{\sin x \cos x}{2} + \frac{1}{2} \int 1 dx \right)$$

$$= \frac{\sin x \cos^5 x}{6} + \frac{5}{24} \sin x \cos^3 x + \frac{5}{16} \sin x \cos x + \frac{5}{16} x + C$$

$$\text{vii)} \int (\ln(x))^7 dx$$

$$\int \ln^n x dx = x \ln^n x - n \int \ln^{n-1} x dx$$

$$= x \ln^7 x - 7 \int \ln^6 x dx = x \ln^7 x - 7 \left(x \ln^6 x - 6 \int \ln^5 x dx \right)$$

$$= x \ln^7 x - 7x \ln^6 x + 42 \int \ln^4 x dx$$

$$= x \ln^7 x - 7x \ln^6 x + 42 \left(x \ln^5 x - 5 \int \ln^3 x dx \right)$$

$$= x \ln^7 x - 7x \ln^6 x + 42x \ln^5 x - 210 \left(x \ln^4 x - 4 \int \ln^2 x dx \right)$$

$$= x \ln^7 x - 7x \ln^6 x + 42x \ln^5 x - 210x \ln^4 x + 840 \left(x \ln^3 x - 3 \int \ln x dx \right)$$

$$= x \ln^7 x - 7x \ln^6 x + 42x \ln^5 x - 210x \ln^4 x + 840x \ln^3 x - 2520 \left(x \ln^2 x - 2 \int \ln x dx \right)$$

$$= x \ln^7 x - 7x \ln^6 x + 42x \ln^5 x - 210x \ln^4 x + 840x \ln^3 x - 2520x \ln^2 x + 5040 \left(x \ln x - \int 1 dx \right)$$

$$= x \ln^7 x - 7x \ln^6 x + 42x \ln^5 x - 210x \ln^4 x + 840x \ln^3 x - 2520x \ln^2 x + 5040x \ln x - 5040x$$

$$\begin{aligned}
 \text{P 48(vi)} \quad \int \tan^7 x \, dx &= \frac{\tan^6 x}{6} - \int \tan^5 x \, dx \\
 &= \frac{\tan^6 x}{6} - \left(\frac{\tan^4 x}{4} - \int \tan^3 x \, dx \right) \\
 &= \frac{\tan^6 x}{6} - \frac{\tan^4 x}{4} + \int \tan^3 x \, dx \\
 &= \frac{\tan^6 x}{6} - \frac{\tan^4 x}{4} + \left(\frac{\tan^2 x}{2} - \int \tan x \, dx \right) \\
 &= \frac{\tan^6 x}{6} - \frac{\tan^4 x}{4} + \frac{\tan^2 x}{2} - \int \tan x \, dx \\
 &= \frac{\tan^6 x}{6} - \frac{\tan^4 x}{4} + \frac{\tan^2 x}{2} - \left(- \int \right) \\
 &= \frac{\tan^6 x}{6} - \frac{\tan^4 x}{4} + \frac{\tan^2 x}{2} - (-\ln(\cos u)) \\
 &= \frac{\tan^6 x}{6} - \frac{\tan^4 x}{4} + \frac{\tan^2 x}{2} + \ln(\cos u) + C
 \end{aligned}$$

Q5) $y = x^{2/3}$ for $1 \leq x \leq 8$

Arc Length Formula

$$L = \int_a^b \sqrt{1 + (y')^2} dx$$

$$y' = \frac{2}{3} x^{-1/3} \quad (y')^2 = \left(\frac{2}{3} x^{-1/3} \right)^2 = \frac{4}{9} x^{-2/3}$$

$$a = 1$$

$$b = 8$$

$$L = \int_1^8 \sqrt{1 + \frac{4}{9x^{2/3}}} dx = \int_1^8 \sqrt{\frac{9x^{2/3} + 4}{9x^{2/3}}} dx$$

$$= \int_1^8 \frac{\sqrt{9x^{2/3} + 4}}{\sqrt{9x^{2/3}}} dx = \frac{1}{3} \int_1^8 \frac{\sqrt{9x^{2/3} + 4}}{x^{1/3}} dx$$

$$\text{let } U = 9x^{2/3} + 4$$

$$dU = 6x^{-1/3} dx$$

$$dx = \frac{dU}{6x^{-1/3}} \quad dx = \frac{x^{1/3} dU}{6}$$

$$= \frac{1}{3} \int_1^8 \frac{\sqrt{U}}{x^{1/3}} \cdot \frac{x^{1/3} dU}{6} = \frac{1}{18} \int_1^8 U^{1/2} dU$$

$$= U = 9x^{2/3} + 4$$

$$\text{Upper limit } U = 9(8)^{2/3} + 4$$

$$\text{lower limit } U = 9(1) + 4$$

$$U = 13$$

$$U = 40$$

$$= \frac{1}{18} \int_{13}^{40} U^{1/2} dU = \frac{1}{18} \left[\frac{2U^{3/2}}{3} \right]_{13}^{40}$$

$$L = 7.633705416 \text{ units}$$

ii) $y = \frac{1}{3}\sqrt{x}(3-x)$ for $0 \leq x \leq 3$

$$y = \sqrt{x} - \frac{1}{3}x^{3/2}$$

$$y = x^{1/2} - \frac{1}{3}x^{3/2}$$

$$y' = \frac{1}{2}x^{-1/2} - \frac{1}{2}x^{1/2}$$

$$y'^2 = \left(\frac{1}{2\sqrt{x}} - \frac{1\sqrt{x}}{2} \right) \left(\frac{1}{2\sqrt{x}} - \frac{1\sqrt{x}}{2} \right) = \frac{1}{2x} - \frac{1}{4} - \frac{1}{4} + \frac{1}{4}x$$

$$= \frac{1}{2x} - \frac{1}{2} + \frac{1}{4}x = \frac{1 - 2x + x^2}{4x}$$

$$L = \int_0^3 \sqrt{\frac{1 + 1 - 2x + x^2}{4x}} dx$$

$$L = \int_0^3 \sqrt{\frac{4x + 1 - 2x + x^2}{4x}} dx$$

$$L = \int_0^3 \frac{\sqrt{x^2 + 2x + 1}}{\sqrt{4x}} dx = \int_0^3 \frac{\sqrt{(x+1)^2}}{2\sqrt{x}} dx$$

$$L = \frac{1}{2} \int_0^3 \frac{x+1}{x^{1/2}} dx = \frac{1}{2} \int_0^3 x^{1/2} + x^{-1/2} dx$$

$$L = \frac{1}{2} \left[\frac{2}{3}x^{3/2} + 2x^{1/2} \right]_0^3 = 3.4641 \text{ units}$$

iii) $y = \frac{x^5}{15} + \frac{1}{4x^3}$ for $1 \leq x \leq 2$

$$y = \frac{x^5}{15} + \frac{1}{4}x^{-3} \quad (y')^2 = \left(\frac{x^4}{3} - \frac{3}{4x^4} \right) \left(\frac{x^4}{3} - \frac{3}{4x^4} \right)$$

$$y' = \frac{x^4}{3} - \frac{3}{4x^4}$$

$$y' = \frac{x^4}{3} - \frac{3}{4x^4}$$

$$= \frac{x^8}{9} - \left(\frac{x^4}{3} \right) \left(\frac{3}{4x^4} \right) - \left(\frac{3}{4x^4} \right) \left(\frac{x^4}{3} \right) + \frac{9}{16x^8}$$

$$= \frac{x^8}{9} - \frac{1}{2} + \frac{9}{16x^8} = \frac{32x^{16} - 144x^8}{288x^8}$$

$$= \frac{32x^{16} - 144x^8 + 162}{288x^8} = \frac{16x^{16} - 72x^8 + 81}{144x^8}$$

$$L = \int_1^2 \sqrt{\frac{16x^{16} - 72x^8 + 81}{144x^8}} dx$$

$$L = \int_1^2 \sqrt{\frac{144x^8 + 16x^{16} - 72x^8 + 81}{144x^8}} dx$$

$$L = \int_1^2 \sqrt{\frac{16x^{16} + 72x^8 + 81}{144x^8}} dx$$

$$L = \int_1^2 \frac{\sqrt{(4x^8 + 9)^2}}{\sqrt{144x^8}} dx = \int_1^2 \frac{4x^8 + 9}{12x^4} dx$$

$$\left[\frac{x^5}{15} - \frac{1}{4x^3} \right]_1^2 = 0.96875 \text{ units}$$

$$i) \quad 6xy = x^4 + 3 \quad \text{for } 1 \leq x \leq 2$$

$$y = \frac{x^4 + 3}{6x}$$

$$u = x^4 + 3 \quad v = 6x$$

$$u' = 4x^3 \quad v' = 6$$

$$y' = \frac{24x^4 - (6x^4 + 18)}{36x^2}$$

$$y' = \frac{24x^4 - 6x^4 - 18}{36x^2}$$

$$y' = \frac{18x^4 - 18}{36x^2} = \frac{x^4 - 1}{2x^2}$$

$$L = \int_1^2 \sqrt{1 + \left(\frac{x^4 - 1}{2x^2}\right)^2} dx$$

$$L = \int_1^2 \sqrt{1 + \frac{x^{16} - 2x^4 + 1}{4x^4}} dx$$

$$L = \int_1^2 \sqrt{\frac{4x^4 + x^{16} - 2x^4 + 1}{4x^4}} dx$$

$$L = \int_1^2 \sqrt{\frac{x^{16} + 2x^4 + 1}{4x^4}} dx$$

$$L = \int_1^2 \frac{\sqrt{(x^4 + 1)^2}}{\sqrt{4x^4}} dx$$

$$L = \int_1^2 \frac{x^4 + 1}{2x^2} dx = \frac{1}{2} \int_1^2 \frac{x^4}{x^2} + \frac{1}{x^2} dx$$

$$L = \frac{1}{2} \int_1^2 x^4 + x^{-2} dx$$

$$L = \frac{1}{2} \left[\frac{x^5}{5} - \frac{1}{x} \right]_1^2$$

$$L = \frac{1}{2} \left[\left(\frac{2^5}{5} - \frac{1}{2} \right) - \left(\frac{1^5}{5} - \frac{1}{1} \right) \right]$$

$$L = \frac{1}{2} \left[(6.04 - 0.5) - (0.2 - 1) \right]$$

$$L = \frac{1}{2} [5.09 + 0.8]$$

$$L = 3.035 \text{ units}$$

Area Bounded By Given Curves

The area that lies in between two intersecting curves can be calculated using integral calculus.

Examples

i) $y = \sqrt{x}$ and $y = x^3$

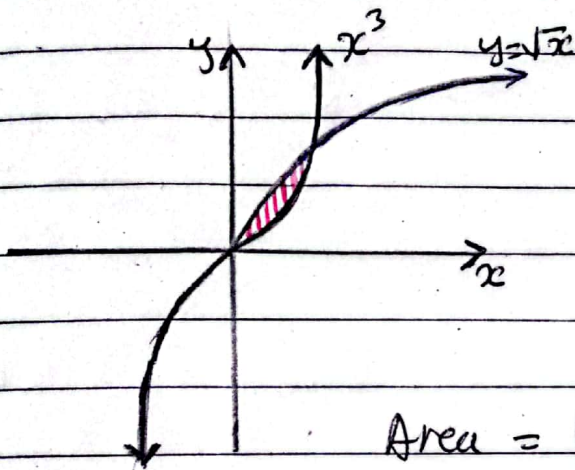
$$x^3 = \sqrt{x}$$

$$x^2 = \frac{1}{x^{1/2}}$$

$$x^{5/2} = 1$$

$$x = 1$$

$$x = 0$$



$$\text{Area} = \int_0^1 \sqrt{x} - x^3 dx$$

$$= \int_0^1 x^{1/2} - x^3 dx \quad \left(\frac{2}{3} (1)^{3/2} - \frac{1}{4} \right) - \left(\frac{2}{3} (0)^{3/2} - \frac{(0)^4}{4} \right)$$

$$\frac{x^{1/2+1}}{\frac{1}{2}+1} - \frac{x^4}{4}$$

$$\frac{2}{3} - \frac{1}{4} = \frac{8-3}{12} = \frac{5}{12} \text{ units}^2$$

$$\frac{2}{3} x^{3/2} - \frac{x^4}{4} \Big|_0^1$$

ii) $x = -y^2$ and $y = x+6$

ii) $x = -y^2$ and $y = x + 6$

$x = y - 6$

points of intersection

$y = -y^2 + 6$

$x = -y^2$

$x = -(-3)^2$

$y^2 + y - 6 = 0$

$x = -(2)^2$

$x = -4$

$y^2 + 3y - 2y - 6 = 0$

$x = -4$

$y(y+3) - 2(y+3) = 0$

$(y-2)(y+3) = 0$

$(-4, 2)$

$(-9, -3)$

$y = 2$ & $y = -3$

Graph.

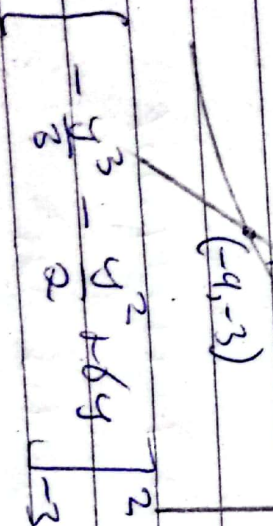
$y = x + 6$

$x = -y^2$

Integrate wrt y

$\int_{-3}^2 (-y^2 - (y-6)) dy$

$\int_{-3}^2 -y^2 - y + 6 dy$



$\left[-\frac{y^3}{3} - \frac{y^2}{2} + 6y \right]_{-3}^2 = 20.83 \text{ units}^2$

Ex) $y = x^2 + x$ and $y = 2x^2 - 2$

points of intersection

$$x^2 + x = 2x^2 - 2$$

$$y = (-1)^2 + (-1)$$

$$y = 2^2 + 1$$

$$x^2 + x - 2x^2 - 2 = 0$$

$$y = 0$$

$$y = 6$$

$$x^2 - x - 2 = 0$$

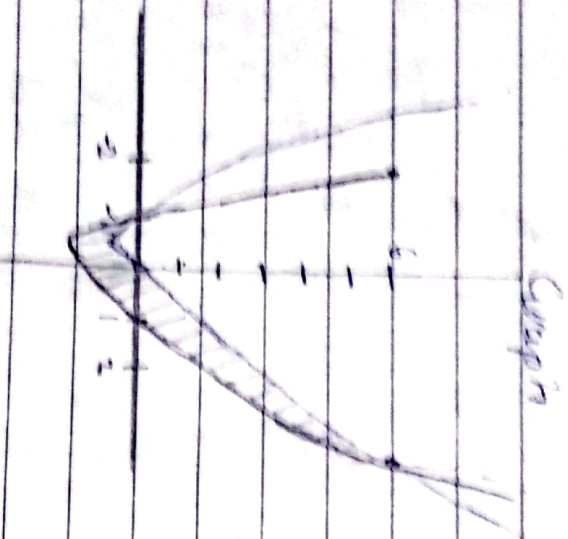
$$(-1, 0) \quad (2, 6)$$

$$x^2 - 2x + x - 2 = 0$$

$$x(x-2) + (x-2) = 0$$

$$(x+1)(x-2) = 0$$

$$x = -1 \quad x = 2$$



Graph

For integrating w.r.t x

$$\int_{-1}^2 (x^2 + x - (2x^2 - 2)) dx$$

$$\int_{-1}^2 (x^2 + x - 2x^2 + 2) dx$$

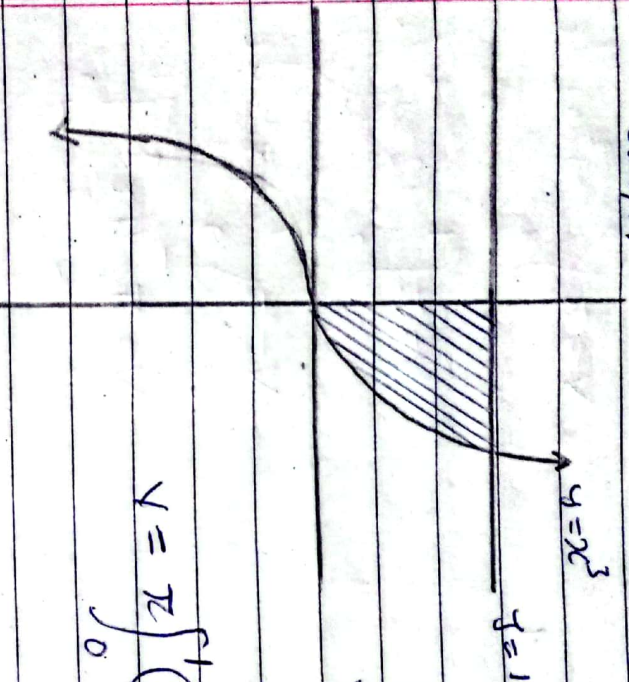
$$\left[\frac{x^3}{3} - \frac{x^3}{3} + 2x \right]_{-1}^2$$

$$= 4.499996667$$

$$= 4.5 \text{ units}^2$$

Q 6 (LC) lower curve $y = x^3$ limits $b=1$
 (b) upper curve $y=1$ $a=0$

Graph



using disc method

About the x-axis

$$V = \pi \int_a^b (R^2 - r^2) dx$$

$$V = \pi \int_0^1 (1^2 - (x^3)^2) dx$$

$$V = \pi \int_0^1 (1 - x^6) dx = \pi \left[x - \frac{x^7}{7} \right]$$

$$V = \pi \left[1 - \frac{1}{7} \right] = \frac{6}{7} \pi$$

$$= 0.857 \pi \text{ units}^2$$

about the y-axis

$$V = \pi \int_0^1 y^{1/3} dy = \pi \left[\frac{3y^{4/3}}{4} \right]_0^1$$

$$V = \pi \left[\frac{3}{4} \right]$$

$$V = \frac{3}{4} \pi \text{ units}^2$$

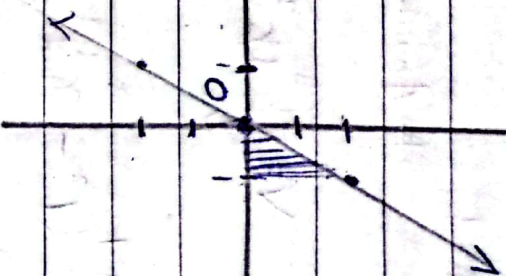
b

 $y = 2x$, above the x -axis

$b = 1$

$a = 0$

Graph

# About the x -axis

$$V = \pi \int_a^b y^2 dx$$

$$V = \pi \int_0^1 (2x)^2 dx$$

$$V = \pi \int_0^1 4x^2 dx$$

$$V = \pi \left[\frac{4x^3}{3} \right]_0^1$$

$$V = \frac{4}{3} \pi \text{ units}^2$$

About the x -axis

$$y = 2x \quad x = \frac{y}{2}$$

$a = 0$

$$V = \pi \int_0^2 x^2 dy$$

$b = 2$

$$V = \pi \int_0^2 \left(\frac{y}{2} \right)^2 dy$$

$$V = \pi \int_0^2 \frac{y^2}{4} dy$$

$$V = \pi \left[\frac{y^3}{12} \right]_0^2$$

$$V = \pi \frac{2^3}{12} = \frac{2}{3} \pi \text{ units}^2$$

Q7 ② $y = x^2$ for $0 \leq x \leq \frac{1}{2}$ $b = \frac{1}{2}$ $a = 0$

i) $y = x^2$ $(y')^2 = 4x^2$

$$S = 2\pi \int_a^b y \sqrt{1 + (y')^2} dx$$

$$S = 2\pi \int_0^{\frac{1}{2}} x^2 \sqrt{1 + 4x^2} dx$$

$$S = 2\pi \int_0^{\frac{1}{2}} x^2 \sqrt{\frac{1}{4} + x^2} dx = 4\pi \int_0^{\frac{1}{2}} x^2 \sqrt{\frac{1}{4} + x^2} dx$$

$x = \frac{1}{2} \tanh \theta$ For limits $\frac{1}{2} = \frac{1}{2} \tanh \theta$ $\tanh \theta = 1 \Rightarrow 4S^2 = \frac{\pi}{4}$
 $dx = \frac{1}{2} \sec^2 \theta d\theta$

$$S = 4\pi \int_0^{\frac{\pi}{4}} \frac{\tanh^2 \theta}{4} \sqrt{\frac{1}{4} + \frac{1}{4} \tanh^2 \theta} \cdot \frac{1}{2} \sec^2 \theta d\theta$$

$$S = \pi \int_0^{\frac{\pi}{4}} \tanh^2 \theta \cdot \frac{1}{2} \sqrt{1 + \tanh^2 \theta} \cdot \frac{1}{2} \sec^2 \theta d\theta$$

$$S = \frac{\pi}{4} \int_0^{\frac{\pi}{4}} \tanh^2 \theta \sec \theta \sec^2 \theta d\theta = \frac{\pi}{4} \int_0^{\frac{\pi}{4}} \tanh^2 \theta \sec^3 \theta d\theta$$

$$S = \frac{\pi}{4} \int_0^{\frac{\pi}{4}} (\sec^2 \theta - 1) \sec^3 \theta d\theta = \frac{\pi}{4} \int_0^{\frac{\pi}{4}} \sec^5 \theta - \sec^3 \theta d\theta$$

$$S = \frac{\pi}{4} \int_0^{\frac{\pi}{4}} \sec^5 \theta d\theta - \frac{\pi}{4} \int_0^{\frac{\pi}{4}} \sec^3 \theta d\theta$$

Q7②

ii) $y = \cos x$ For $0 \leq x \leq \frac{1}{4}$

$$y = x^2$$

when i) $x=0, y=0$

$$x = \sqrt{y}$$

ii) $x = \frac{1}{2}, y = \frac{1}{4}$

$$\frac{dx}{dy} = \frac{1}{2}$$

$$\left(\frac{dx}{dy}\right)^2 = \frac{1}{4y}$$

$$S = 2\pi \int_a^b x \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy$$

$$S = 2\pi \int_0^{1/4} \sqrt{y} \sqrt{1 + \frac{1}{4y}} dy = 2\pi \int_0^{1/4} \sqrt{y} \sqrt{\frac{4y+1}{4y}} dy$$

$$S = 2\pi \int_0^{1/4} \sqrt{y} \frac{\sqrt{4y+1}}{2\sqrt{y}} dy = \pi \int_0^{1/4} \sqrt{4y+1} dy$$

Let $m = 4y+1$

$$dm = 4 dy$$

$$S = \frac{\pi}{4} \int_1^2 m^{1/2} dm$$

$$dy = \frac{dm}{4}$$

$$S = \frac{\pi}{4} \left[\frac{m^{3/2}}{\frac{3}{2}} \right]_1^2 = \frac{\pi \times 2}{4 \times 3} \left[m^{3/2} \right]_1^2$$

$$m = 4\left(\frac{1}{4}\right) + 1$$

$$m = \frac{5}{4}$$

$$m = 4(0) + 1$$

$$m = 1$$

$$S = \frac{\pi}{6} \left[m^{3/2} \right]_1^{5/4} \Rightarrow \frac{1}{4}$$

$$S = \frac{\pi}{6} \left[(4y+1)^{3/2} \right]_0^{1/4}$$

$$S = 0.47140452\pi$$

$$S = \frac{\pi}{6} \left[(4\left(\frac{1}{4}\right) + 1)^{3/2} \right]$$

$$S = 0.47\pi \text{ units}^2$$

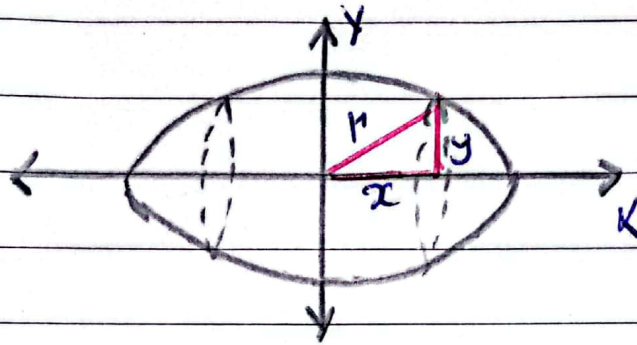
$$S = \frac{\pi}{6} \left[(2)^{3/2} \right]$$

Appointments

WK-03 • 018-347

Volume Of Sphere Using Integral Calculus

#



By pythagoras theorem

$$r^2 = y^2 + x^2$$

$$y = r^2 - x^2$$

$$A = \pi r^2$$

$$r = y$$

$$= \pi y^2$$

$$A = \pi (r^2 - x^2) dx$$

$$\int A = \int \pi (r^2 - x^2) dx$$

$$= \pi \int_0^r r^2 - x^2 dx$$

$$= \pi \left[r^2 x - \frac{x^3}{3} \right]_0^r$$

$$V = \frac{4\pi r^3}{3}$$

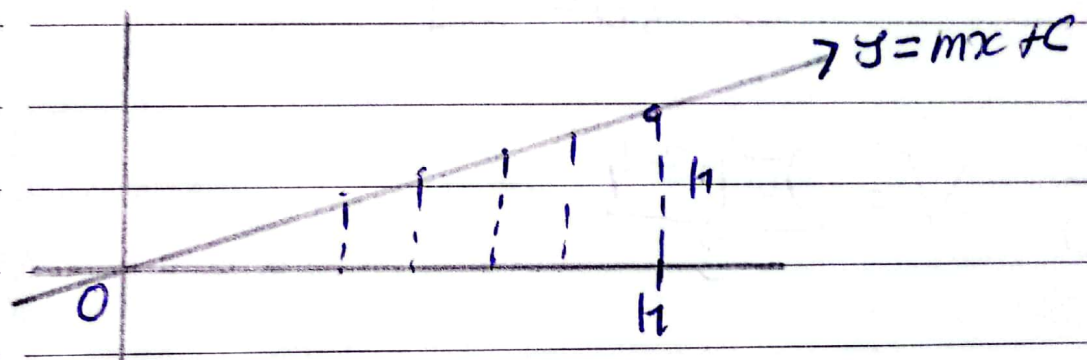
Appointments

Volumes of Revolution

$$V = \pi \int_a^b [f(x)]^2 dx$$

Disc method ↗

Volume of right circular cone,



Rotating $y = mx + c$ about the x-axis yields a cone with radius r and height h

$$f(x) = y = \frac{h}{h} x \quad \begin{matrix} b = h \\ a = 0 \end{matrix}$$

$$V = \pi \int_0^h \left[\frac{h}{h} x \right]^2 dx$$

14

SATURDAY - JANUARY

WK-02 • 014-351

15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31

Appointments

$$V = \pi \int_0^h \frac{r^2 x^2}{h^2} dx$$

$$V = \pi \left[\frac{r^2 x^3}{3h^2} \right]_0^h$$

$$V = \pi \left[\frac{r^2 h^3}{3h^2} \right]$$

15

$$V = \frac{\pi r^2 h}{3}$$

SUNDAY 015-350

∴ Volume of right circular cone is

$$V = \frac{\pi r^2 h}{3}$$