# ENG 3165 LECTURE 5

THERMODYNAMICS COMPONENT

# Ideal Gas Laws

# Introduction

- This lecture provides a link between **Classical Thermodynamics definitions** and an **ideal gas a working fluid.**
- Gases are an easy way to introduce thermodynamic concepts because they allow for the simplified analysis.

# PHYSICAL CHARACTERISTICS OF GASES

| Physical Characteristics | Typical Units |
|---|---|
| Volume, **V** | Liters (**L**), Cubic Metres ($cm^3$) |
| Pressure, **P** | atmosphere<br>(1 **atm** = $1.015 \times 10^5$ $N/m^2$) |
| Temperature, **T** | Kelvin (**K**) |
| Number of atoms or molecules, **n** | **mole** (1 mol = $6.022 \times 10^{23}$ atoms or molecules) |

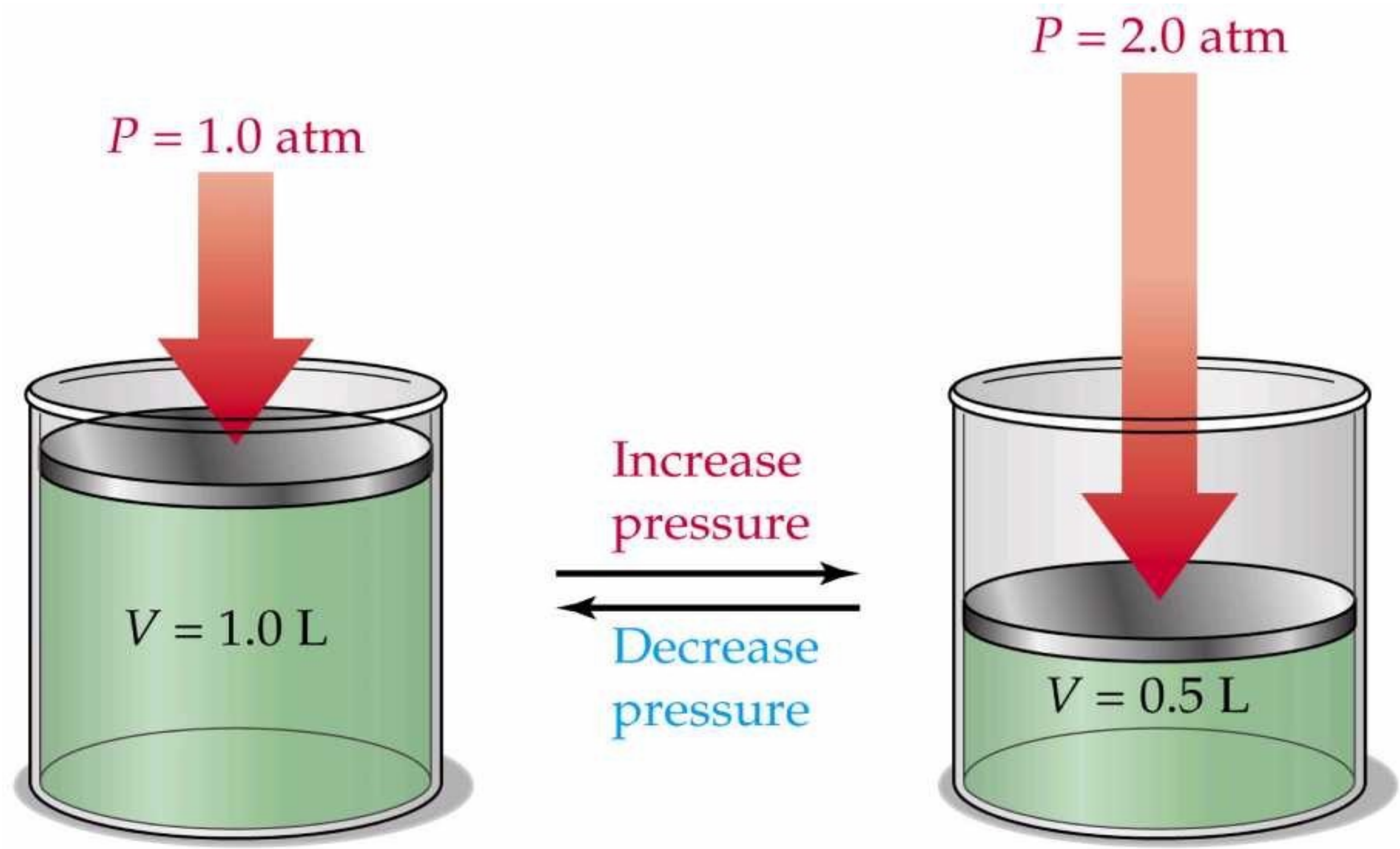
# BOYLE'S LAW

- **Pressure and volume are inversely related** at **constant temperature**.
- **PV = K**
- As one goes up, the other goes down.
- $P_1V_1 = P_2V_2$

**"Father of Modern Chemistry"**
**Robert Boyle**
**Chemist & Natural Philosopher**
Listmore, Ireland
*January 25, 1627 – December 30, 1690*

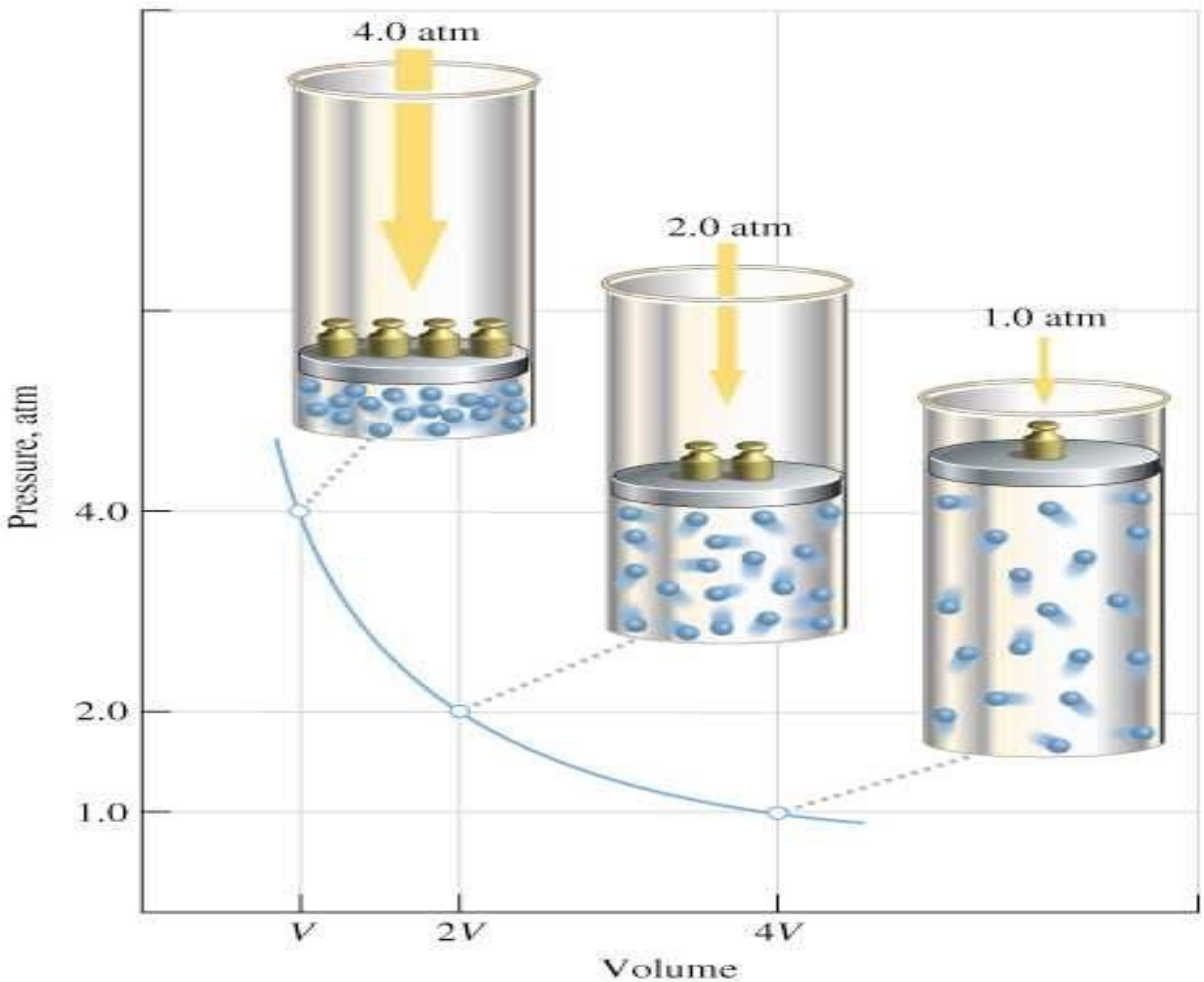
# BOYLE'S LAW

**Boyle's Law:** $P_1V_1 = P_2V_2$



# BOYLE'S LAW

## $P_1V_1 = P_2V_2$



# BOYLE'S LAW

***Example 5.1*** *During an experiment on Boyle's law, the original volume of air trapped in the apparatus, with the two mercury levels of the same, as 20 000 mm$^3$. The apparatus was then modified such that the volume of air became 17 000 mm$^3$, while the temperature remained constant. If the barometer reading was 765 mm Hg, what was the new pressure exerted on the air in mm Hg? Also, what was the difference in the two mercury column levels?*

SOLUTION

Since both levels of mercury are the same at the beginning, then

$$P_1 = \text{atmospheric pressure} = \mathbf{765\ mm\ Hg}$$

Now Boyle's law states that $PV = C$, a constant, from this

$$P_1V_1 = P_2V_2$$

$$\therefore \quad P_2 = P_1\frac{V_1}{V_2} = 765 \times \frac{20\,000}{17\,000}$$

$$= \mathbf{900\ mm\ Hg}$$

The final pressure $P_2 = 900$ mm Hg and the atmospheric pressure = 765 mm Hg, so

Difference in height of the two mercury columns = 900 − 765
= **135 mm**

# BOYLE'S LAW

***Example 5.2*** *A gas whose original pressure and volume were 300 $kN/m^2$ and 0.14 $m^3$ is expanded until its new pressure is 60 $kN/m^2$ while its temperature remains constant. What is its new volume?*

SOLUTION

The temperature remains constant, so this is an expansion according to Boyle's law.

$$\therefore \quad P_1V_1 = P_2V_2 \quad \text{or} \quad V_2 = V_1\frac{P_1}{P_2}$$

$$\therefore \quad V_2 = 0.14 \times \frac{300}{60}$$

$$= \mathbf{0.7\ m^3}$$

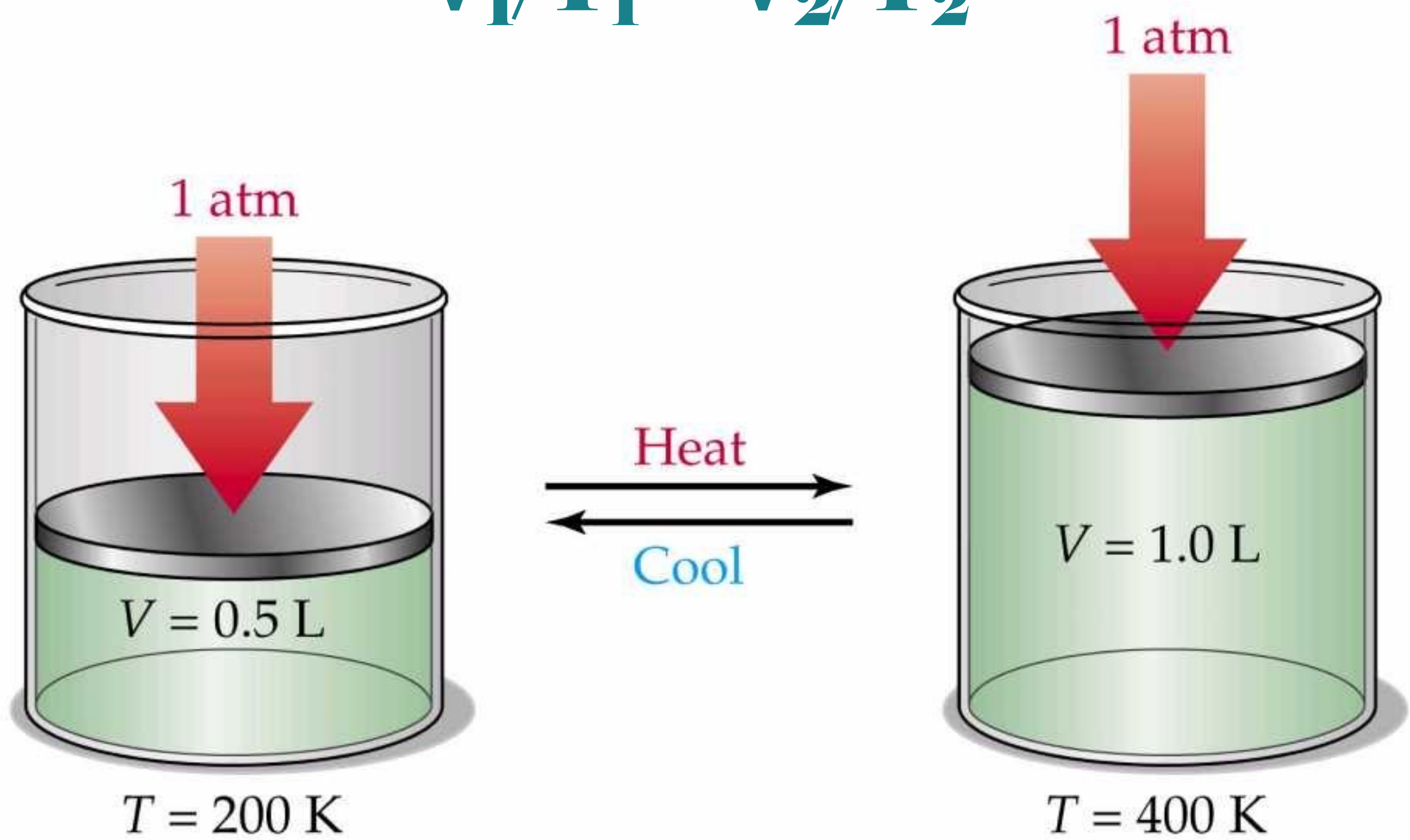
# CHARLES' LAW

- **Volume** of a gas **varies directly with** the absolute **temperature** at **constant pressure.**
- **V = KT**
- $V_1 / T_1 = V_2 / T_2$

**Jacques-Alexandre Charles**
**Mathematician, Physicist, Inventor**
Beaugency, France
*November 12, 1746 – April 7, 1823*

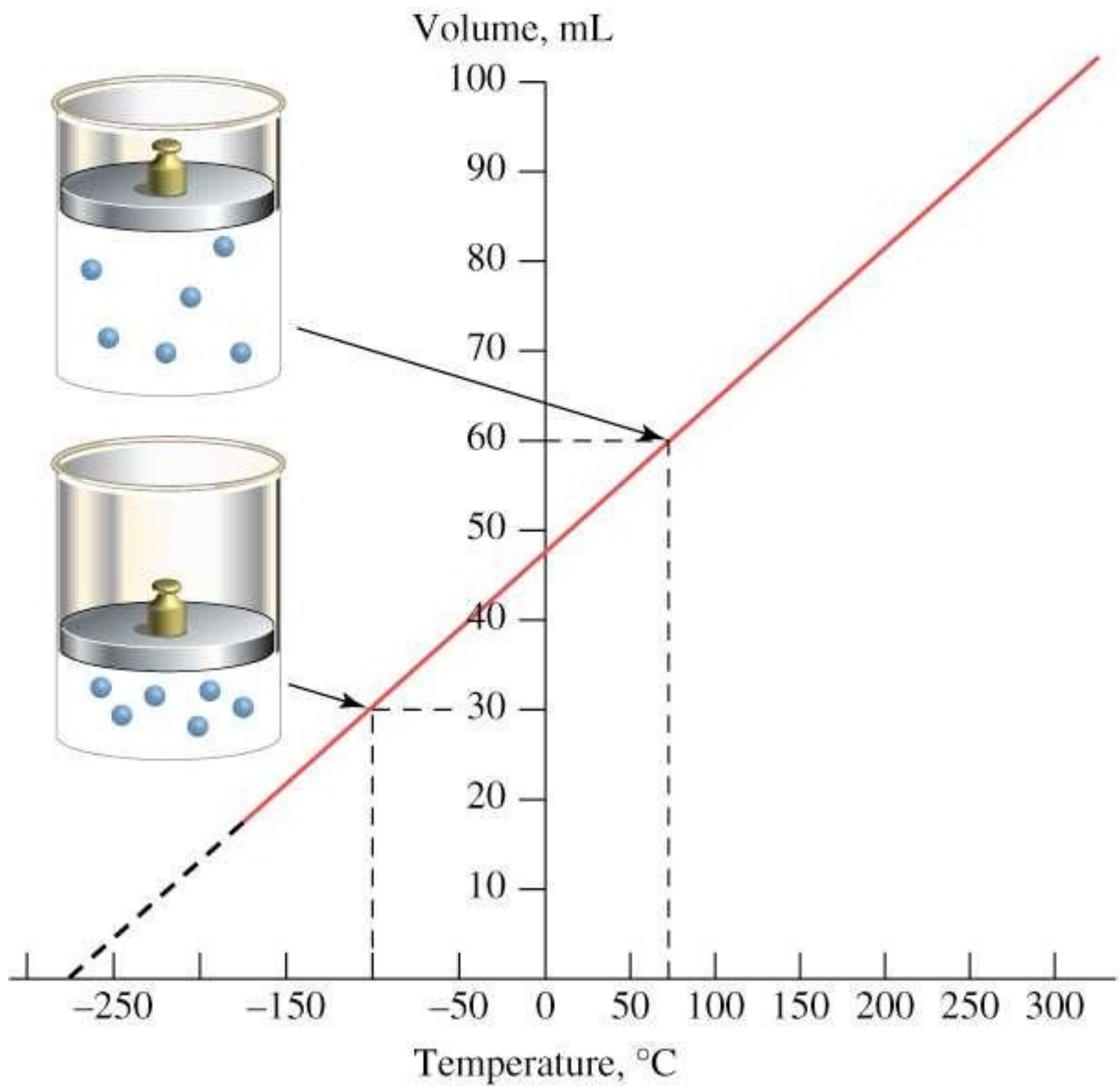
# CHARLES' LAW

$$V_1/T_1 = V_2/T_2$$



# CHARLES' LAW

## $V_1/T_1 = V_2/T_2$



# CHARLES' LAW

***Example 5.3** During an experiment on Charles' law, the volume of gas trapped in the apparatus was 10 000 mm$^3$ when the temperature was 18 °C. The temperature of the gas was then raised to 85 °C. Determine the new volume of gas trapped in the apparatus if the pressure exerted on the gas remained constant.*

SOLUTION

Now according to Charles' law

$$\frac{V}{T} = C, \text{ a constant}$$

From this

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} \qquad [1]$$

In order to use this equation the temperatures $T_1$ and $T_2$ must be absolute temperatures.

$$\therefore \quad T_1 = 18 + 273 = \mathbf{291\ K}$$

and

$$T_2 = 85 + 273 = \mathbf{358\ K}$$

From equation [1]

$$V_2 = V_1 \frac{T_2}{V_1} \qquad [2]$$

$$V_2 = 10\,000 \times \frac{358}{291}$$

$$= \mathbf{12\,302\ mm^3}$$

# CHARLES' LAW

***Example 5.4*** *A quantity of gas whose original volume and temperature are $0.2\ m^3$ and $303\ °C$, respectively, is cooled at constant pressure until its volume becomes $0.1\ m^3$. What will be the final temperature of the gas?*

SOLUTION

Again, this is a change according to Charles' law.

$$\therefore \quad \frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$T_2 = T_1 \frac{V_2}{V_1}$$

The temperature is in degrees Celsius this time.

$$\therefore \quad T_1 = 303 + 273 = 576\ K$$

and from equation [1]

$$T_2 = 576 \times \frac{0.1}{0.2}$$

$$= 288\ K$$

$$\therefore \quad t_2 = 288 - 273 = 15\ °C$$

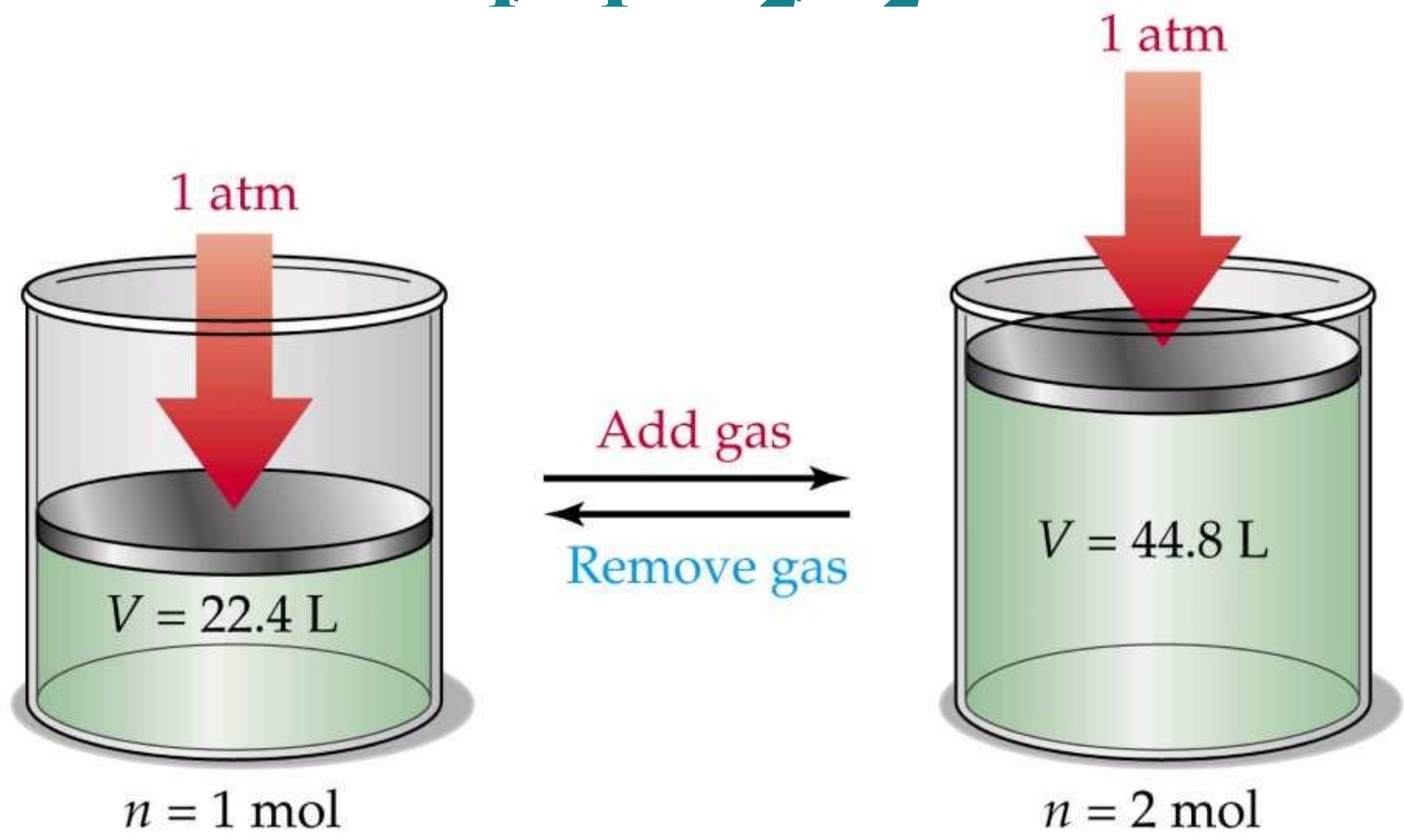
# AVOGADRO'S LAW

**Amedeo Avogadro**
**Physicist**
Turin, Italy
*August 9, 1776 – July 9, 1856*

- At **constant temperature and pressure**, the **volume** of a gas is **directly related to the number of moles**.
- $V = K n$
- $V_1 / n_1 = V_2 / n_2$

# AVOGADRO'S LAW

$$V_1/n_1 = V_2/n_2$$



# GAY-LUSSAC LAW

- At constant volume, pressure and absolute temperature are directly related.
  - $P = k T$
  - $P_1 / T_1 = P_2 / T_2$

- Joseph-Louis Gay-Lussac
  - Experimentalist
  - Limoges, France
- *December 6, 1778 – May 9, 1850*

# DALTON'S LAW

- The **total pressure** in a container is the **sum of the pressure each gas** would exert if it were alone in the container.
- The total pressure is the sum of the partial pressures.
- $P_{Total} = P_1 + P_2 + P_3 + P_4 + P_5 \ldots$

**(For each gas $P = nRT/V$)**

**John Dalton**
**Chemist & Physicist**
Eaglesfield, Cumberland, England
*September 6, 1766 – July 27, 1844*

# THE CHARACTERISTIC EQUATION OF A PERFECT GAS

**Consider the Boyle's Law change from 1 to A.** In this case the temperature remains constant at $T_1$. Also:

$$P_1V_1 = P_AV_A \text{..............}(1)$$

All the pressure change must take place during this process because there willl be no change in pressure during the Charles's Law process which follows. In this case, $P_A = P_2$

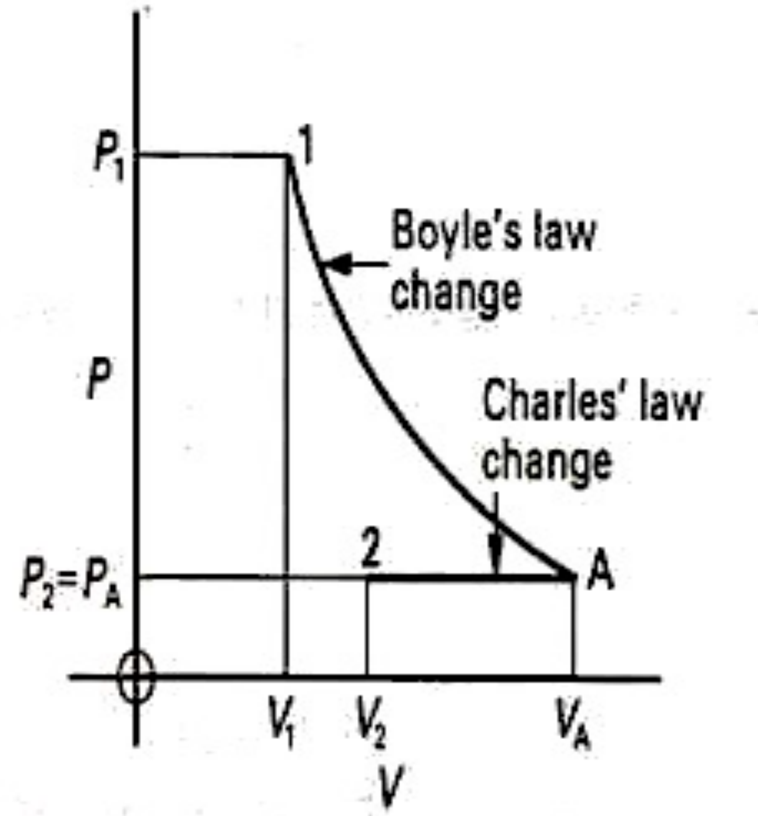
$$\therefore \ Equation\ (1)\ becomes\ P_1V_1 = P_AV_A$$

Or

$$V_A = \frac{P_1V_1}{P_2} \text{............}(2)$$

**Consider now the Charle's Law change from A to 2.** In this case the pressure remains constant at $P_2$ . Also:

$$\frac{V_A}{T_A} = \frac{V_2}{T_2} \text{..................}(3)$$



# THE CHARACTERISTIC EQUATION OF A PERFECT GAS

**During the boyle's Law change from 1 to A, the temperature remained constant**

$$\therefore\ T_A = T_1$$

From which equation (3) becomes:

$$\frac{V_A}{T_1} = \frac{V_2}{T_2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

But:

$$V_A = \frac{P_1 V_1}{P_2}$$

From equation (2) and substituting this in equation (4):

$$\frac{P_1 V_1}{P_2 T_1} = \frac{V_2}{T_2}$$

From which:

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5)$$

# THE CHARACTERISTIC EQUATION OF A PERFECT GAS

Equation (5) can be extended:

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} = \frac{P_3V_3}{T_3} = \frac{P_4V_4}{T_4} \ldots\ldots\ldots\ldots, etc \ldots\ldots\ldots\ldots\ldots..\ldots(6)$$

Where 3 and 4 represent other new conditions of state of the same mass of gas. From equation (6) it follows that for any fixed mass of gas , changes of state are connected by the equation:

$$\frac{PV}{T} = constant \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(7)$$

Taking into account the actual mass of the gas during any particular prrocess and consifering the specific volume v of the gas, this equation becomes:

$$\frac{Pv}{T} = a\ constant \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(8)$$

When 1kg of gas is considered, the constant written as ***R*** is called the **characteristic gas constant** or sometimes the **specific gas constant**:

$$\frac{Pv}{T} = R \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(9)$$

# THE CHARACTERISTIC EQUATION OF A PERFECT GAS

Now consider a case when there are ***m*** kg of gas, multiply both sides of equation (9) by m, then:

$$\frac{P(mv)}{T} = mR$$

But ***mv*** is the total volume of the gas being used, ***V,*** so for ***m*** of of gas it folllows that:

$$\frac{PV}{T} = mR$$

***or***

$$PV = mRT \ldots \ldots \ldots \ldots \ldots (10)$$

This is known as the **Characteristic Equation of a Perfect Gas.** The units of ***R*** can be obtained fron equation (9). If pressure is ***N/m²*** **,** specific volume is ***m³/kg*** and temperature ***K,*** then the units for ***R*** are $^{J}/_{kg\cdot K}$.

***For Air the value is of R is usually of the order of 0.287 kJ/kg.K***

# THE UNIVERSAL GAS CONSTANT $\bar{R}$

- For any **1 mole of gas**, the mass of that mole is equal to the gases Molecular Weight. Therefore dividing by the molecular weight, M, throughout:

$$\frac{Pv}{M} = \frac{mRT}{M}$$

$$\Longrightarrow P\bar{v} = m\bar{R}T$$

- Where $\bar{v}$ is the volume per mole and $\bar{R}$ is the Universal Gas constant.
- The value has been determined experimentally as $8.314\,\frac{kJ}{kmol.K}$

# THE CHARACTERISTIC EQUATION OF A PERFECT GAS

***Example 5.5*** *A gas whose original pressure, volume and temperature were 140 kN/m², 0.1 m³ and 25 °C, respectively, is compressed such that its new pressure is 700 kN/m² and its new temperature is 60 °C. Determine the new volume of the gas.*

SOLUTION

By the characteristic equation

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \quad \text{and} \quad T_1 = 25 + 273 = \mathbf{298\ K}$$

$$\text{also} \quad T_2 = 60 + 273 = \mathbf{333\ K}$$

$$\therefore \quad V_2 = \frac{P_1}{P_2}\frac{T_2}{T_1} V_1 = \frac{140}{700} \times \frac{333}{298} \times 0.1 = \mathbf{0.022\,3\ m^3}$$

# THE CHARACTERISTIC EQUATION OF A PERFECT GAS

***Example 5.6*** *A quantity of gas has a pressure of 350 kN/m² when its volume is 0.03 m³ and its temperature is 35 °C. If the value of R = 0.29 kJ/kg K, determine the mass of gas present. If the pressure of this gas is now increased to 1.05 MN/m² while the volume remains constant, what will be the new temperature of the gas?*

SOLUTION

By the characteristic equation

$$PV = mRT$$

$$\therefore \quad m = \frac{PV}{RT} = \frac{350 \times 10^3 \times 0.3}{0.29 \times 10^3 \times 308} = \mathbf{0.118\ kg}$$

For the second part of the problem

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \quad \text{and in this case, } V_1 = V_2$$

$$\therefore \quad \frac{P_1}{T_1} = \frac{P_2}{T_2} \quad \text{or} \quad T_2 = T_1 \frac{P_2}{P_1} = 308 \times \frac{1.05 \times 10^6}{0.35 \times 10^6}$$

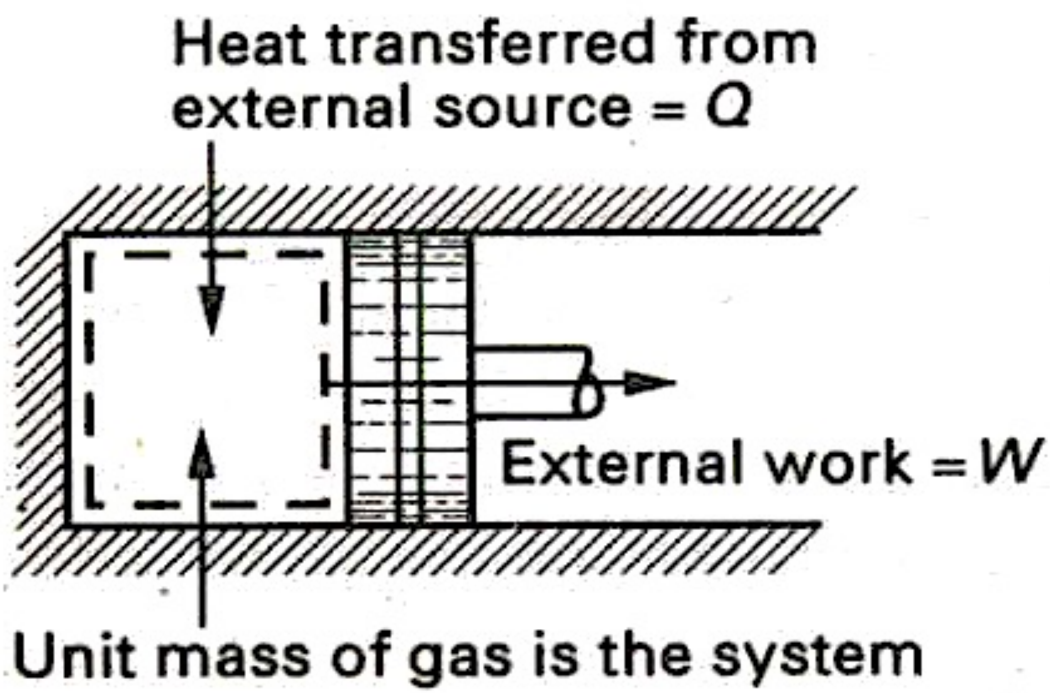
$$= 308 \times 3 \quad (T_1 = 35 + 273 = 308\ \text{K})$$

$$= \mathbf{924\ K}$$

$$\therefore \quad t_2 = 924 - 273 = \mathbf{651\ °C}$$

# SPECIFIC HEAT CAPACITIES OF A GAS

- A **specific heat capacity** may be defined as the amount of heat transfer required to raise a unit mass of substance through 1 degree difference in temperature.
- The following figure shows how a piston can be used to determine specific heat of a gas



# SPECIFIC HEAT CAPACITIES OF A GAS

- If the **specific heat capacity** of a gas is quoted, it is necessary to define the conditions under which that specific heat capacity is being measured.
- Two important cases are called the Principle Specific Heat Capacities.

1. **The specific heat capacity at constant volume:** This is defines as the amount of heat which transfers to or from a unit mass of gas while the temperature changes by 1 degree and the volume remains constant. It is written as $c_v$.
2. **The specific heat capacity at constant pressure:** This is defines as the amount of heat which transfers to or from a unit mass of gas while the temperature changes by 1 degree and the pressure remains constant. It is written as $c_p$.

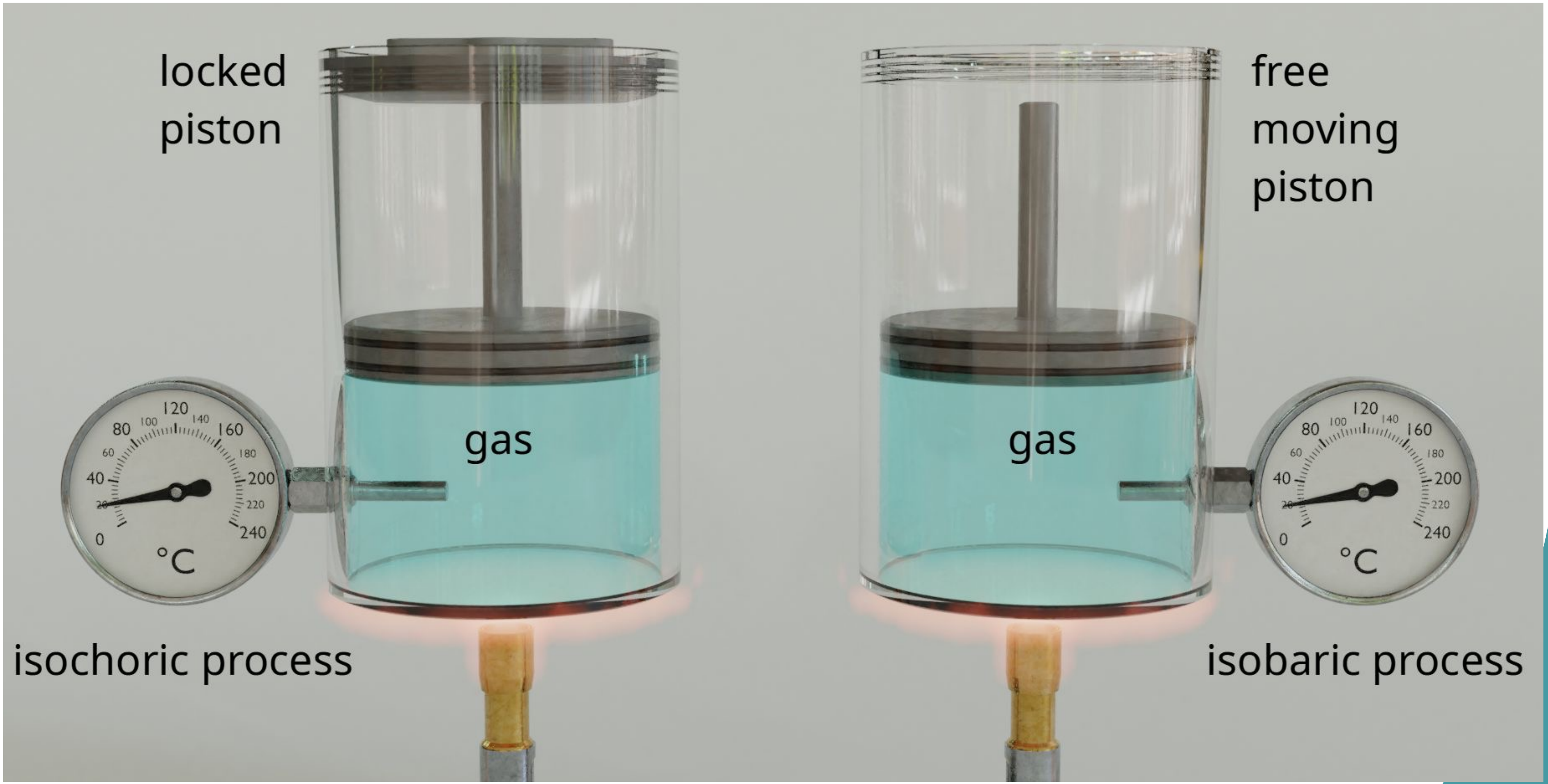
The specific heat capacities at constant volume or pressure rise in value with temperature so for calculations an average value if assumed within the temperature range being considered.

# SPECIFIC HEAT CAPACITIES OF A GAS

Table of average specific heat capacities of gases

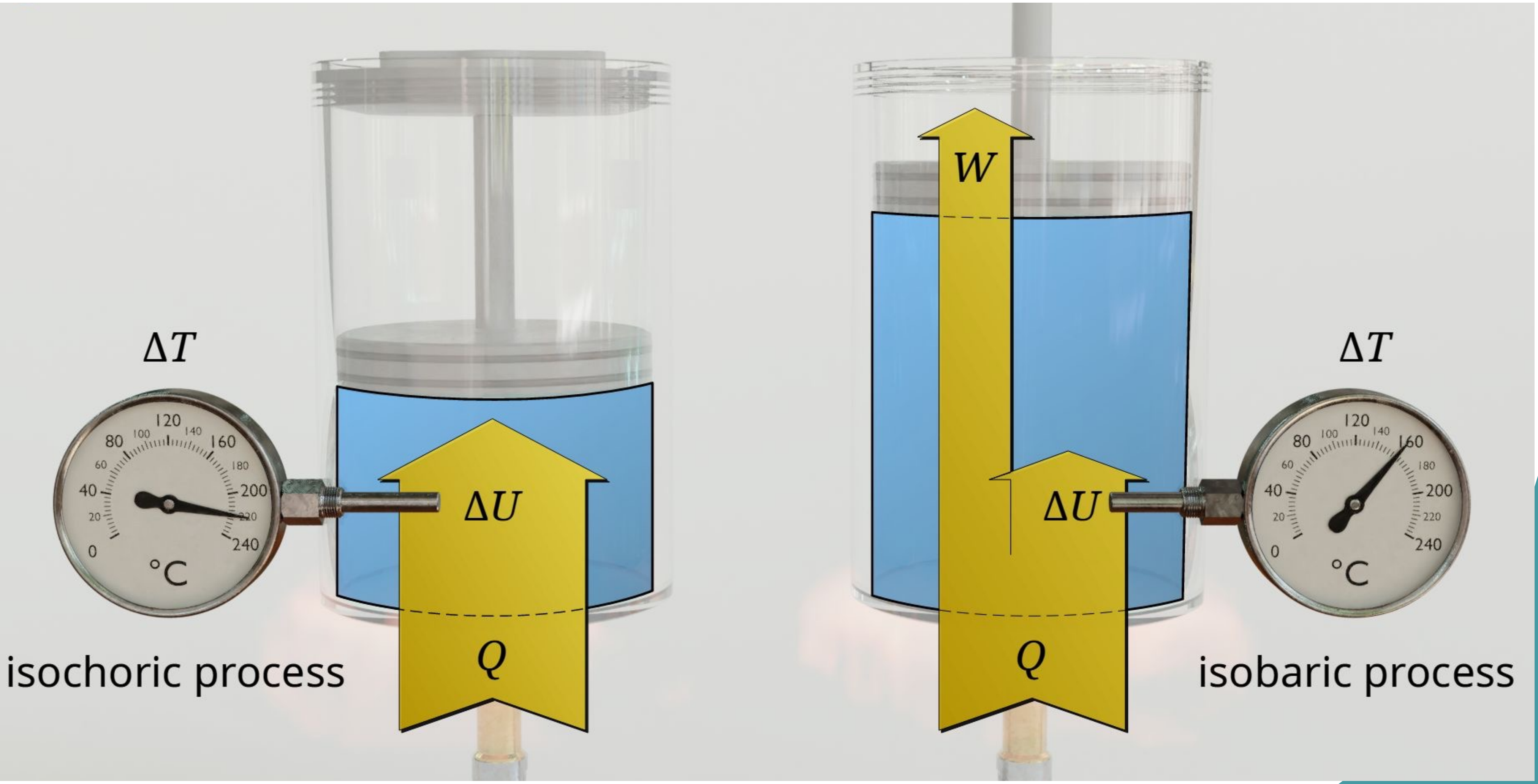
| Gas | $c_p$(kJ/kg K) | $c_v$(kJ/kg K) |
|---|---|---|
| Air | 1.006 | 0.718 |
| Carbon dioxide | 0.87 | 0.67 |
| Carbon monoxide | 1.04 | 0.74 |
| Hydrogen | 14.4 | 10.2 |
| Nitrogen | 1.04 | 0.74 |
| Oxygen | 0.92 | 0.66 |
| Methane | 2.29 | 1.74 |
| Sulphur dioxide | 0.65 | 0.52 |

# THE CONSTANT PRESSURE HEATING OF A GAS



# THE CONSTANT PRESSURE HEATING OF A GAS

For an ideal gas at constant pressure, it **takes more heat to achieve the same temperature change than it does at constant volume**. At constant volume all the heat added goes into raising the temperature. At constant pressure some of the heat goes to doing work.



# THE CONSTANT VOLUME HEATING OF A GAS

- Let a mass of gas m be heated at constant volume such that its temperature rises from $T_1$ to $T_2$ and its pressure rises from $P_1$ to $P_2$, then:

$$\boldsymbol{Heat\ received\ by\ the\ gas}$$
$$\boldsymbol{= mass \times specific\ heat\ capacity\ at\ constant\ volume}$$
$$\boldsymbol{\times rise\ in\ temperature}$$

$$\boldsymbol{= mc_v(T_2 - T_1)}$$

- Constant volume heating is a particular cas of a non-flow process carried out on a gas.

$$\boldsymbol{Q = \Delta U + W\ , where\ W = 0}$$

- No external work is done during the process

$$\boldsymbol{\therefore \Delta U = U_2 - U_1 = mc_v(T_2 - T_1)}$$

# THE CONSTANT VOLUME HEATING OF A GAS

***Example 5.7*** *2 kg of gas, occupying 0.7 $m^3$, had an original temperature of 15 °C. It was then heated at constant volume until its temperature became 135 °C. Determine the heat transferred to the gas and its final pressure. Take $c_v = 0.72$ kJ/kg K and R = 0.29 kJ/kg K.*

SOLUTION

Heat transferred at constant volume $= mc_v(T_2 - T_1)$

$= 2 \times 0.72 \times (135 - 15)$

$= 2 \times 0.72 \times 120$

$= \mathbf{172.8\ kJ}$

Now $P_1V_1 = mRT_1$ and $T_1 = (15 + 273)\,\text{K} = \mathbf{288\ K}$

$$\therefore \quad P_1 = \frac{mRT_1}{V_1} = \frac{2 \times 0.29 \times 288}{0.7} = \frac{167.04}{0.7} = \mathbf{238.6\ kN/m^2}$$

# THE CONSTANT VOLUME HEATING OF A GAS

## Example continued…

Since the volume remains constant, then

$$\frac{P_1}{T_1} = \frac{P_2}{T_2} \quad \therefore \quad P_2 = P_1 \frac{T_2}{T_1} \quad \text{and} \quad T_2 = (135 + 273)\ \text{K} = 408\ \text{K}$$

$$= 238.6 \times \frac{408}{288} = 338.02\ \text{kN/m}^2$$

# THE CONSTANT PRESSURE HEATING OF A GAS

- Let a mass of gas m be heated at constant pressure such that its temperature rises from $T_1$ to $T_2$ and its pressure rises from $V_1$ to $V_2$, then:

$$\textit{Heat received by the gas}$$
$$= \textit{mass} \times \textit{specific heat capacity at constant pressure} \times \textit{rise in temperature}$$

$$= mc_p(T_2 - T_1)$$

- Also

$$\Delta U = Q - W$$

$$or. \qquad \Delta U = U_2 - U_1 = mc_p(T_2 - T_1) - P(V_2 - V_1)$$

$$U_2 - U_1 = mc_p(T_2 - T_1) - mR(T_2 - T_1)$$

**Or applying the characteristic gas equation:**

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}, \qquad where\ P_2 = P_1$$

$$\Rightarrow \frac{V_1}{T_1} = \frac{V_2}{T_2} \quad \text{or} \quad V_2 = V_1\frac{V_2}{T_2}$$

# THE CONSTANT PRESSURE HEATING OF A GAS

**Example 5.8** *A gas whose pressure, volume and temperature are 275 kN/m², 0.09 m³ and 185 °C, respectively, has its state changed at constant pressure until its temperature becomes 15 °C. Determine the heat transferred from the gas and the work done on the gas during the process. Take* R = *0.29 kJ/kg K,* $c_p$ = *1.005 kJ/kg K.*

Now $P_1V_1 = mRT_1$ and $T_1 = (185 + 273)\text{ K} = \mathbf{458\ K}$

$$\therefore \quad m = \frac{P_1V_1}{RT_1} = \frac{275 \times 10^3 \times 0.09}{0.29 \times 10^3 \times 458} = \mathbf{0.186\ kg}$$

Heat transferred $= mc_p(T_2 - T_1)$

$$\therefore \quad \text{Heat transferred} = 0.186 \times 1.005 \times (15\text{-}185)$$
$$= 0.186 \times 1.005 \times (-170)$$
$$= \mathbf{-31.78\ kJ}$$

# THE CONSTANT PRESSURE HEATING OF A GAS

## Example continued....

Notice the negative sign, indicating that the heat has been transferred from the gas.

Since the pressure remains constant, then

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$\therefore \quad V_2 = V_1 \frac{T_2}{T_1} = 0.09 \times \frac{288}{458} = \mathbf{0.056\,6\ m^3}$$

$$\begin{aligned} \text{Work done} &= P(V_2 - V_1) \\ &= 275 \times (0.056\,6 - 0.009) \\ &= 275 \times (-0.0334) \\ &= \mathbf{-9.19\ kJ} \end{aligned}$$

# THE DIFFERENCE OF THE SPECIFIC HEAT CAPACITIES OF A GAS

- It has been shown that if a mass of gas m has its temperature changed from $T_1$ to $T_2$ then the change in its internal energy can be determined by the expressions:

$$U_2 - U_1 = mc_v(T_2 - T_1) \ldots\ldots\ldots\ldots\ldots\ldots..\textbf{(1)}$$

And

$$U_2 - U_1 = mc_p(T_2 - T_1) - mR(T_2 - T_1)\textbf{....(2)}$$

- If the temperature change is the same for both expressions, then it follows that equation (1) quals equation (2) because the change in internal energy is a function of temperature alone by Joule’s Law:

$$\therefore mc_v(T_2 - T_1) = mc_p(T_2 - T_1) - mR(T_2 - T_1)$$

Since $(T_2 - T_1)$ is common throughout, we get:

$$c_v = c_p - R$$

$$\therefore c_p - c_v = R \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\textit{(3)}$$

$$= \frac{Pv}{T} \left( since \ \frac{Pv}{T} = R \right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots.(4)$$

And.

$$\frac{c_p}{c_v} = \gamma \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.(5)$$

# “The Second Law of Thermodynamics: if you think things are a in a mess now, just wait!!”

Jim Warner

# Thank You

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