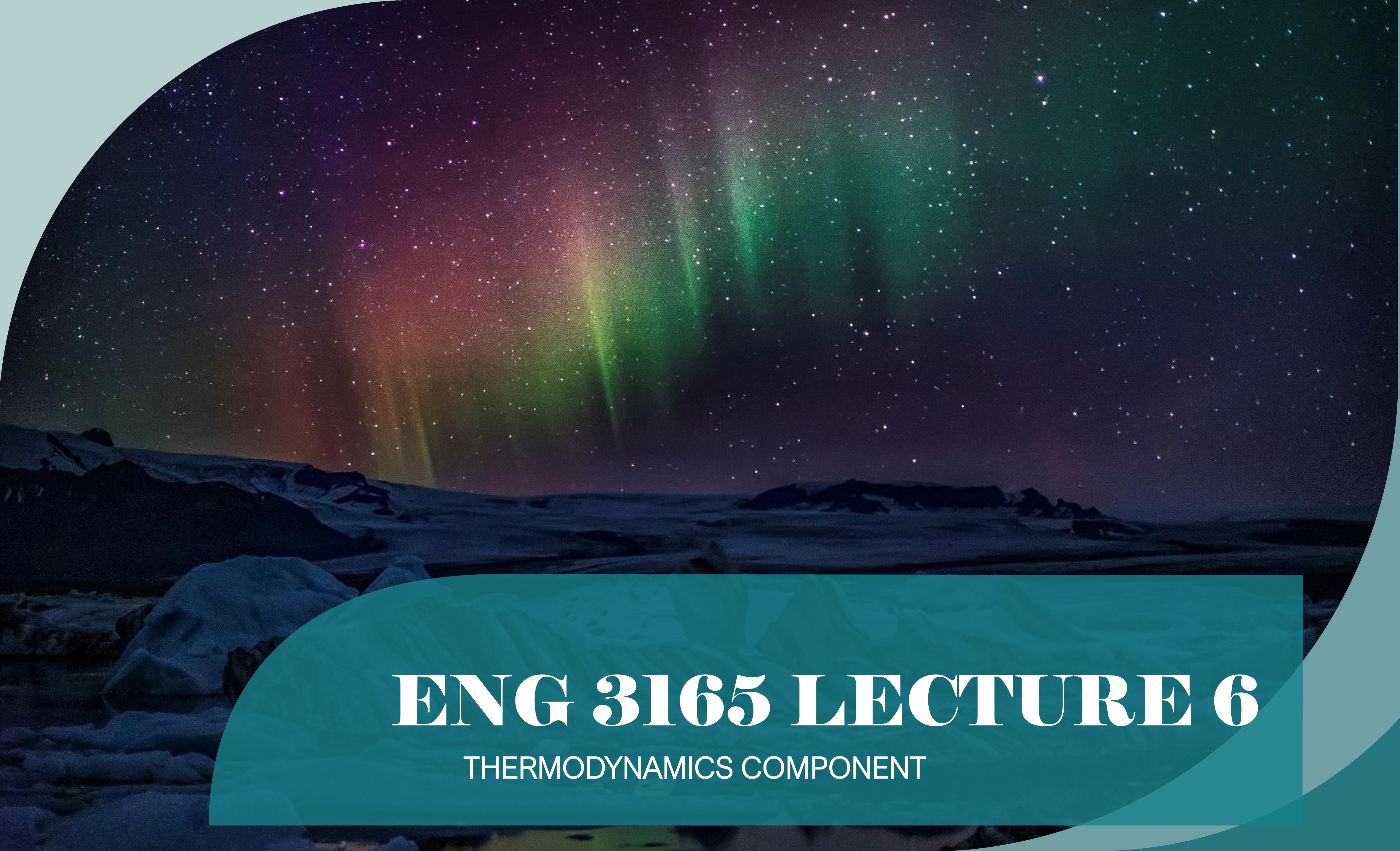


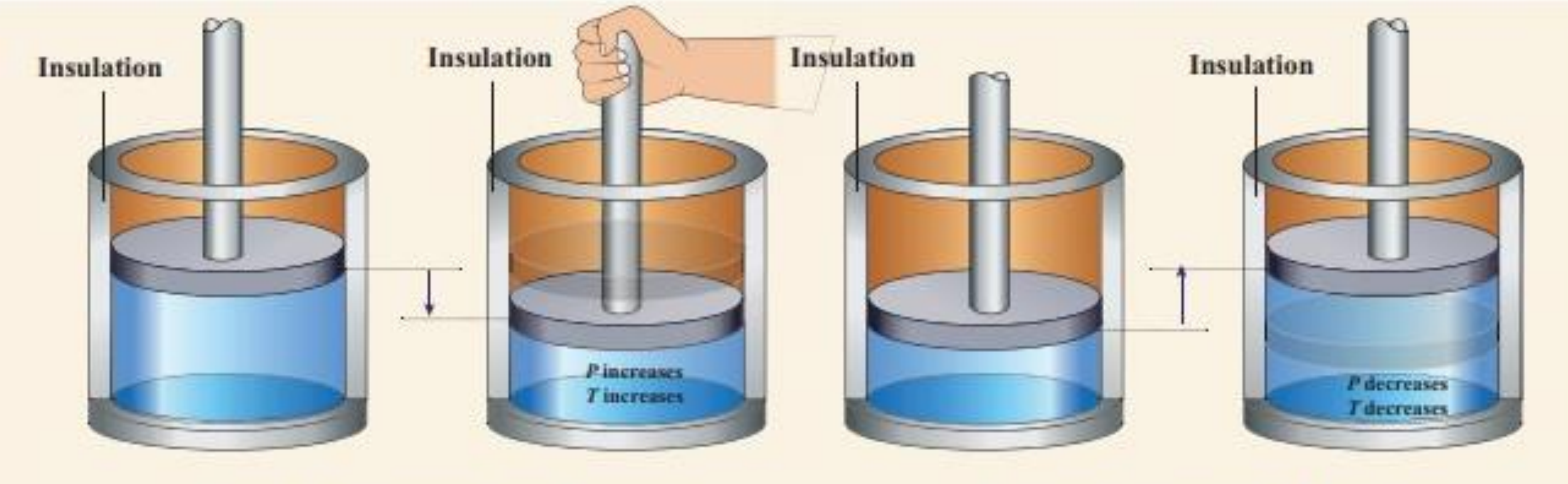
ENG 3165 LECTURE 6

THERMODYNAMICS COMPONENT

Ideal Gas Laws Continued …

THE ADIABATIC PROCESS AND A GAS

Adiabatic Compression and Expansion



(a) Adiabatic compression (P and T increases)

(b) Adiabatic expansion (P and T decreases)

THE ADIABATIC PROCESS AND A GAS

- When dealing with the general case of a polytropic expansion or compression, it was stated that the process followed the law: $\boldsymbol{PV^n = C}$
- Now the Adiabatic process can be a particular case of the Polytropic process where no heat exchange during the progress of the process.
- Consider an adiabatic process in which a change of state occurs from P_1, V_1, T_1 to P_2, V_2, T_2

THE ADIABATIC PROCESS AND A GAS

$$\text{Change of internal energy} = mc_v(T_2 - T_1) \quad (1)$$

Also

$$\text{Work done during the process} = \frac{P_1V_1 - P_2V_2}{(\gamma - 1)} \quad (2)$$

$$= \frac{mR(T_1 - T_2)}{(\gamma - 1)} \quad (3)$$

where γ (gamma) is the particular index which will satisfy the case of an adiabatic process (sometimes the adiabatic index is written k).

From the polytropic law then, if γ is the adiabatic index

$$P_1V_1^\gamma = P_2V_2^\gamma \quad (4)$$

Also, from the polytropic law,

$$\frac{T_1}{T_2} = \left(\frac{P_1}{P_2}\right)^{(\gamma-1)/\gamma} = \left(\frac{V_2}{V_1}\right)^{(\gamma-1)} \quad (5)$$

and by the characteristic equation

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \quad (6)$$

Applying the non-flow energy equation

$$Q = \Delta U + W$$

THE ADIABATIC PROCESS AND A GAS

For an adiabatic process $Q = 0$

$$\therefore \quad 0 = \Delta U + W$$

or

$$W = -\Delta U \tag{7}$$

i.e. Work is done at the expense of internal energy during an adiabatic expansion. Internal energy increases at the expense of work during an adiabatic compression.

Substituting equations (1) and (3) in equation (7)

$$\frac{mR(T_1 - T_2)}{(\gamma - 1)} = -mc_v(T_2 - T_1)$$

$$\therefore \quad \frac{mR(T_1 - T_2)}{(\gamma - 1)} = mc_v(T_1 - T_2)$$

THE ADIABATIC PROCESS AND A GAS

from which

$$\frac{R}{(\gamma-1)} = c_v \tag{8}$$

since $m(T_1 - T_2)$ is a common term to both sides.

From this,

$$\frac{R}{c_v} = (\gamma - 1) \tag{9}$$

or,

$$\gamma = \frac{R}{c_v} + 1 = \frac{R+c_v}{c_v} \tag{10}$$

Now

$R = c_p - c_v$ and hence, substituting in equation (8)

$$\gamma = \frac{c_p - c_v + c_v}{c_v}$$

or

$$\gamma = \frac{c_p}{c_v} \tag{11}$$

From this then the Law for an Adiabatic Expansion or Compression of a Gas is:

$$\boldsymbol{PV^{\gamma} = C} \qquad where\ \gamma = \frac{c_p}{c_v}$$

Example

A gas expands adiabatically from a pressure and volume of 700 kN/m^2 and 0·015 m^3, respectively, to a pressure of 140 kN/m^2. Determine the final volume and the work done by the gas. What is the change of internal energy in this case?

Take, $c_p = 1{\cdot}046$ kJ/kg K, $c_v = 0{\cdot}752$ kJ/kg K.

Adiabatic index $= \gamma = c_p/c_v = 1{\cdot}046/0{\cdot}752 = \underline{1{\cdot}39}$

For an adiabatic expansion,

$$P_1V_1^\gamma = P_2V_2^\gamma$$

$$\therefore \left(\frac{V_2}{V_1}\right)^\gamma = \frac{P_1}{P_2} \quad \text{or} \quad \frac{V_2}{V_1} = \left(\frac{P_1}{P_2}\right)^{1/\gamma}$$

from which

$$V_2 = V_1\left(\frac{P_1}{P_2}\right)^{1/\gamma} = 0{\cdot}015 \times \left(\frac{700}{140}\right)^{1/1{\cdot}39} = 0{\cdot}015 \times 5^{1/1{\cdot}39}$$

$$= 0{\cdot}015 \times \sqrt[1{\cdot}39]{5} = 0{\cdot}015 \times 3{\cdot}18 = \underline{0{\cdot}048\ \text{m}^3} = \text{final volume}$$

Example continued....

$$\text{Work done} = \frac{P_1V_1 - P_2V_2}{\gamma - 1}$$

$$= \frac{(700 \times 0{\cdot}015) - (140 \times 0{\cdot}048)}{1{\cdot}39 - 1}$$

$$= \frac{10{\cdot}5 - 6{\cdot}72}{0{\cdot}39} = \frac{3{\cdot}78}{0{\cdot}39}$$

$$= \underline{9{\cdot}69 \text{ kJ}}$$

For an adiabatic process,

$$W = -\Delta U$$

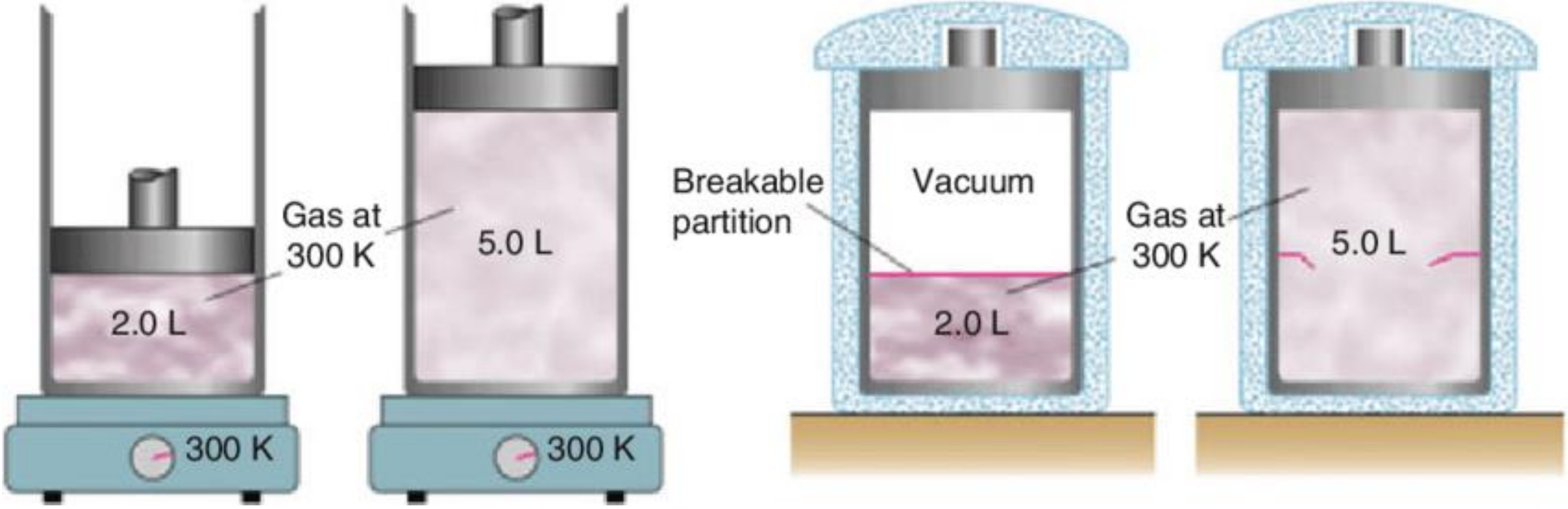
or

$$\Delta U = -W$$

$$\therefore \text{ Change of internal energy } = -\underline{9{\cdot}69 \text{ kJ}}$$

This is a loss of internal energy from the gas.

THE ISOTHERMAL PROCESS AND A GAS



(a) Slow, controlled isothermal expansion of a gas from an initial state 1 to a final state 2 with the same temperature but lower pressure.
(b) Rapid, uncontrolled expansion of the same has starting at the same state 1 and ending at the same state 2

THE ISOTHERMAL PROCESS AND A GAS

- It follows that during an isothermal gas expansion, all the heat transferred is converted into external work.
- Conversely, during an isothermal gas compression, all the work done on the gas is rejected by the gas as heat transfer
- Since the temperature remains constant throughout the process, then the internal energy before is equal to the internal energy after
- Any heat transferred to the gas is immediately dissipated in carrying out work (expansion)
- Energy input in the form of work done on the gas is immediately rejected as heat transfer (compression)

THE ISOTHERMAL PROCESS AND A GAS

An isothermal process is defined as a process carried out such that the temperature remains constant throughout the process. This is evidently the same as a process carried out according to Boyle's Law. The law for an isothermal expansion or compression of a gas is therefore

$$PV = C, \text{ a constant} \tag{1}$$

Thus, for a change of state from 1 to 2,

$$P_1V_1 = P_2V_2 \tag{2}$$

$$T_1 = T_2 = T = \text{constant temperature} \tag{3}$$

Now the law $PV = C$ is that of a rectangular hyperbola.

In section 1.22 this law was discussed under the heading Work and the Hyperbolic Process. There it was shown that,

$$\text{Work done} = PV \ln \frac{V_2}{V_1} \tag{4}$$

This, therefore, is the expression which will give the work done during an isothermal process on a gas.

From the characteristic equation,

$$PV = mRT \tag{5}$$

THE ISOTHERMAL PROCESS AND A GAS

Substituting equation (5) in (4)

$$\text{Work done} = mRT \ln \frac{V_2}{V_1} \qquad (6)$$

Applying the non-flow energy equation,

$$Q = \Delta U + W.$$

Since for an isothermal process T = constant, and by Joule's Law, the internal energy of a gas is a function of temperature only then, if T = constant, then there is no change of internal energy.

Hence, for an isothermal process,

$$\Delta U = 0$$

$\therefore$ The energy equation becomes,

$$Q = W \qquad (7)$$

$$= PV \ln \frac{V_2}{V_1} = mRT \ln \frac{V_2}{V_1} \qquad (8)$$

Example

A quantity of gas occupies a volume of $0{\cdot}4\ \text{m}^3$ at a pressure of $100\ \text{kN/m}^2$ and a temperature of 20°C. The gas is compressed isothermally to a pressure of $450\ \text{kN/m}^2$ and then expanded adiabatically to its initial volume. Determine, for this quantity of gas:

(a) the heat transferred during the compression,
(b) the change of internal energy during the expansion,
(c) the mass of gas.

Assume for this gas that $\gamma = 1{\cdot}4$ and $c_p = 1{\cdot}0\ \text{kJ/kg K}$.

(a) For the isothermal compression,

$$P_1V_1 = P_2V_2$$

$$\therefore\ V_2 = V_1\frac{P_1}{P_2} = 0{\cdot}4 \times \frac{100}{450} = \underline{0{\cdot}089\ \text{m}^3}$$

Now $Q = \Delta U + W$ and for an isothermal process on a gas $\Delta U = 0$

$$\therefore\ Q = W$$

$$= PV\ln r = PV\ln\frac{P_1}{P_2}$$

$$= 100 \times 0{\cdot}4 \times \ln\frac{100}{450} = -100 \times 0{\cdot}4 \times \ln\frac{450}{100}$$

$$= -40 \times \ln 4{\cdot}5 = -40 \times 1{\cdot}5$$

$= \underline{-60\ \text{kJ}}$. This is heat rejected.

Example continued.....

(b) For the adiabatic expansion,

$$P_2V_2^\gamma = P_3V_3^\gamma$$

$$\therefore\ P_3 = P_2\left(\frac{V_2}{V_3}\right)^\gamma = 450 \times \left(\frac{0{\cdot}089}{0{\cdot}4}\right)^{1\cdot4} = \frac{450}{4{\cdot}5^{1\cdot4}}$$

$$= \frac{450}{8{\cdot}21} = \underline{54{\cdot}8\ \mathrm{kN/m^2}}$$

Now $Q = \Delta U + W$ and for an adiabatic process $Q = 0$

$$\therefore\ 0 = \Delta U + W$$

or

$$\Delta U = -W = \frac{-(P_2V_2 - P_3V_3)}{\gamma - 1}$$

$$= \frac{-(450 \times 0{\cdot}089 - 54{\cdot}8 \times 0{\cdot}4)}{1{\cdot}4 - 1} = \frac{-(40{\cdot}05 - 21{\cdot}9)}{0{\cdot}4}$$

$$= \frac{-18{\cdot}15}{0{\cdot}4} = \underline{-45{\cdot}4\ \mathrm{kJ}}$$

This is a loss of internal energy.

Example continued…..

(c) $c_p - c_v = R$ and $c_p/c_v = \gamma$

from which, $c_v = c_p/\gamma$

$$\therefore\ (c_p - c_p/\gamma) = R = c_p\left(1 - \frac{1}{\gamma}\right)$$

$$\therefore\ R = 1{\cdot}0\left(1 - \frac{1}{1{\cdot}4}\right) = 1{\cdot}0(1 - 0{\cdot}714) = \underline{0{\cdot}286\ \text{kJ/kg K}}$$

$$P_1V_1 = mRT_1 \qquad \text{and} \qquad T_1 = (20 + 273)\ \text{K} = \underline{293\ \text{K}}$$

$$\therefore\ m = \frac{P_1V_1}{RT_1} = \frac{100 \times 10^3 \times 0{\cdot}4}{0{\cdot}286 \times 10^3 \times 293} = \underline{0{\cdot}477\ \text{kg}}$$

THE POLYTROPIC PROCESS OF A GAS

- **The polytropic process equation describes expansion and compression processes which include heat transfer.**
- A polytropic process is a thermodynamic process that obeys the relation:

$$PV^n = C\ldots\ldots\ldots\ldots(1)$$

- where p is the pressure, V is volume, n is the polytropic index, and C is a constant.

$$P_1V_1^n = P_2V_2^n \ldots\ldots\ldots\ldots\ldots.(2)$$

- Furthermore,

$$Work\ done = \frac{P_1V_1 - P_2V_2}{n-1}\ldots\ldots\ldots\ldots\ldots(3)$$

THE POLYTROPIC PROCESS OF A GAS

By the Characteristic Equation:

$$PV = mRT \ldots\ldots\ldots\ldots.(4)$$

Substituting (4) in (3):

$$Work\ Done = \frac{mR(T_1 - T_2)}{n-1} \ldots\ldots\ldots\ldots\ldots.(5)$$

❑Applying the Non-flow Energy Equation:

$$Q = \Delta U + W = (U_2 - U_1) + W$$

$$= mc_v(T_2 - T_1) + \frac{P_1V_1 - P_2V_2}{n-1} \ldots\ldots\ldots\ldots(6)$$

$$= mc_v(T_2 - T_1) + \frac{mR(T_1 - T_2)}{n-1} \ldots\ldots\ldots\ldots\ldots(7)$$

THE COMBINATIPON OF POLYTROPIC LAW $PV^n=C$ AND THE CHARACTERISTIC EQUATION OF A PERFACT GAS

- Having developed these expressions, it might be useful to note that, initially, it may be difficult to know which one to use for the solution of a particular problem:

$$\frac{P_1V_1}{T_1}=\frac{P_2V_2}{T_2}\ \ldots\ldots\ldots\ldots.(8)$$

- There is no hard and fast rule, but notice that requires five conditions of state to be known before solving the sixth. If five conditions are not known, another expression may become more appropriate such as:

$$\frac{P_1}{P_2}=\left(\frac{V_2}{V_1}\right)^n or\ \frac{T_1}{T_2}=\left(\frac{P_1}{P_2}\right)^{(n-1)/n}\ \ldots\ldots\ldots\ldots\ldots..(9)$$

- Finally:

$$\frac{T_1}{T_2}=\left(\frac{P_1}{P_2}\right)^{(n-1)/n}=\left(\frac{V_2}{V_1}\right)^{(n-1)}\ \ldots\ldots\ldots\ldots\ldots\ldots.(10)$$

Example 1

A gas whose original pressure and temperature were 300 kN/m² and 25 °C, respectively, is compressed according to the law $PV^{1.4}=C$ until its temperature becomes 180 °C. Determine the new pressure of the gas.

SOLUTION

It has been shown that for a polytropic compression, the relationship between pressure and temperature is

$$\frac{T_1}{T_2}=\left(\frac{P_1}{P_2}\right)^{(n-1)/n}$$

THE COMBINATIPON OF POLYTROPIC LAW $PV^n=C$ AND THE CHARACTERISTIC EQUATION OF A PERFACT GAS

Example 1 continued..

From this

$$\frac{P_1}{P_2} = \left(\frac{T_1}{T_2}\right)^{n/(n-1)}$$

$$\therefore \quad P_2 = P_1\left(\frac{T_2}{T_1}\right)^{n/(n-1)}$$

Now

$$T_1 = (25 + 273)\ \text{K} = 298\ \text{K}$$

and

$$T_2 = (180 + 273)\ \text{K} = 453\ \text{K}$$

Hence

$$P_2 = 300 \times \left(\frac{453}{298}\right)^{1.4/0.4}$$

$$= 300 \times 1.52^{3.5}$$

$$= 300 \times 4.33$$

$$= 1299\ \text{kN/m}^2 \quad \text{or} \quad 1.299\ \text{MN/m}^2$$

Example 2

A gas whose original volume and temperature were $0.015\ m^3$ and $285\ °C$, respectively, is expanded according to the law $PV^{1.35} = C$ until its volume is $0.09\ m^3$. Determine the new temperature of the gas.

SOLUTION

The relationship between volume and temperature during a polytropic expansion of a gas is

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{(n-1)} \quad \text{and} \quad T_1 = (285 + 273)\,\text{K} = \mathbf{558\ K}$$

$$\therefore \quad T_2 = T_1\left(\frac{V_1}{V_2}\right)^{(n-1)} = 558 \times \left(\frac{0.015}{0.09}\right)^{(1.35-1)}$$

$$= \frac{558}{6^{0.35}}$$

$$= \frac{558}{1.87}$$

$$= \mathbf{298.4\ K}$$

$$t_2 = 298.4 - 273 = \mathbf{25.4\ °C}$$

Example 3

0.675 kg of gas at 1.4 MN/m^2 and 280 °C is expanded to four times the original volume according to the law $PV^{1.3} = C$. Determine

(a) the original and final volume of the gas

(b) the final pressure of the gas

(c) the final temperature of the gas

Take R = 0.287 *kJ/kg K.*

(a)

Now $P_1V_1 = mRT_1$ and $T_1 = (280 + 273)\text{ K} = \mathbf{553\ K}$

$$\therefore \quad V_1 = \frac{mRT_1}{P_1} = \frac{0.675 \times 0.287 \times 10^3 \times 553}{1.4 \times 10^6}$$

$$= \mathbf{0.076\,5\ m^3}$$

The original volume is 0.076 5 m^3.
Since the gas is expanded to four times its original volume, then

$$V_2 = 4V_1 = 4 \times 0.076\,5 = \mathbf{0.306\ m^3}$$

The final volume is 0.306 m^3.

Example 3 Continued….

(b)

$$P_1 V_1^n = P_2 V_2^n$$

$$\therefore \quad P_2 = P_1 \left(\frac{V_1}{V_2}\right)^n = 1.4\left(\frac{1}{4}\right)^{1.3}$$

$$= \frac{1.4}{4^{1.3}}$$

$$= \frac{1.4}{6.06}$$

$$= \mathbf{0.231\ MN/m^2}$$

$$= \mathbf{231\ kN/m^2}$$

The final pressure is 231 kN/m^2.

(c)

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

$$\therefore \quad T_2 = \frac{P_2}{P_1}\frac{V_2}{V_1} T_1 = \frac{0.231}{1.4} \times 4 \times 553 = 365\ \mathrm{K}$$

$$t_2 = 365 - 273 = \mathbf{92\ ^\circ C}$$

The final temperature is 92 °C.

Example 4

0.25 kg of air at a pressure of 140 kN/m² occupies 0.15 m³ and from this condition it is compressed to 1.4 MN/m² according to the law PV^1.25 = C. Determine

(a) the change of internal energy of the air

(b) the work done on or by the air

(c) the heat received or rejected by the air

Take $c_p = 1.005$ *kJ/kg K,* $c_v = 0.718$ *kJ/kg K*

(a)

Now $c_p - c_v = R$

$$\therefore \quad R = 1.005 - 0.718 = \mathbf{0.287\ kJ/kg\ K}$$

Also $P_1 V_1 = mRT_1$

$$\therefore \quad T_1 = \frac{P_1 V_1}{mR} = \frac{140 \times 10^3 \times 0.15}{0.25 \times 0.287 \times 10^3} = \mathbf{292.7\ K}$$

Also $$\frac{T_1}{T_2} = \left(\frac{P_1}{P_2}\right)^{(n-1)/n}$$

$$\therefore \quad T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{(n-1)/n} = 292.7 \times \left(\frac{1.4 \times 10^6}{140 \times 10^3}\right)^{0.25/1.25}$$

$$= 292.7 \times 10^{1/5} = 292.7 \times \sqrt[5]{10}$$

$$= 292.7 \times 1.585$$

$$= \mathbf{463.9\ K}$$

Example 4 continued…..

Change of internal energy

$$\Delta U = U_2 - U_1 = mc_v(T_2 - T_1)$$
$$= 0.25 \times 0.718 \times (463.9 - 292.7)$$
$$= 0.25 \times 0.718 \times 171.2$$
$$= \mathbf{30.73\ kJ}$$

This is positive, so it is a gain of internal energy to the air.

(b)

$$\text{Work done, W} = \frac{mR(T_1 - T_2)}{n-1} = \frac{0.25 \times 0.287 \times (292.7 - 463.9)}{1.25 - 1}$$
$$= \frac{0.25 \times 0.287 \times (-171.2)}{0.25}$$
$$= \mathbf{-49.1\ kJ}$$

This is negative, so the work is done on the air.

(c)

$$Q = \Delta U + W$$
$$\therefore \quad Q = 30.73 - 49.1 = \mathbf{-18.37\ kJ}$$

This is negative, so the heat is rejected by the air.

THE NON FLOW ENERGY EQUATION AND THE POLYTROPIC LAW $PV^n = C$

$$Q = mc_v(T_2 - T_1) + \frac{mR(T_1 - T_2)}{n - 1}$$

$$= m\left(c_v + \frac{R}{1 - n}\right)(T_2 - T_1)$$

- **Using the relationship** $c_p - c_v = R \text{ and } \gamma = \frac{c_p}{c_v}$ **, it yields** $Q = m\left(c_v + \frac{c_p - c_v}{1 - n}\right)(T_2 - T_1)$

$$Q = m\left(1 + \frac{\gamma - n}{1 - n}\right)c_v(T_2 - T_1) = m\left(\frac{1 - n + \gamma - 1}{1 - n}\right)c_v(T_2 - T_1)$$

$$Q = m\left(\frac{\gamma - n}{1 - n}\right)c_v(T_2 - T_1)$$

- The last equation gives the heat transfer for polytropic process between states 1 and 2.

Furthermore, the **Polytropic Specific Heat Capacity**

$$c_n = c_v\left(\frac{\gamma - n}{1 - n}\right)$$

THE NON FLOW ENERGY EQUATION AND THE POLYTROPIC LAW $PV^n = C$

or

$$Q = \frac{\gamma - n}{\gamma - 1} \times \text{Polytropic work} \tag{9}$$

Now from this equation it is possible to examine what happens to the heat received or rejected during an expansion or compression of a gas if the value of the index n is varied. For a compression the work done is negative. In this case then,

If $n > \gamma$, then $\frac{\gamma - n}{\gamma - 1}$ is negative and hence Q is positive, i.e. heat is received.

If $n < \gamma$, then $\frac{\gamma - n}{\gamma - 1}$ is positive and hence Q is negative, i.e. heat is rejected.

For an expansion the work done is positive. In this case then,

If $n > \gamma$, then $\frac{\gamma - n}{\gamma - 1}$ is negative and hence Q is negative, i.e. heat is rejected.

If $n < \gamma$, then $\frac{\gamma - n}{\gamma - 1}$ is positive and hence Q is positive, i.e. heat is received.

THE NON FLOW ENERGY EQUATION AND THE POLYTROPIC LAW $PV^n = C$

Note that,

If $n = \gamma$, then $\dfrac{\gamma - n}{\gamma - 1} = 0$ and hence $Q = 0$, i.e. this is the adiabatic case.

If $n = 1$, then $\dfrac{\gamma - n}{\gamma - 1} = 1$ and hence $Q =$ Work done, i.e. this is the isothermal case.

Note that this analysis has shown how to control the value of the index n. The control of the index n is obtained by the extent to which heat is allowed, or not allowed, to pass out of, or into, the gas during the compression or expansion.

Example

A gas expands according to the law $PV^{1\cdot3} = C$ from a pressure of 1 MN/m^2 and a volume 0·003 m^3 to a pressure of 0·1 MN/m^2. How much heat was received or rejected by the gas during this process? Determine, also, the polytropic specific heat capacity.

Take, $\gamma = 1\cdot4$, $c_v = 0\cdot718$ kJ/kg K

Now, $P_1 V_1^n = P_2 V_2^n$

$$\therefore \quad V_2 = V_1 \left(\frac{P_1}{P_2}\right)^{1/n} = 0\cdot003 \times \left(\frac{1}{0\cdot1}\right)^{1/1\cdot3} = 0\cdot003 \times 10^{1/1\cdot3}$$

$$= 0\cdot003 \times 5\cdot88 = \underline{0\cdot0176\ \text{m}^3}$$

Heat received or rejected

$$= Q = \frac{(\gamma - n)}{(\gamma - 1)} \times \text{Work done}$$

$$= \frac{(\gamma - n)}{(\gamma - 1)} \times \frac{(P_1 V_1 - P_2 V_2)}{n - 1}$$

$$= \frac{(1\cdot4 - 1\cdot3)}{(1\cdot4 - 1)} \times \frac{(1 \times 0\cdot003 - 0\cdot1 \times 0\cdot0176)}{1\cdot3 - 1}$$

$$= \frac{0\cdot1}{0\cdot4} \times \frac{(0\cdot003 - 0\cdot00176)}{0\cdot3}$$

$$= \frac{1}{4} \times \frac{0\cdot00124}{0\cdot3} = \frac{0\cdot00124}{1\cdot2} = \underline{0\cdot00103\ \text{MJ}} = \underline{1\cdot03\ \text{kJ}}$$

This is positive and hence heat is received by the gas.

$$c_n = c_v \frac{(\gamma - n)}{(n - 1)} = 0\cdot718 \times \frac{(1\cdot4 - 1\cdot3)}{(1\cdot3 - 1)} = 0\cdot718 \times \frac{0\cdot1}{0\cdot3}$$

$$c_n = \underline{0\cdot239\ \text{kJ/kg K}}$$

AIR - STEAM MIXTURES

- The problem of air – steam mixtures is solved by the Dalton's Law of Partial Pressures.

❑ The **total pressure** in a container is the **sum of the pressure each gas** would exert if it were alone in the container.

❑ The total pressure is the sum of the partial pressures.

$$P_{Total} = P_1 + P_2 + P_3 + P_4 + P_5 \ldots$$

- **(For each gas P = nRT/V)**
- **Hence for an Air – Steam Mixure:**

$$P_{Mixture} = P_{Air} + P_{Steam}$$

Example

A container is filled with a mixture of air and wet steam at a temperature of 39°C and a pressure of 100 kN/m² (1 bar). The temperature is then raised to 120·2°C, the steam remaining wet. Determine:

(a) the initial partial pressures of the steam and air,
(b) the final partial pressures of the steam and air,
(c) the total pressures in the container after heating.

(a) From steam tables, the pressure of wet steam at

$$39°\text{C} = \underline{7\ \text{kN/m}^2} = \text{initial partial pressure of the steam.}$$

From this, by Dalton's law of partial pressures, initial partial pressure of the air $= 100 - 7 = \underline{97\ \text{kN/m}^2}$

(b) At 120·2°C, the pressure of wet steam

$$= \underline{0{\cdot}20\ \text{MN/m}^2} = \underline{200\ \text{kN/m}^2}$$

$=$ final partial pressure of the steam.

For air, $P_1/T_1 = P_2/T_2$ for a constant volume process.

$$\therefore\ P_2 = P_1 \frac{T_1}{T_2} = 97 \times \frac{393{\cdot}2}{312} = \underline{122{\cdot}24\ \text{kN/m}^2}$$

$=$ final partial pressure of the air.

(c) Total pressure after heating

$= 200 + 122{\cdot}24$

$= \underline{322{\cdot}24\ \text{kN/m}^2}$

Example

A vacuum gauge on a condenser reads 660 mm Hg and the barometer height is 765 mm Hg. Steam enters the condenser with a dryness fraction of 0·8 and has a temperature of 41·5°C. Determine the partial pressures of the air and steam in the condenser. If the steam is condensed at the rate of 1 500 kg/h, determine the mass of air which will be associated with this steam. Take R for air = 0·29 kJ/kg K.

$$\text{Absolute pressure in condenser} = 765 - 660$$
$$= \underline{105 \text{ mm Hg}}$$
$$= \underline{14 \text{ kN/m}^2}$$

At 41·5°C, from steam tables,

$$\text{Partial pressure of the steam} = \underline{8 \text{ kN/m}^2}$$

$$\therefore \text{ Partial pressure of the air} = 14 - 8 = \underline{6 \text{ kN/m}^2}$$

At 41·5°C and with a dryness fraction 0·8, specific volume of the steam

$$= xv_g = 0{\cdot}8 \times 18{\cdot}1 = \underline{14{\cdot}48 \text{ m}^3/\text{kg}}$$

The air associated with 1 kg of the steam will occupy this same volume.

For air, $PV = mRT$ and $T = (41{\cdot}5 + 273)\text{ K} = \underline{314{\cdot}5 \text{ K}}$

$$\therefore m = \frac{PV}{RT} = \frac{6 \times 14{\cdot}48}{0{\cdot}29 \times 314{\cdot}5} = \frac{86{\cdot}88}{91{\cdot}2}$$

$$= \underline{0{\cdot}953 \text{ kg/kg steam}}$$

$$\therefore \text{ mass of air/h} = 0{\cdot}953 \times 1\,500 = \underline{1\,429{\cdot}5 \text{ kg}}$$

Example

A cylinder contains a mixture of air and wet steam at a pressure of 130 kN/m^2 and a temperature of 75·9°C. The dryness fraction of the steam is 0·92. The air–steam mixture is then compressed to one-fifth of its original volume the final temperature being 120·2°C. Determine:

(a) the final pressure in the cylinder;
(b) the final dryness fraction of the steam.

(a) At 75·9°C,

Partial pressure of the wet steam = $\underline{40\ kN/m^2}$

$\therefore$ Partial pressure of the air = 130 − 40 = $\underline{90\ kN/m^2}$

Example continued....

Specific volume of the wet steam at 75·9°C and 0·92 dry $= 0·92 \times 3·99 = \underline{3·67 \text{ m}^3/\text{kg}}$

For air,

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2} \qquad \text{and} \qquad T_1 = (75·9 + 273)\text{ K} = \underline{348·9\text{ K}}$$

$$T_2 = (120·2 + 273)\text{ K} = \underline{393·2\text{ K}}$$

$$\therefore \quad P_2 = P_1\frac{V_1}{V_2}\frac{T_2}{T_1} = 90 \times 5 \times \frac{393·2}{348·9} = \underline{507·1\text{ kN/m}^2}$$

= final pressure of the air

For the steam at 120·2°C,

Pressure $= \underline{200\text{ kN/m}^2}$

$\therefore$ Final pressure in the cylinder $= 507·1 + 200$
$= \underline{707·1\text{ kN/m}^2}$

(b) Final specific volume of steam in the cylinder

$$= \frac{3·67}{5} = \underline{0·734\text{ m}^3/\text{kg}}$$

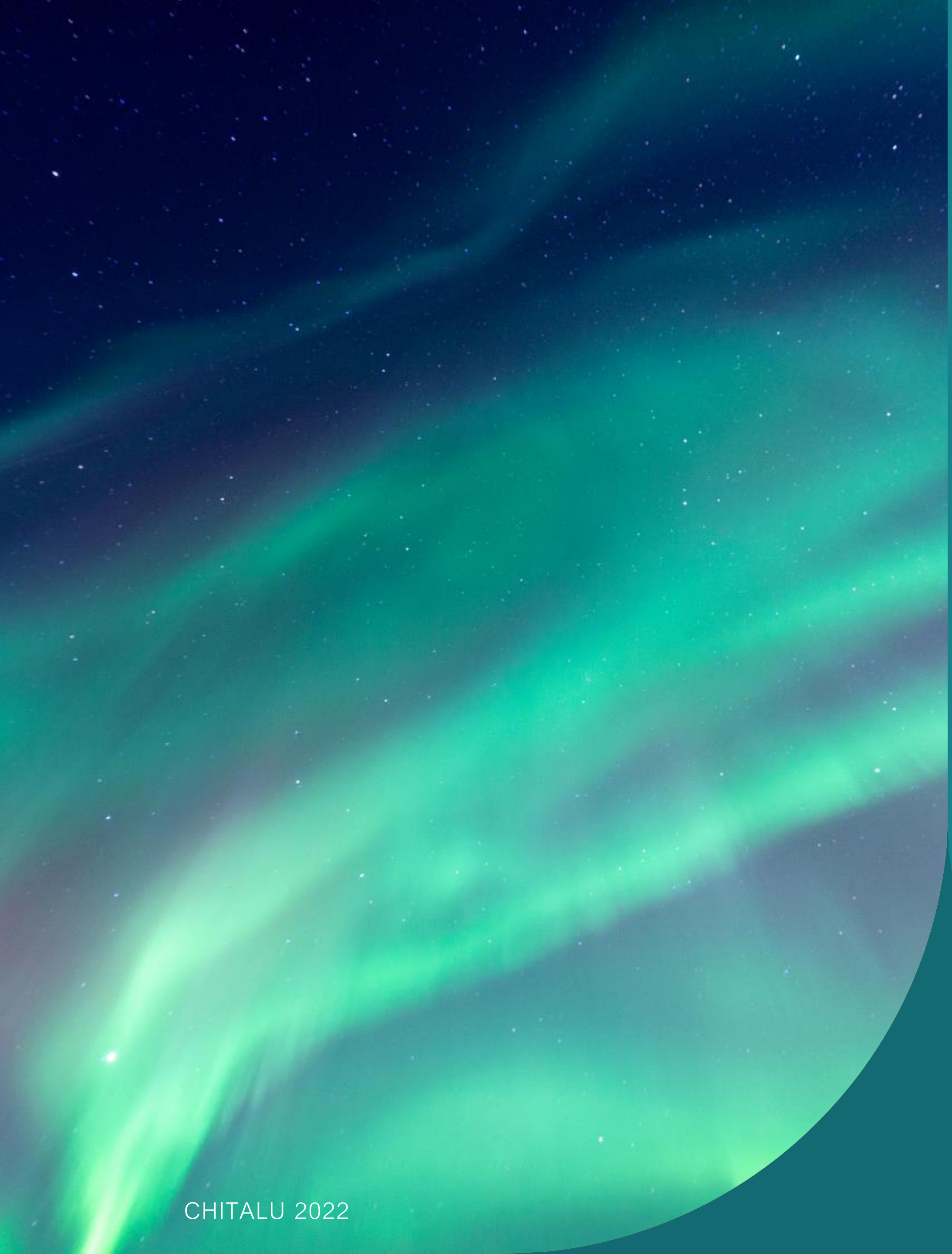
At 200 kN/m²,

$$v_g = 0·885\text{ m}^3/\text{kg}$$

$\therefore$ Final dryness fraction of the steam $= \dfrac{0·734}{0·885}$
$= \underline{0·83}$

“When a body acts upon another body, it is either directly or through some intermediate body; this intermediate body in general is what we call a **machine**”

Lazare Carnot



Thank You

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