ENG 3165 LECTURE 7

THERMODYNAMICS COMPONENT

Ideal Gas Power Cycles – The Carnot Cycle

Introduction

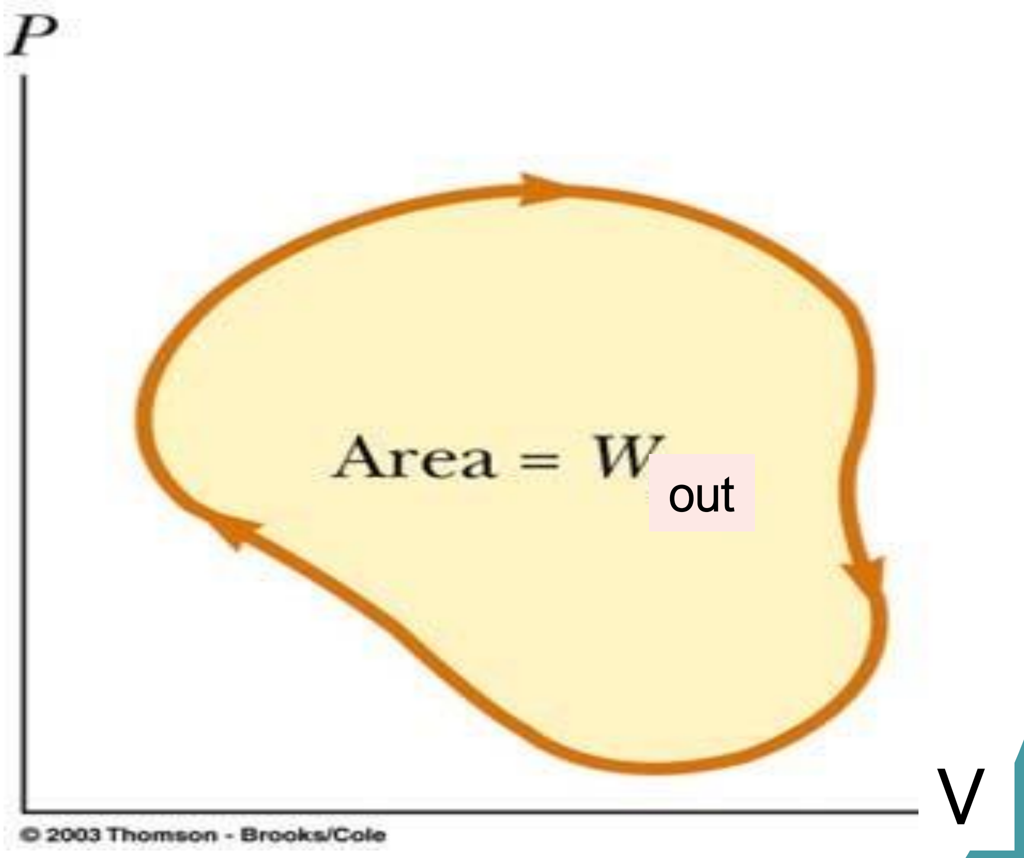
- This lecture provides an introduction to thermodynamic cycles as **a linked sequence of thermodynamic processes that involve the transfer of heat and work into and out of the system**.
- **The Carnot Engine** as the **ideal thermodynamic cycle** is explained.

THE HEAT ENGINE

- A **heat engine** is a device that takes in energy by heat and, operating in a cyclic process, expels a fraction of that energy by means of work
- It produces work, which can be mechanical, electrical, chemical, etc., in nature
- A heat engine carries some working substance through a cyclical process
- Most heat engines use either a vapor or a gas as the internal energy transfer medium
- We call the device that actually produces the heat engine's output work the **prime mover.** A prime mover can be a reciprocating piston-cylinder steam engine, a steam turbine, an internal combustion engine, and so forth.

THE HEAT ENGINE

- The working substance absorbs energy by heat from a high temperature energy reservoir **(Q_H)**
- Work is done by the engine (**W_{out}**)
- Energy is expelled as heat to a lower (colder) temperature reservoir **(Q_L)**
- Since it is a cyclical process, **$\Delta E_{int} = 0$**



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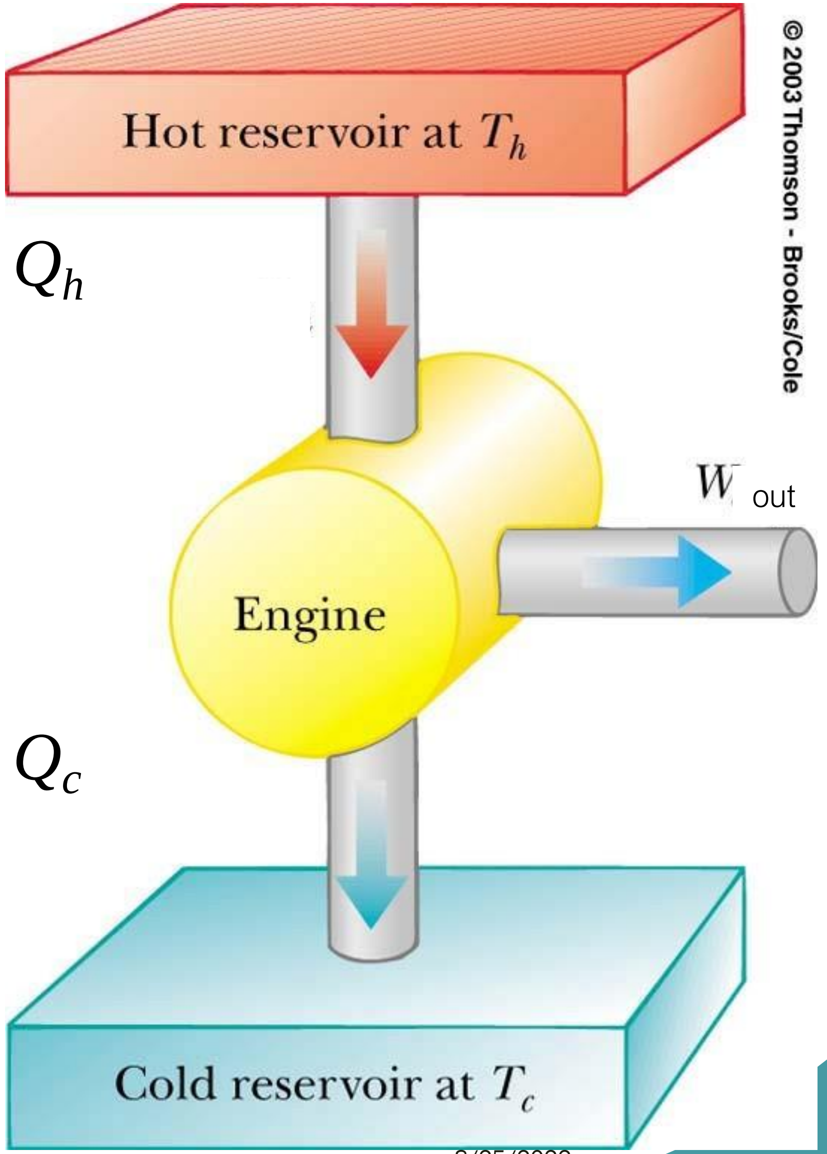
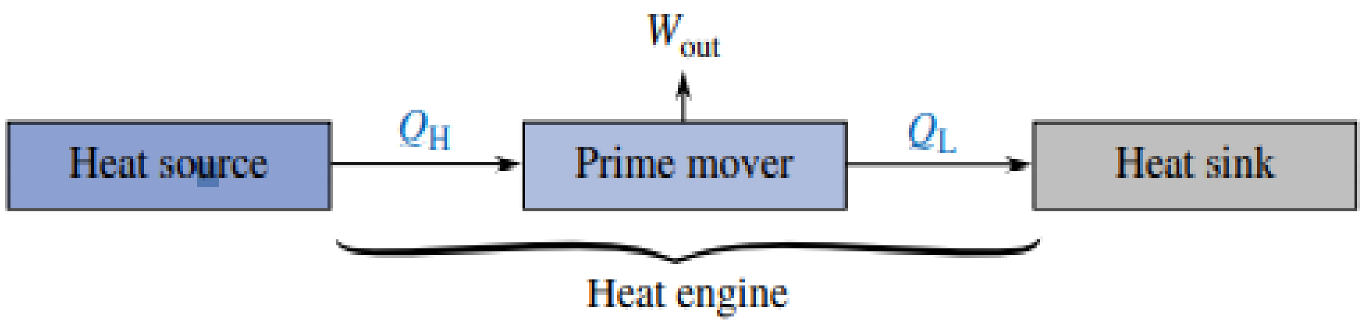
THE HEAT ENGINE

- $\Delta E_{int} = 0$
- $Q_{net} = W_{out}$
- The work done by the engine equals the net energy absorbed by the engine
- Q_c can be thought of the heat loss to the environment, that is Q_h is not transformed 100% to work



TYPICAL HEAT ENGINE CHARCTERISTICS

Heat Sources	Heat Sinks	Working Fluid	Work Output Prime Movers	Cycle Types	Uses
Combustion	Atmosphere	Gas	Engine	Power	Transportation
Nuclear	Oceans, lakes	Vapor	Turbine	Refrigeration	Generate electricity
Atmosphere	Rivers	Vapor	Reversed engine	Heat pump	Heating and cooling
Ocean	Groundwater	Vapor	Solid state (e.g., thermoelectric)	Power	Generate electricity



HEAT AND WORK TRANSFER DURING A CYCLE OF A HEAT ENGINE

If a substance passes through a series of processes such that it is eventually returned to its original state then the substance is said to have been taken through a cycle (see section 1.13).

During a cycle there will be some heat transfer and some work transfer to and from the substance.

Since after performing a cycle, the substance is returned to its original state, then, by the first law of thermodynamics,

$$\oint Q = \oint W \tag{1}$$

Thus, for a cycle, the net work transfer can be determined by an analysis of the net heat transfer, or,

$$\text{Net work done} = \text{Net heat received} - \text{Net heat rejected} \tag{2}$$

HEAT AND WORK TRANSFER DURING A CYCLE OF A HEAT ENGINE

Theoretical and Actual Thermal Efficiency

The ratio, $\dfrac{\text{Net work done}}{\text{Net heat received}}$

is called the *thermal efficiency* or,

$$\text{Thermal } \eta = \frac{\oint W}{\oint Q} \qquad (3)$$

where $\oint W$ = Net work done, $\oint Q$ = Net heat received.

Note also that, because the area under a process illustrated on a pressure–volume graph is equal to the work done, then, the net area of a pressure–volume diagram of a cycle is equal to the net work of the cycle.

This, therefore, gives another method by which the net work of a cycle can be determined.

HEAT AND WORK TRANSFER DURING A CYCLE OF A HEAT ENGINE

Theoretical and Actual Thermal Efficiency

The equation,

$$\text{Thermal } \eta = \frac{\oint W}{\oint Q}$$

gives the *theoretical* or *ideal* thermal efficiency. The *actual* thermal efficiency of a practical cycle is given by the equation,

$$\text{Actual thermal } \eta = \frac{\text{Actual work done}}{\text{Thermal energy from fuel}} \qquad (4)$$

This is always less than the theoretical thermal efficiency.

A practical cycle is carried out in an engine or turbine and will incur many losses which will include heat transfer loss, fuel combustion loss, non-uniform energy distribution in the working substance, friction, leakage, the need to keep temperatures within practical working limits, the running of auxiliary equipment such as pumps, alternators, valve gear and cooling equipment.

HEAT AND WORK TRANSFER DURING A CYCLE OF A HEAT ENGINE

Relative Efficiency and Work Ratio

The ratio of the actual thermal efficiency and the ideal thermal efficiency is called the *relative efficiency* or *efficiency ratio*, thus,

$$\text{Relative efficiency} = \frac{\text{Actual thermal efficiency}}{\text{Ideal thermal efficiency}} \tag{5}$$

Another useful concept is the consideration of a cycle is that of the *work ratio*. This is defined as:

$$\text{Work ratio} = \frac{\text{Net work done}}{\text{Positive work done}} = \frac{\oint W}{\text{Positive work done}} \tag{6}$$

where, net work done = positive work done − negative work done = $\oint W$.

Note that, from equation (6), if the negative work is reduced then the work ratio → 1.

HEAT AND WORK TRANSFER DURING A CYCLE OF A HEAT ENGINE

Relative Efficiency and Work Ratio

A cycle with good ideal thermal efficiency together with a good work ratio suggests good overall efficiency potential in a practical power producing plant using the cycle.

Work ratio can give comparative indication of plant size. A plant with a low work ratio would suggest that the work components of the plant are larger when compared with a plant which has a higher work ratio and similar power output.

HEAT AND WORK TRANSFER DURING A CYCLE OF A HEAT ENGINE

Specific Fuel Consumption

Another form of comparison between cycles is by means of the *specific steam or fuel consumption.*

In the case of steam plant,

$$\text{Specific steam consumption} = \frac{\text{Mass flow of steam in kg/h}}{\text{Power output in kW}} \qquad (7)$$

This gives the mass of steam used per unit power output in kg/kW h.

In the case of internal combustion engines such as the gas turbine and the petrol or diesel engine the specific fuel consumption is used where,

$$\text{Specific fuel consumption} = \frac{\text{Mass of fuel used in kg/h}}{\text{Power output in kW}} \qquad (8)$$

HEAT AND WORK TRANSFER DURING A CYCLE OF A HEAT ENGINE

Specific Work Output

This gives the mass of fuel used per unit power output in kg/kW h.

Thus a cycle which has a lower specific steam or fuel consumption indicates that it has better energy conversion performance than a cycle with a higher specific steam or fuel consumption.

The specific steam or fuel consumption can also be determined from a knowledge of the specific work output.

If

$$\text{Specific work output} = w \frac{\text{kJ}}{\text{kg (steam or fuel)}}$$

then,

$$\frac{1}{w}\frac{\text{kg}}{\text{kJ}} = \frac{1}{w}\frac{\text{kg}}{\text{kW s}} = \frac{3\,600}{w}\frac{\text{kg}}{\text{kW h}}$$

$$= \text{specific steam or fuel consumption} \qquad (9)$$

(Note that 1 kW = 1 kJ/s. ∴ 1 kJ = 1 kW s.)

THE CARNOT ENGINE

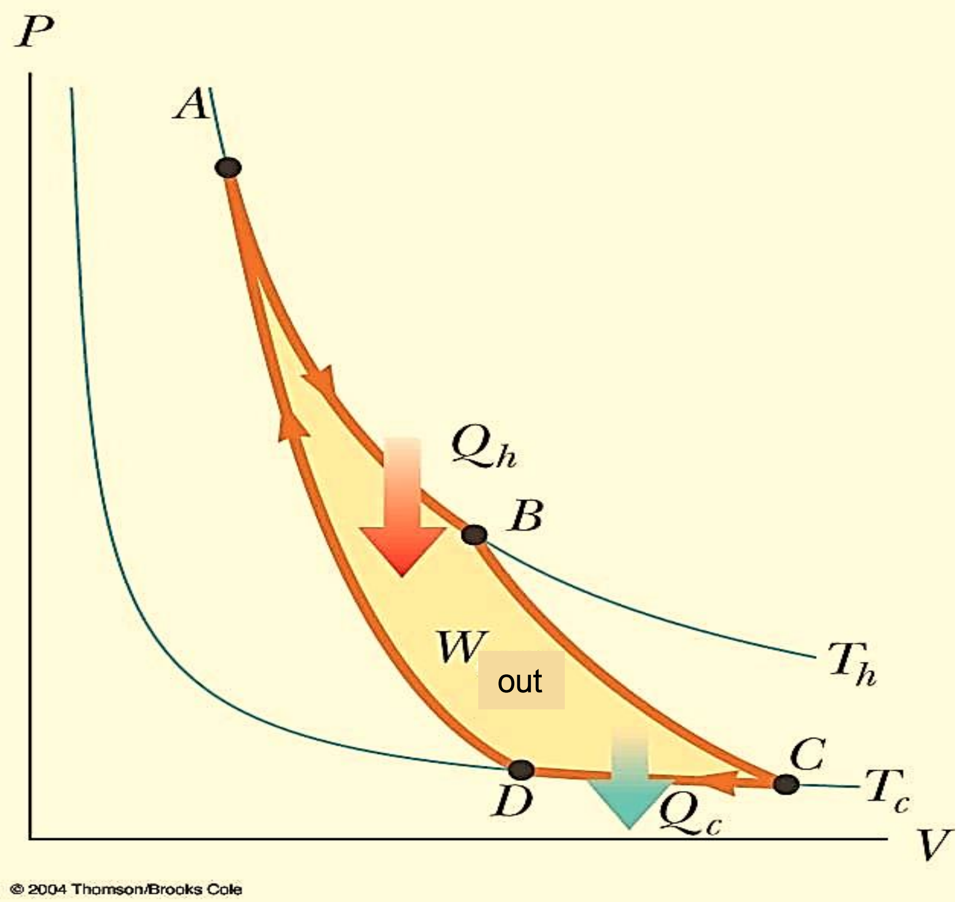
- A theoretical engine developed by **Sadi Carnot**
- A heat engine operating in an **ideal, reversible cycle** (now called a *Carnot cycle*) between **two reservoirs is the most efficient engine possible**
- This sets an **upper limit** on the efficiencies of all other engines
- **No real heat engine operating between two energy reservoirs can be more efficient** than a Carnot engine operating between the same two reservoirs
- All real engines are **less efficient** than a Carnot engine because they do not operate through a reversible cycle

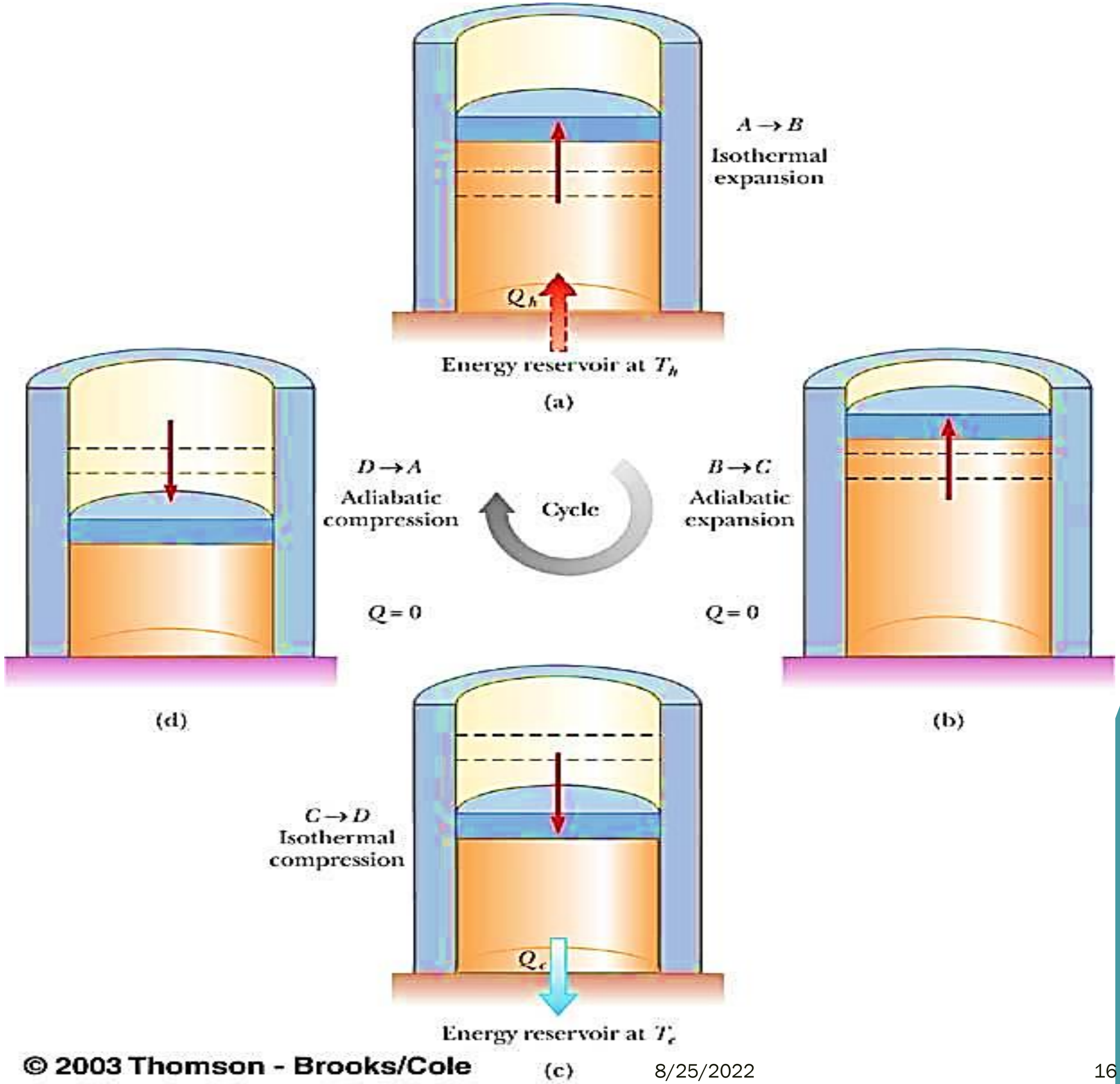
Sadi Carnot

THE CARNOT CYCLE

$\Delta E_{int} = 0$ for the entire cycle

$W_{out} = |Q_h| - |Q_c|$





THE CARNOT CYCLE

- The Carnot cycle was introduced as the most efficient heat engine that operate between two fixed temperatures T_H and T_L.
- Carnot showed that the efficiency of the engine depends **only** on the temperatures of the reservoirs
- The thermal efficiency of Carnot cycle is given by

$$\eta_{th,\,Carnot} = 1 - \frac{T_L}{T_H}$$

- Upon derivation the performance of the real cycle is often measured in terms of its thermal efficiency.:

$$\boxed{\eta_{th} = \frac{W_{net}}{Q_{in}}}$$

THE CARNOT CYCLE

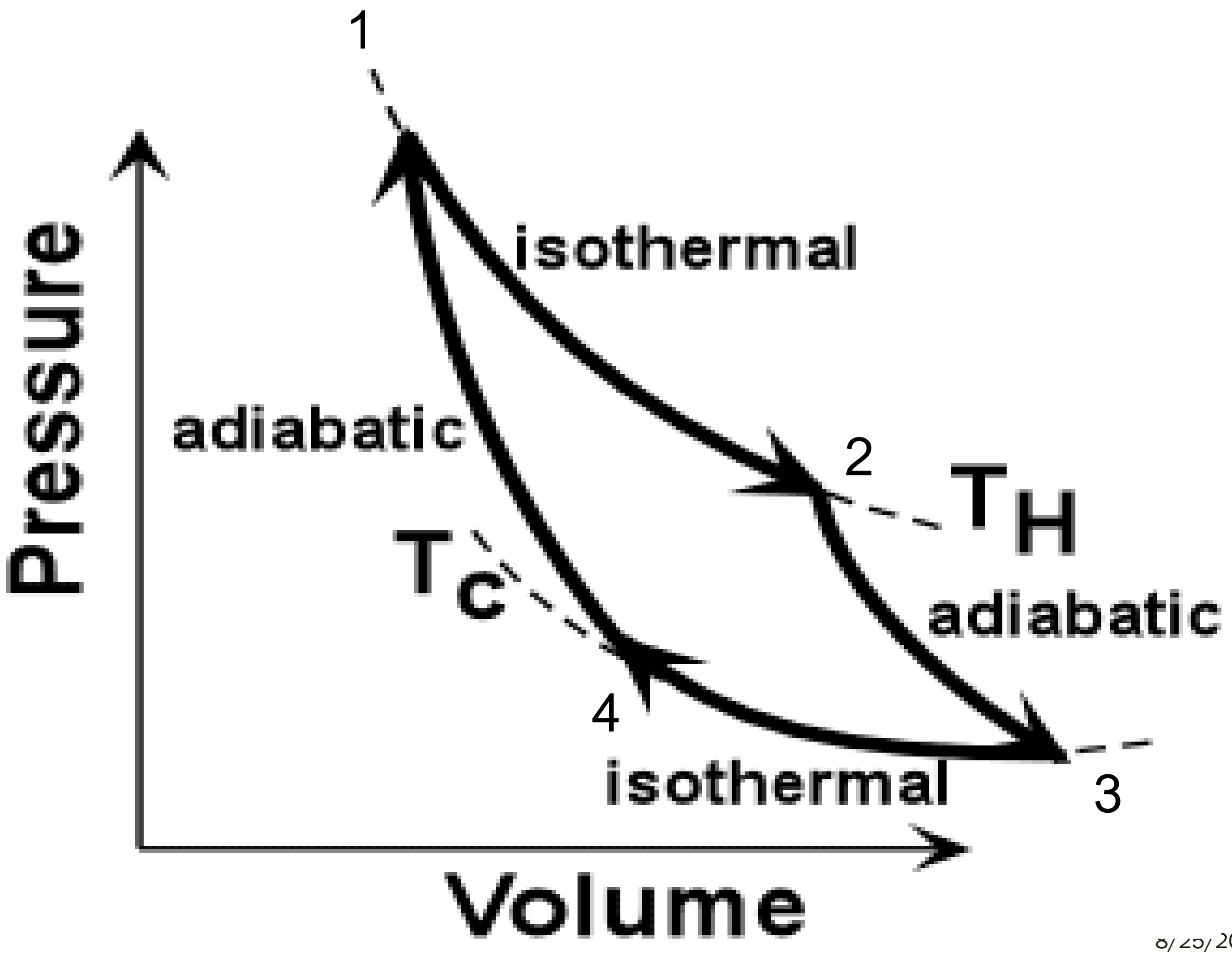
- Temperatures must be in Kelvins
- All Carnot engines operating between the same two temperatures will have the same efficiency
- Efficiency is 0 if $T_H = T_L$
- Efficiency is 100% only if $T_L = 0$
- Such reservoirs are not available, as the absolute zero temperature cannot be reached
- Efficiency is always less than 100%

THE CARNOT CYCLE

- The efficiency increases as T_L is lowered and as T_H is raised
- In most practical cases, T_L is near room temperature, 300 K
- So generally T_H is raised to increase efficiency
- Theoretically, a Carnot-cycle heat engine can run in **reverse**
- This would constitute the most **effective heat pump** available. This would determine the maximum possible COPs for a given combination of hot and cold reservoirs

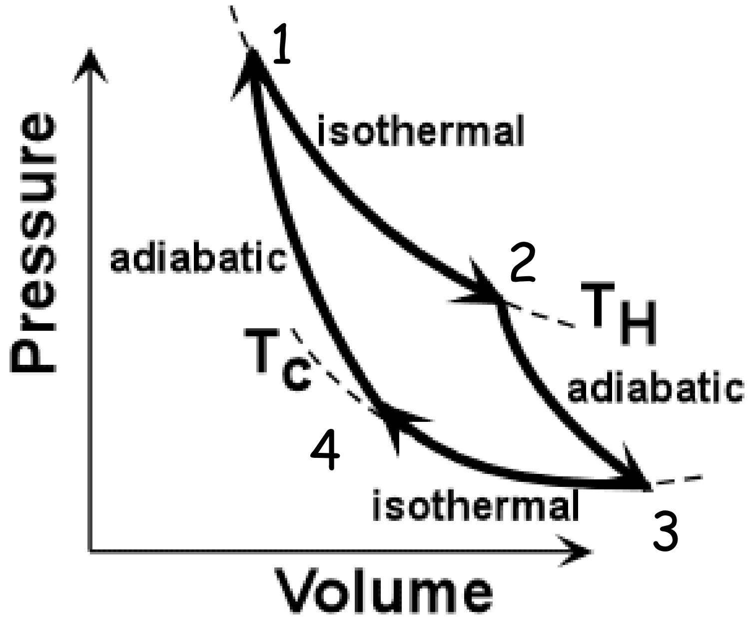
THE CARNOT CYCLE

Cycle made with reversible isothermal and adiabatic processes.



THE CARNOT CYCLE

- Where in the cycle is heat absorbed?



A. At point 1

B. Between 1 and 2

C. Between 4 and 1

- ❑ To absorb heat, we need a process between two points
- ❑ No heat transfer between 4 and 1 (adiabatic)
- ❑ Between 1 and 2, temperature is always T_H

$$\Delta U_{1\to 2} = 0$$

$$Q_{1\to 2} = W_{1\to 2} > 0$$

THE CARNOT CYCLE

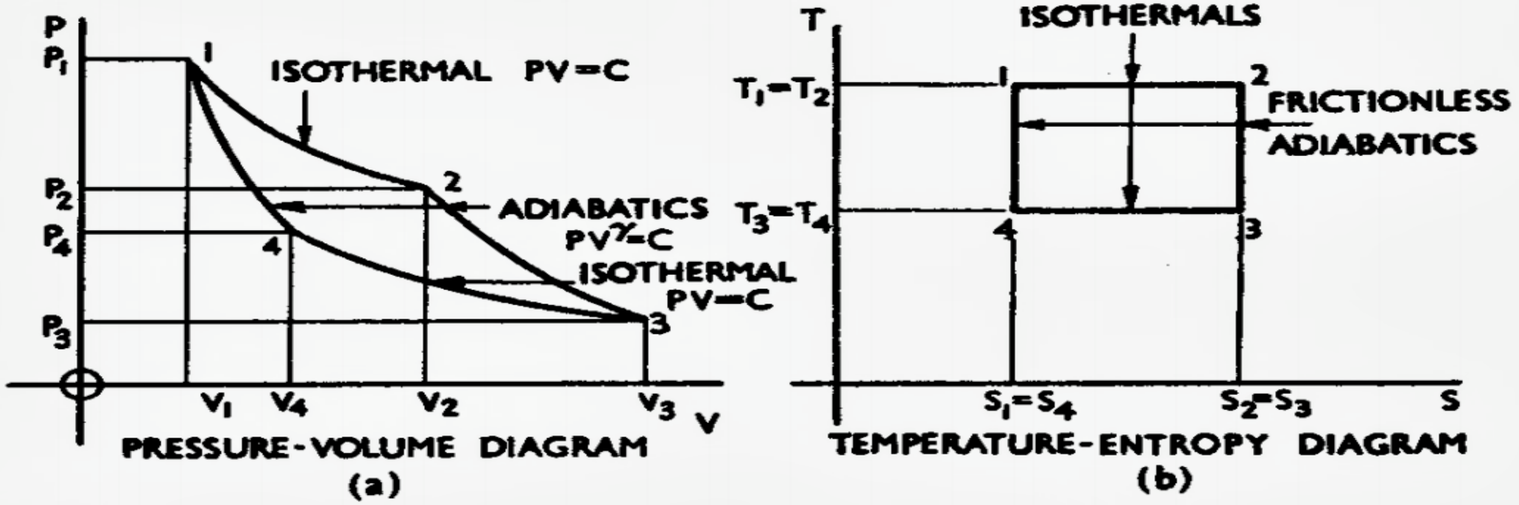


Fig. 13.13

$$\text{Work done} = P_1 V_1 \ln \frac{V_2}{V_1} = mRT_1 \ln \frac{V_2}{V_1}$$

For the isothermal, $Q = W$,

$$\therefore \ \text{Heat received} = mRT_1 \ln \frac{V_2}{V_1}$$

THE CARNOT CYCLE

Process 1-2

It will be observed that the cycle is made up of four reversible processes. The processes are as follows:

1–2 Isothermal expansion.

Pressure falls from P_1 to P_2.
Volume increases from V_1 to V_2.
Temperature remains constant at $T_1 = T_2$.

THE CARNOT CYCLE

Process 2-3

2–3 Adiabatic expansion.

Pressure falls from P_2 to P_3.
Volume increases from V_2 to V_3.
Temperature falls from T_2 to T_3.

$$\text{Work done} = \frac{P_2V_2 - P_3V_3}{(\gamma - 1)} = \frac{mR(T_2 - T_3)}{(\gamma - 1)}$$

For the adiabatic, $Q = 0$,

$\therefore$ no heat transfer during this process

THE CARNOT CYCLE

Process 3-4

3–4 Isothermal compression.

Pressure increases from P_3 to P_4.
Volume reduced from V_3 to V_4.
Temperature remains constant at $T_3 = T_4$.

$$\text{Work done} = P_3 V_3 \ln \frac{V_4}{V_3} = -P_3 V_3 \ln \frac{V_3}{V_4}$$

$$= -mRT_3 \ln \frac{V_3}{V_4}$$

For the isothermal, $Q = W$,

$$\therefore \text{ Heat rejected} = mRT_3 \ln \frac{V_3}{V_4}$$

THE CARNOT CYCLE

Process 4-1

4–1 Adiabatic compression.

Pressure increases from P_4 to P_1.
Volume reduced from V_4 to V_1.
Temperature increases from T_4 to T_1.

$$\text{Work done} = \frac{P_4V_4 - P_1V_1}{(\gamma - 1)} = -\frac{(P_1V_1 - P_4V_4)}{(\gamma - 1)}$$

$$= -\frac{mR(T_1 - T_4)}{(\gamma - 1)}$$

For the adiabatic $Q = 0$.

$\therefore$ no heat transfer during this process.

Note that this process returns the gas to its original state at 1.

THE CARNOT CYCLE

Work Done

The net work done during this cycle may be determined by summing the areas beneath the various processes taking the expansion as positive areas and compressions as negative areas. Thus,

$$
\begin{aligned}
\text{Net work done/cycle} &= \oint W \\
&= \text{area under } 1\text{–}2 + \text{area under } 2\text{–}3 \\
&\quad - \text{area under } 3\text{–}4 - \text{area under } 4\text{–}1 \\
&= \text{area } 1\ 2\ 3\ 4 \\
&= \text{area enclosed by cycle}
\end{aligned}
$$

THE CARNOT CYCLE

Work Done

or

$$\oint W = mRT_1 \ln\frac{V_2}{V_1} + \frac{mR(T_2 - T_3)}{(\gamma - 1)}$$

$$-mRT_3 \ln\frac{V_3}{V_4} - \frac{mR(T_1 - T_4)}{(\gamma - 1)} \qquad (1)$$

Now $T_1 = T_2$ and $T_3 = T_4$, from the isothermals.

$$\therefore \frac{mR(T_2 - T_3)}{(\gamma - 1)} = \frac{mR(T_1 - T_4)}{(\gamma - 1)}$$

Hence, from equation (1),

$$\oint W = mRT_1 \ln\frac{V_2}{V_1} - mRT_3 \ln\frac{V_3}{V_4} \qquad (2)$$

THE CARNOT CYCLE

Now for the adiabatic 1–4,

$$\frac{T_1}{T_4} = \left(\frac{V_4}{V_1}\right)^{(\gamma-1)} \qquad (3)$$

for the adiabatic 2–3,

$$\frac{T_2}{T_3} = \left(\frac{V_3}{V_2}\right)^{(\gamma-1)} \qquad (4)$$

But $T_1 = T_2$ and $T_3 = T_4$,

$$\therefore \quad \frac{T_1}{T_4} = \frac{T_2}{T_3} \qquad (5)$$

Hence, from equations (3) and (4),

$$\frac{V_4}{V_1} = \frac{V_3}{V_2} \quad \text{or} \quad \frac{V_2}{V_1} = \frac{V_3}{V_4} \qquad (6)$$

Substituting equation (6) in equation (2),

$$\oint W = mR \ln \frac{V_2}{V_1} (T_1 - T_3) \qquad (7)$$

THE CARNOT CYCLE

This is positive work done and this is always the case if the processes of a cycle proceed in a clockwise direction. Net external work can thus be obtained from such cycles.

If the processes proceed in an anticlockwise direction then the work done is negative, in which case equation (7) now becomes,

$$\boxed{\oint W = -mR \ln \frac{V_2}{V_1} (T_1 - T_3)} \qquad (8)$$

Negative work means that net external work must be put in to carry out such cycles.

Now

$$\text{Thermal } \eta = \frac{\text{Heat received} - \text{Heat rejected}}{\text{Heat received}}$$

$\therefore$ from the analysis given above,

$$\text{Thermal } \eta = \frac{mRT_1 \ln (V_2/V_1) - mRT_3 \ln (V_3/V_4)}{mRT_1 \ln (V_2/V_1)}$$

$$= \frac{mR \ln(V_2/V_1)(T_1 - T_3)}{mR \ln (V_2/V_1) T_1}$$

THE CARNOT CYCLE

since $V_2/V_1 = V_3/V_4$ from equation (6)

$$\therefore \quad \text{Thermal } \eta = \frac{T_1 - T_3}{T_1} \tag{9}$$

$$= \frac{\text{Max. abs. temp.} - \text{Min. abs. temp.}}{\text{Max. abs. temp.}} \tag{10}$$

Now from equation (9),

$$\text{Thermal } \eta = 1 - \frac{T_3}{T_1} \tag{11}$$

and from equations (3), (4) and (5),

$$\frac{T_1}{T_3} = \left(\frac{V_4}{V_1}\right)^{(\gamma-1)} = \left(\frac{V_3}{V_2}\right)^{(\gamma-1)} = r_v^{(\gamma-1)}$$

where r_v = adiabatic compression and expansion volume ratio.

$\therefore$ from equation (11),

$$\text{Thermal } \eta = 1 - \frac{1}{r_v^{(\gamma-1)}} \tag{12}$$

THE CARNOT CYCLE

This thermal efficiency gives the maximum possible thermal efficiency obtainable between any two given temperature limits.

From the T–s diagram, Fig. 13.13(b),

$$\text{Heat received from 1 to 2} = T_1(s_1 - s_2) \tag{13}$$

$$= \text{area under 1–2}$$

$$\text{Heat rejected from 3 to 4} = T_3(s_3 - s_4) \tag{14}$$

$$= \text{area under 3–4}$$

$$\text{Thermal efficiency} = \frac{\text{Heat received} - \text{Heat rejected}}{\text{Heat received}}$$

$$= \frac{T_1(s_2 - s_1) - (T_3(s_3 - s_4)}{T_1(s_2 - s_1)}$$

$$= \frac{(T_1 - T_3)(s_2 - s_1)}{T_1(s_2 - s_1)}, \quad \text{since } (s_2 - s_1) = (s_3 - s_4)$$

$$= \frac{T_1 - T_3}{T_1} \tag{15}$$

Note that this solution for the thermal efficiency of the Carnot cycle is rather more simple than that given in the previous work.

THE CARNOT CYCLE

Note also that, $\oint W = \text{Heat received} - \text{Heat rejected}$

$$= (T_1 - T_2)(s_2 - s_1) \qquad (16)$$

Now, since the Carnot cycle has the maximum thermal efficiency obtainable within given temperature limits, then it is possible to suggest that if any engine working between the same temperature limits has a thermal efficiency lower than that of the Carnot cycle, then thermal efficiency improvement for the engine is theoretically possible. All practical engines have a thermal efficiency much lower than the Carnot thermal efficiency.

The ultimate aim, in practice, should be an attempt to reach an efficiency as near 100% as possible. Now how could this be achieved?

$$\text{Carnot thermal } \eta = \frac{\text{Max. abs. temp.} - \text{Min. abs. temp.}}{\text{Max. abs. temp.}}$$

For this equation to be a maximum it must equal unity in which case the thermal efficiency would be 100%.

EXAMPLE

EXAMPLE 118

A Carnot cycle using a gas has the temperature limits of 400°C and 70°C. Determine the thermal efficiency of the cycle.

From section 13.3, equation 15,

$$\text{Thermal } \eta = \frac{T_1 - T_3}{T_1}$$

$$\text{Now,}\quad T_1 = 400 + 273 = \underline{673\ \text{K}}$$

$$T_3 = 70 + 273 = \underline{343\ \text{K}}$$

$$\therefore\ \text{Thermal}\ \eta = \frac{673 - 343}{673} = \frac{330}{673}$$

$$= \underline{0{\cdot}49}$$

$$= 0{\cdot}49 \times 100$$

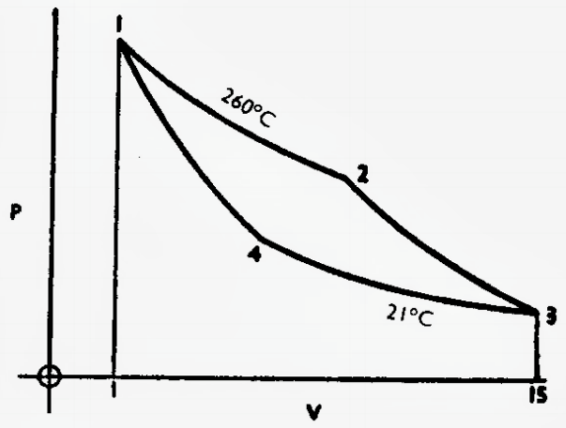
$$= \underline{49\%}$$

EXAMPLE

The overall volume expansion ratio of a Carnot cycle is 15. The temperature limits of the cycle are 260°C and 21°C. Determine:

(a) the volume ratios of the isothermal and adiabatic processes;
(b) the thermal efficiency of the cycle.

Take $\gamma = 1{\cdot}4$



(a) For the adiabatics,

$$\frac{T_1}{T_4} = \frac{T_2}{T_3} = \left(\frac{V_4}{V_1}\right)^{(\gamma-1)} = \left(\frac{V_3}{V_2}\right)^{(\gamma-1)}$$

$$\therefore \frac{V_4}{V_1} = \frac{V_3}{V_2} = \left(\frac{T_1}{T_4}\right)^{1/(\gamma-1)} = \left(\frac{T_2}{T_3}\right)^{1/(\gamma-1)}$$

, and, $T_1 = T_2 = 260 + 273 = \underline{533\text{ K}}$

$T_3 = T_4 = 21 + 273 = \underline{294\text{ K}}$

$$= \left(\frac{533}{294}\right)^{1/(1\cdot4-1)} = 1{\cdot}812^{1/0\cdot4}$$

$$= 1{\cdot}813^{2\cdot5} = \underline{4{\cdot}426}$$

$\therefore$ Volume ratio of adiabatics $= \underline{4{\cdot}426 = r_v}$

$$\text{Volume ratio of isothermals} = \frac{V_3}{V_4} = \frac{V_3}{V_1}\frac{V_1}{V_4}$$

$$= \frac{15}{4{\cdot}426} = \underline{3{\cdot}39}$$

(b) Thermal efficiency $= \dfrac{T_1 - T_4}{T_1}$

$$= \frac{533 - 294}{533} = \frac{239}{533}$$

$$= \underline{0{\cdot}448}$$

$$= 0{\cdot}448 \times 100$$

$$= \underline{44{\cdot}8\,\%}$$

Alternatively,

$$\text{Thermal efficiency} = 1 - \frac{1}{r_v^{(\gamma-1)}} = 1 - \frac{1}{4{\cdot}426^{(1\cdot4-1)}}$$

$$= 1 - \frac{1}{4{\cdot}426^{0\cdot4}}$$

$$= 1 - \frac{1}{1{\cdot}812} = 1 - 0{\cdot}552 = \underline{0{\cdot}448}$$

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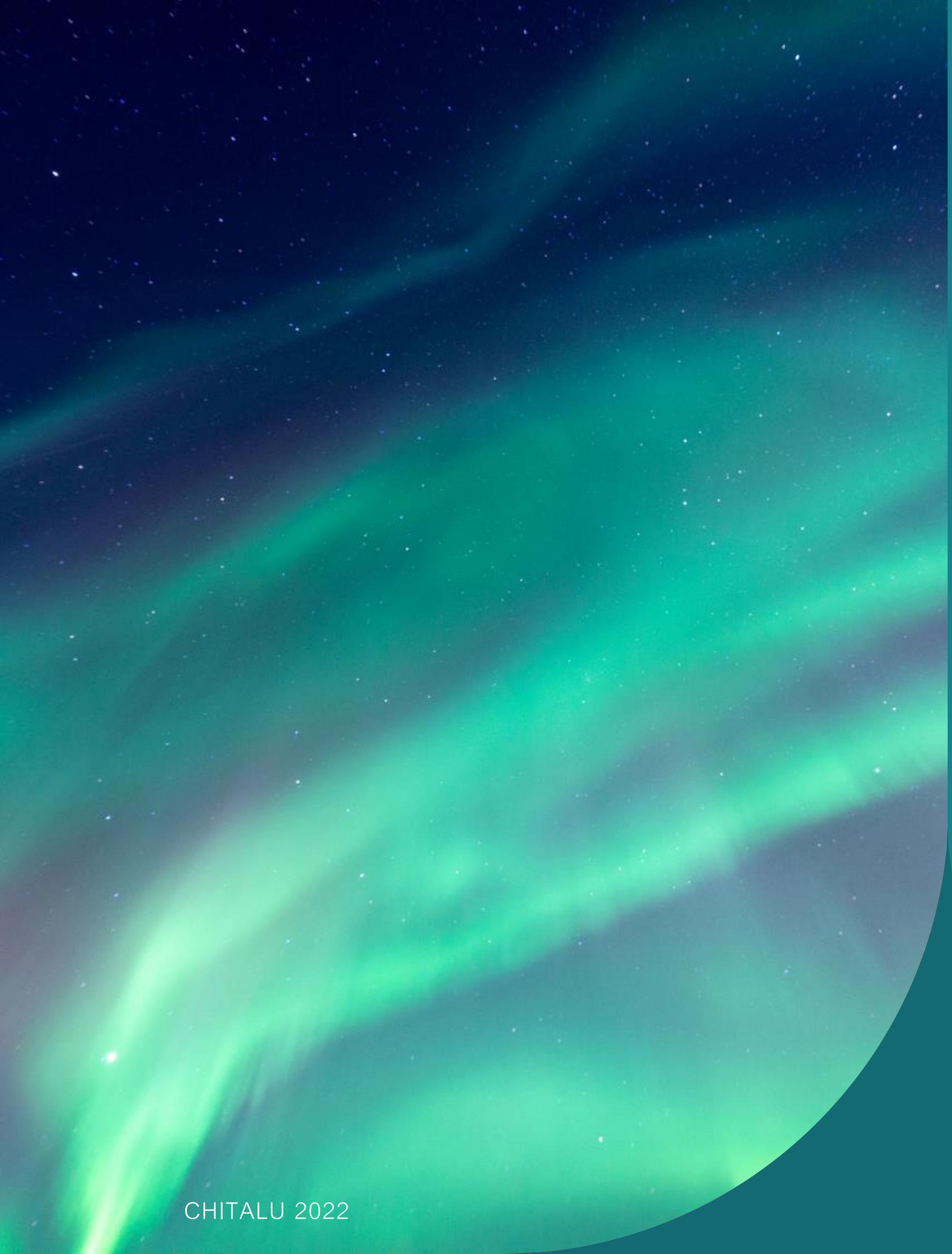
$$= 0{\cdot}448 \times 100$$

$$= \underline{44{\cdot}8\,\%}$$

“The Second Law of Thermodynamics: If you think things are in a mess, just wait! ”

Jim Warner



Thank You

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