EEE 3121 - Signals & Systems

Lecture 1: Signals and Systems

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Course Requirements

- It is an **OBLIGATION** for all students taking this course to attend all lectures and lab sessions.

Prerequisite

- EEE 2019

Simulation Software: Multisim, MATLAB

Time Allocation

Lectures	**4 hours/week**
Labs	**3 hours/week**

Assessment

Assignments (8) /Quizzes	**5%**
Labs/Mini-Projects	**15%**
1 Test (2 hours)	**20%**
1 Final Exam (3 hours)	**60%**

References

Our main reference text book in this course is

[1] B. P. Lathi and R. A. Green, Linear Systems and Signals, 3rd Ed., 2018, Oxford University Press, New York. ISBN 978-0-19-020017-6

[2] Kuo Franklin, F., Network Analysis and Synthesis, 3rd Ed., 1986, J. Wiley (SE), ISBN 0-471-51118-8.

[3] Sundararanjan, D., A Practical Approach to Signals and Systems, 2008, John Wiley & Sons (Asia) Pte Ltd, ISBN 978-0-470-82353-8.

However, feel free to use pretty much any additional text which you might find relevant to our course.

Introduction

- The chief objective of this course is to bring to the fore the fundamentals of electric system theory. Thus, most of the time will be devoted to system analysis and some time on system synthesis and design.
- In system analysis we concern ourselves with determining the response (output), given the excitation (input) and the system (network).
- In system synthesis we concern ourselves with designing the system given the excitation and the desired response.

1.1 Signal Analysis

- For electric systems, the excitation and response are given in terms of voltage and currents as functions of time, t. Generally, these functions of time are called signals.
- In electrical engineering, signals are described using time and frequency. Signals can be described equally well in terms of spectral or frequency information.

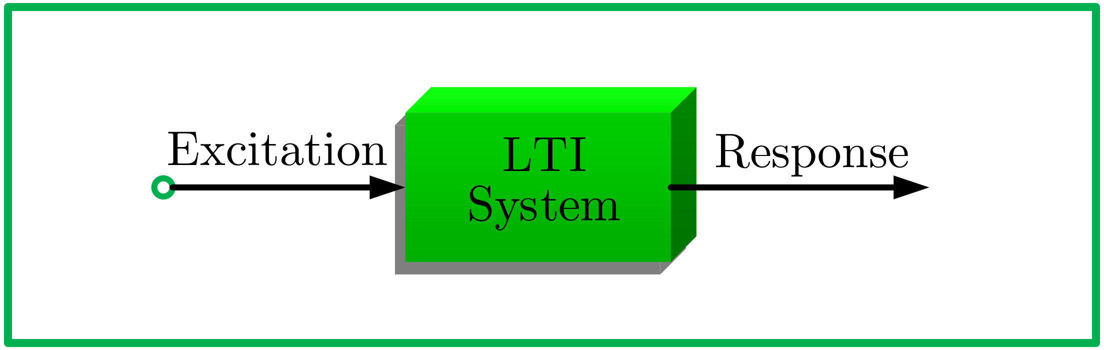

Fig 1.1: Objects of our concern

1.1 Signal Analysis Cont'd

- The signal translation between time and frequency domain is aided by Fourier series, the Fourier integral, and the Laplace transform.
- These terms shall be defined and studied in detail later in this course.

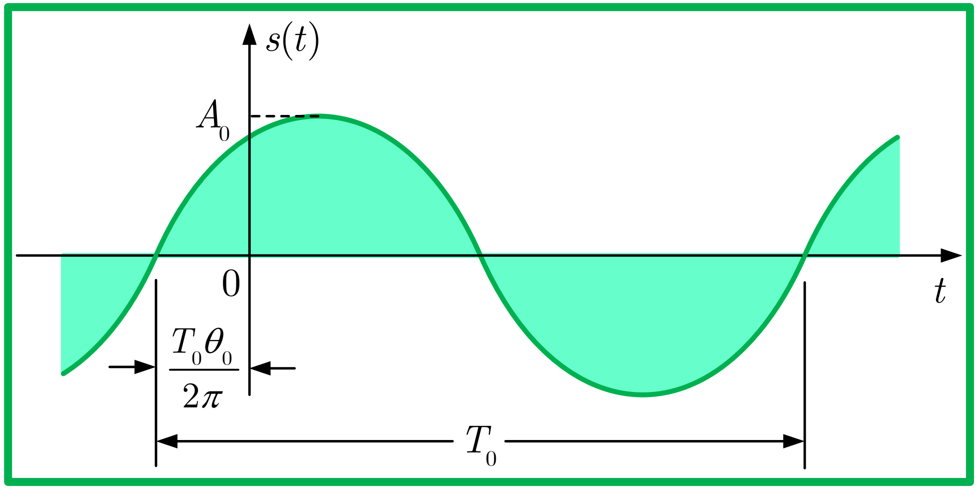

Fig 1.2: Sinusoidal signal

- Let us focus our attention on how to describe a signal in terms of both the frequency and time.

1.1 Signal Analysis Cont'd

- Consider the signal of the form

$$s(t) = A_0 \sin(\omega_0 t + \theta_0) \qquad \textbf{(1.1)}$$

where A_0 is the amplitude, θ_0 is the phase shift, and ω_0 is the angular frequency given by

$$\omega_0 = \frac{2\pi}{T_0} \qquad \textbf{(1.2)}$$

here T_0 is the period of the sinusoid. Fig. 1.2 above depicts the signal plotted against time.

- If we let the angular frequency ω be the independent variable, an equally complete description of the signal is obtained as shown in Figs. 1.3a and b.

1.1 Signal Analysis Cont'd

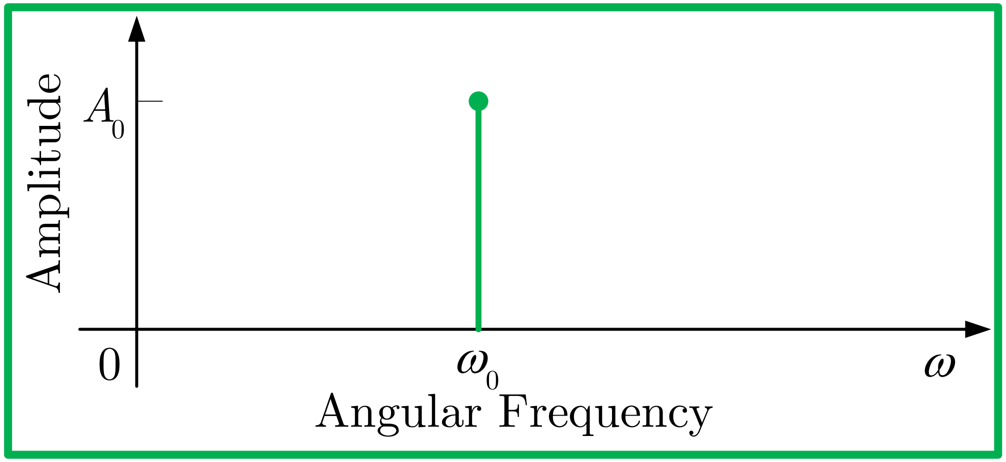

Fig 1.3a: Plot of amplitude *A* versus angular frequency ω.

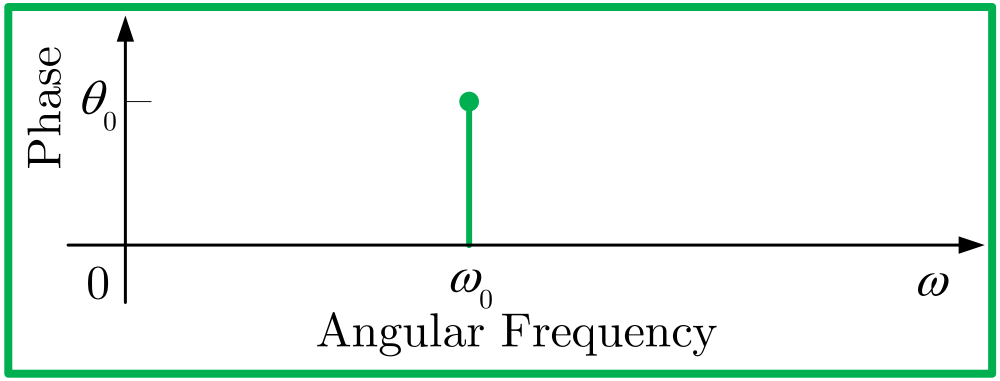

Fig 1.3b: Plot of phase θ versus angular frequency ω.

1.1 Signal Analysis Cont'd

- Now suppose that the signal has $2n+1$ sinusoidal components, such that,

$$s(t) = \sum_{i=-n}^{n} A_i \sin(\omega_i t + \theta_i) \quad \textbf{(1.3)}$$

- Spectral description of the signal would have $2n+1$ line spectra as depicted in Figs. 1.4a and b.

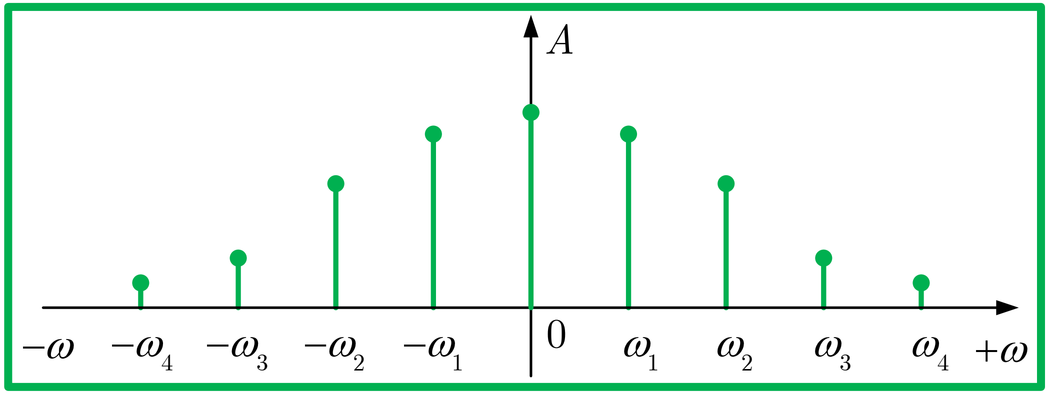

Fig 1.4a: Discrete amplitude spectrum.

1.1 Signal Analysis Cont'd

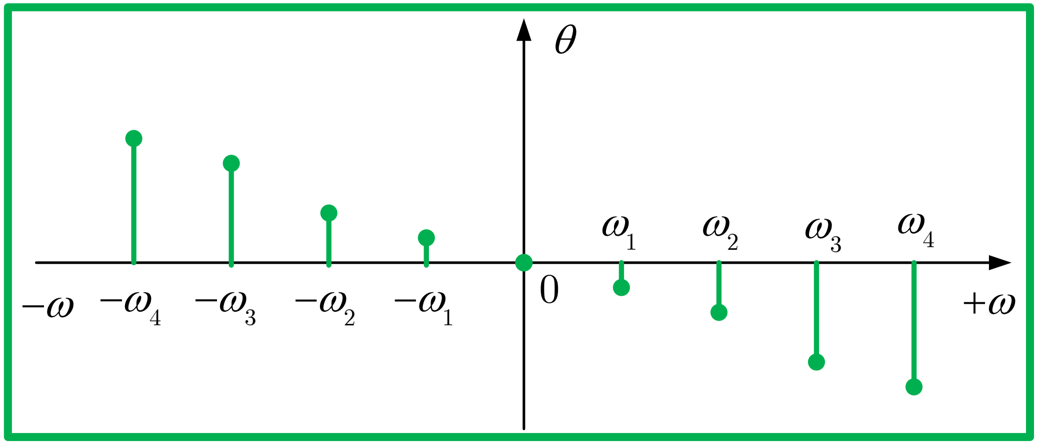

Fig 1.4b: Discrete phase spectrum.

- When the number these spectral lines become infinite, the intervals $\omega_{i+1} - \omega_i$ between the lines approach zero. Thus the discrete line spectra fuse into a continuous spectra as depicted by Figs. 1.5a and b.
- The continuous counterpart of Eq. 1.3 is of the form

$$s(t) = \int_{-\infty}^{\infty} A(\omega)\sin\left[\omega t + \theta(\omega)\right]d\omega \qquad \textbf{(1.4)}$$

here $A(\omega)$ is known as the amplitude spectrum and $\theta(\omega)$ as the phase spectrum.

1.1 Signal Analysis Cont'd

- Later in this course, we shall learn that periodic signals can be described in terms of discrete spectra using Fourier series.

- A nonperiodic signal such as the triangular pulse in Fig. 1.6 can only be described in terms of continuous spectra using the Fourier integral transform.

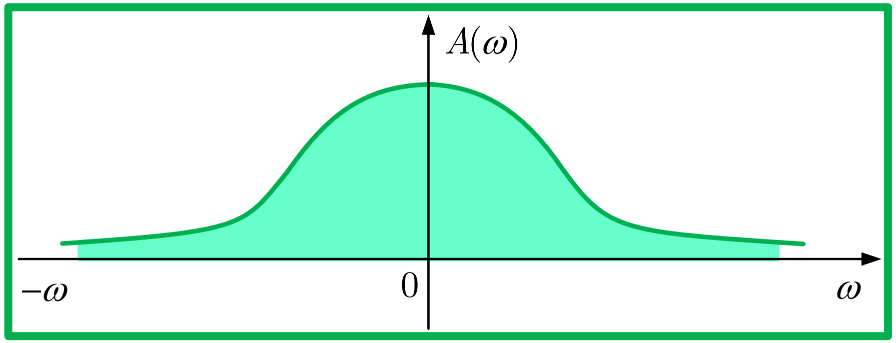

Fig 1.5a: Continuous amplitude spectrum.

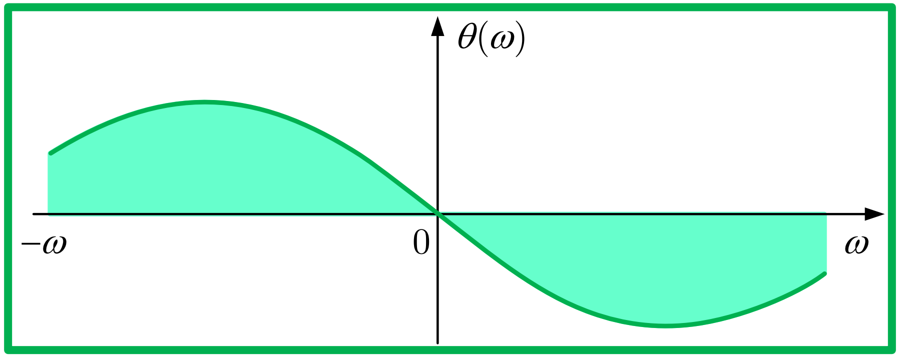

Fig 1.5b: Continuous Phase spectrum.

1.2 Complex Frequency

- The complex frequency variable of the form

$$s = \sigma + j\omega \quad (1.5)$$

is a generalized frequency variable whose real part σ describes growth and decay of the amplitudes of signals, and whose imaginary part $j\omega$ is angular frequency.

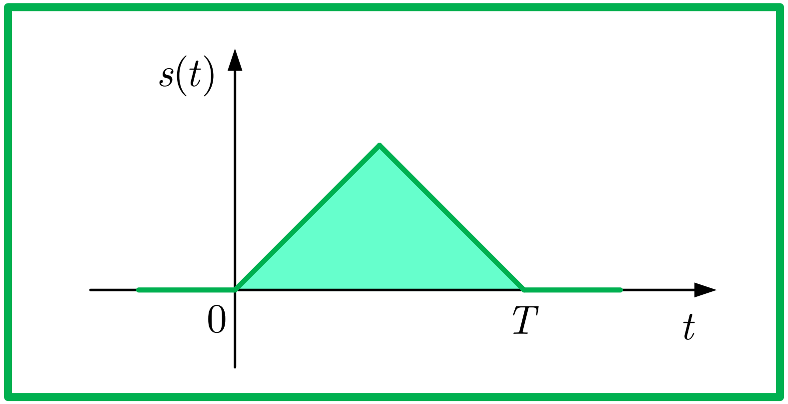

Fig 1.6: Triangular signal.

- The concept of complex frequency is developed by examining the cisoidal signal

$$\mathbf{S}(t) = Ae^{j\omega t} \quad (1.6)$$

1.2 Complex Frequency Cont'd

- Fig. 1.7 shows $\mathbf{S}(t)$ represented as a rotating phasor. Here, the angular frequency ω can be thought of as a velocity at the end of the phasor.

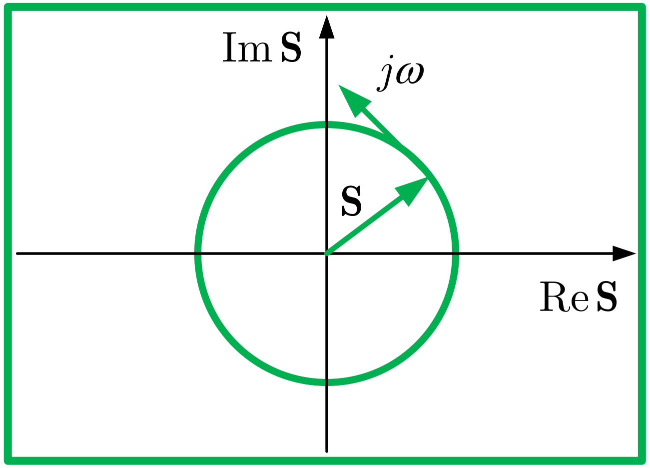

Fig 1.7: Rotating phasor.

- The velocity ω is always at right angles to the phasor. In general, if the velocity s is inclined at any arbitrary angle φ, it has a component ω at right angles to the phasor $\mathbf{S}$ and a component σ, parallel to $\mathbf{S}$.

1.2 Complex Frequency Cont'd

- Fig. 1.8 (a) shows that the phasor $\mathbf{S}$ decreases in amplitude as it spins in a counterclockwise fashion. Thus the signal $\mathbf{S}(t)$ is made of damped sinusoids.

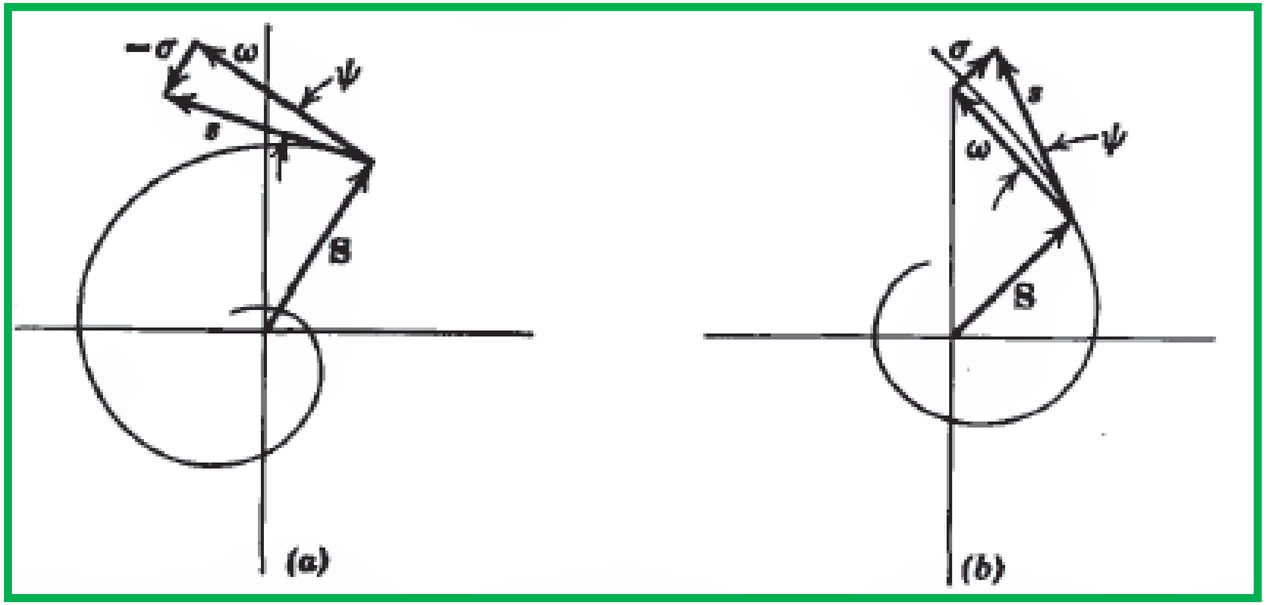

Fig 1.8: (a) Rotating phasor with exponentially decreasing amplitude. (b) Rotating phasor with exponentially increasing amplitude.

1.2 Complex Frequency Cont'd

- The signal $\mathbf{S}(t)$ is thus of the form

$$\begin{aligned} \operatorname{Re}\mathbf{S}(t) &= Ae^{-\sigma t}\cos\omega t \\ \operatorname{Im}\mathbf{S}(t) &= Ae^{-\sigma t}\sin\omega t \end{aligned} \qquad (1.7)$$

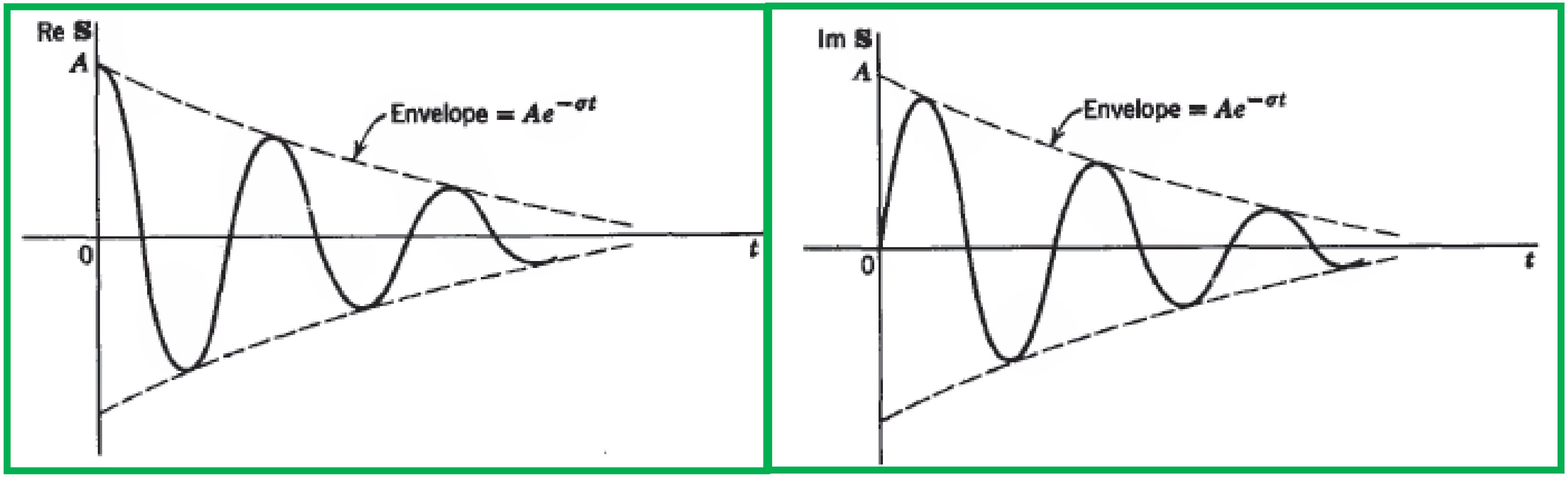

Fig 1.9: Damped sinusoids.

- The plots of Eq. 1.7 are depicted in Fig. 1.9

1.2 Complex Frequency Cont’d

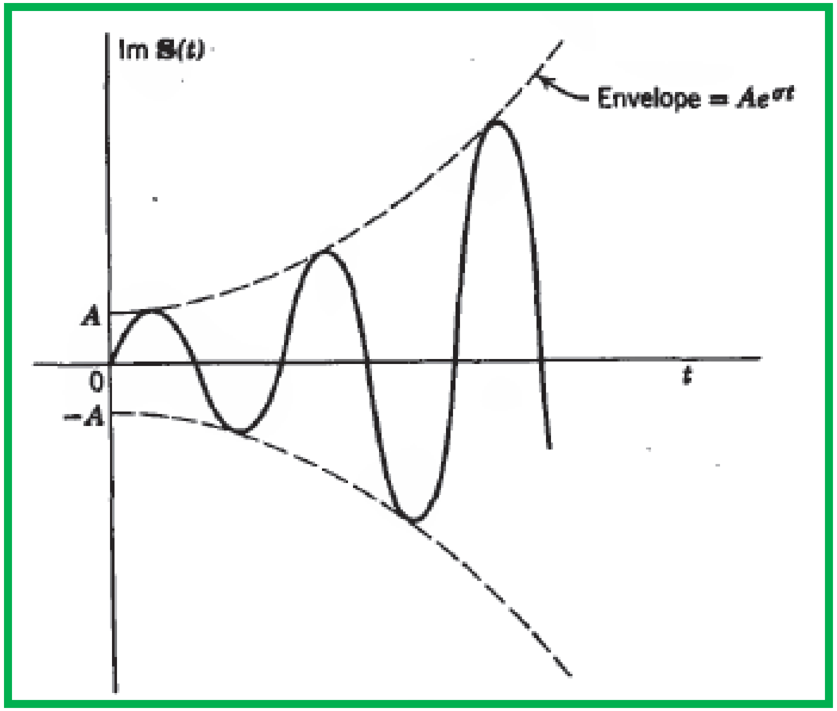

Fig 1.10: Exponentially increasing sinusoid.

- Fig. 1.10 shows an exponentially increasing sinusoid when the real component of velocity is $+\sigma$.

1.2 Complex Frequency Cont'd

- Thus, the generalized cisoidal signal is of the form

$$\mathbf{S}(t) = Ae^{st} = Ae^{(\sigma+j\omega)t} \quad \textbf{(1.8)}$$

which describes the growth and decay of amplitudes apart from angular frequency. If $\sigma = 0$, sinusoid is undamped, and if $j\omega = 0$, the signal is purely exponential, which is of the form

$$\mathbf{S}(t) = Ae^{\pm\sigma t} \quad \textbf{(1.9)}$$

- If $\sigma = j\omega = 0$, the signal is simply a constant.

1.3 System Analysis

- Let us focus our energy on the fundamental properties of linear networks and general characteristics of signal processing by linear systems.

Basic Definitions

- **Linear.** A system is linear if and only if the principles of superposition and proportionality hold. Thus, for a given System, let $[e_1(t), r_1(t)]$ and $[e_2(t), r_2(t)]$ be excitation-response pairs, then for an excitation $e(t) = e_1(t) + e_2(t)$, the response ought to be $r(t) = r_1(t) + r_2(t)$. Likewise, for excitation $C_1e_1(t)$, were C_1 is a constant, the response ought to be $C_1r_1(t)$, implying the constant is preserved by the linear system.
- **Passive.** A linear system is passive if and only if the energy delivered to the System is nonnegative for any arbitrary excitation, and no voltages or currents appear between any two terminals before an excitation is applied.

1.3 System Analysis Cont'd

Basic Definitions

- **Reciprocal.** A linear time-invariant (LTI) System is reciprocal if and only if points of excitation and measurement of response are interchanged, the relationship between excitation and response remains the same.

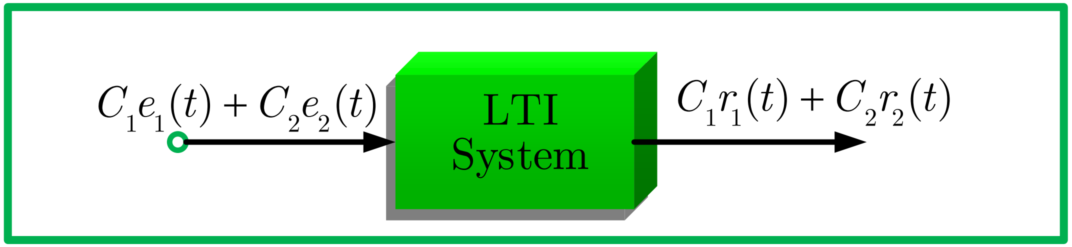

Fig 1.11: Linear System.

- **Causal.** A system is causal if its response is nonanticipatory. Thus, if

$$e(t) = 0 \qquad \forall\, t < T \tag{1.10}$$

then

$$r(t) = 0 \qquad \forall\, t < T$$

Simply put, a system is causal if before an excitation is applied at $t = T$, the response is zero for $-\infty < t < T$.

1.3 System Analysis Cont'd

Basic Definitions

- Time invariant. A system is time-invariant if $e(t) \to r(t)$ implies that $e(t \pm T) \to r(t \pm T)$, here the symbol $\to$ denoting "gives rise to." It is worth noting that linear systems need not be time invariant.
- Derivative. By means of the time-invariant property, it follows that, if an input $e(t)$ gives rise to an output $r(t)$, then for an input $e'(t)$, i.e., the derivative of $e(t)$, an output $r'(t)$ is obtained.

Proof: let the excitation $e(t+\in)$ for a real quantity $\in$, by the time-invariant property, the response would be $r(t+\in)$. Suppose the excitation were

$$e_1(t) = \frac{1}{\in}\left[e(t+\in) - e(t)\right] \quad \textbf{(1.11)}$$

it follows from the linearity and time-invariant properties, that the response is

$$r_1(t) = \frac{1}{\in}\left[r(t+\in) - r(t)\right] \quad \textbf{(1.12)}$$

1.3 System Analysis Cont'd

Taking the limit as $\epsilon \to 0$, yields

$$\lim_{\epsilon \to 0} e_1(t) = \frac{d}{dt} e(t)$$

$$\lim_{\epsilon \to 0} r_1(t) = \frac{d}{dt} r(t) \qquad \textbf{(1.13)}$$

It is worth noting that this idea can extended to higher derivatives and the integrals of $e(t)$ and $r(t)$.

1.3 System Analysis Cont'd

Ideal models

- Some of the idealized models of linear systems are shown below whose properties renders them useful in signal processing.

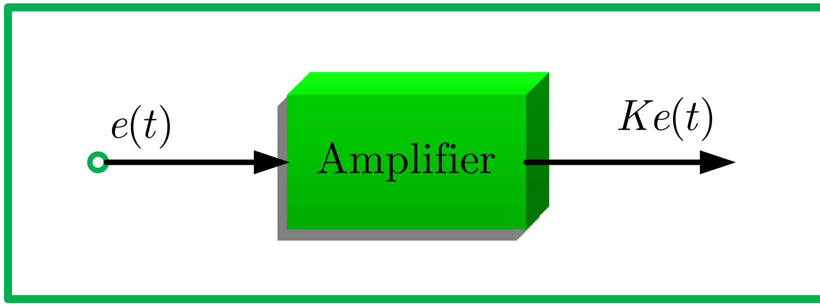

Fig 1.12: Amplifier.

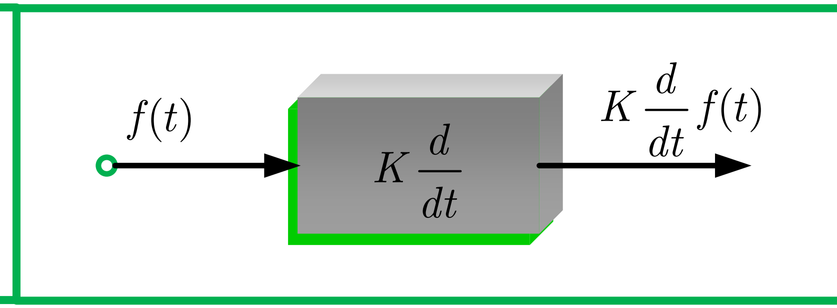

Fig 1.13: Differentiator.

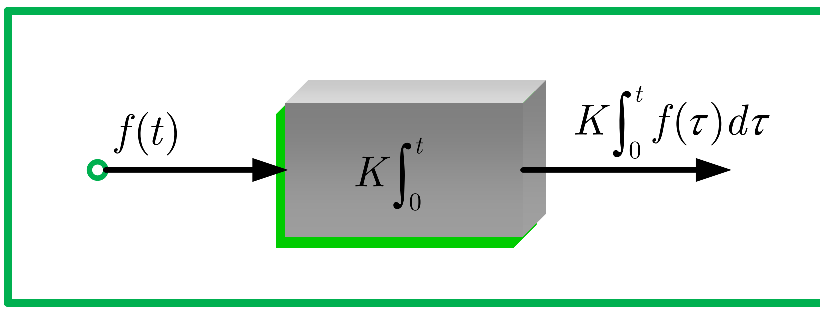

Fig 1.14: Integrator.

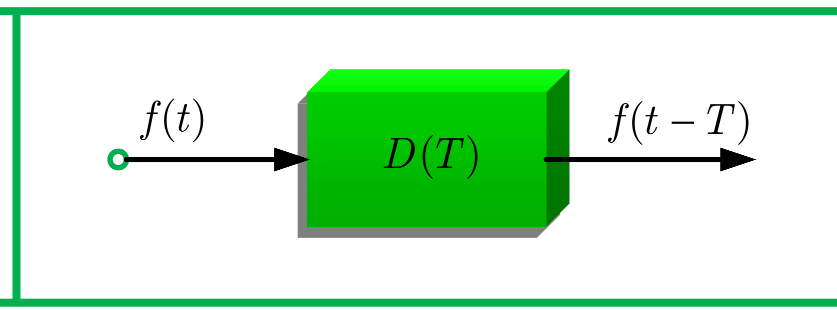

Fig 1.15 Time-delay System.

1.3 System Analysis Cont'd

- Let the triangular pulse in Fig. 1.16 be the excitation to each of the four systems just described. Their respective responses are as shown in Fig. 1.17.

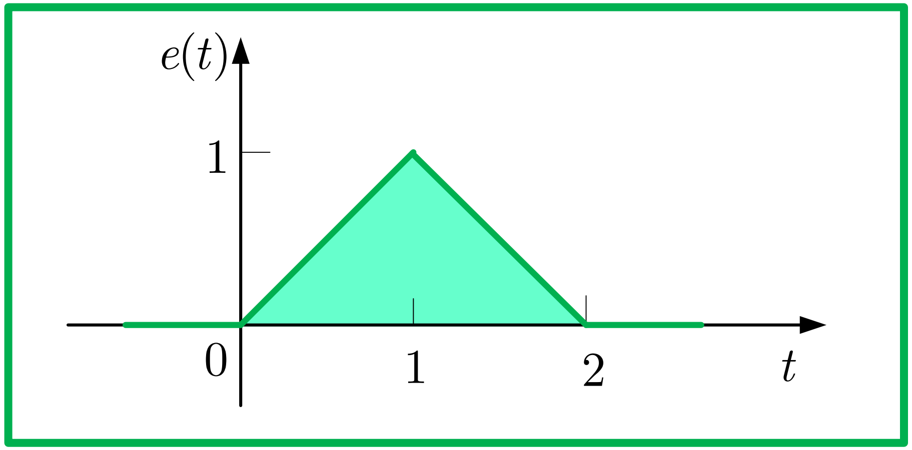

Fig 1.16: Excitation function.

1.3 System Analysis Cont'd

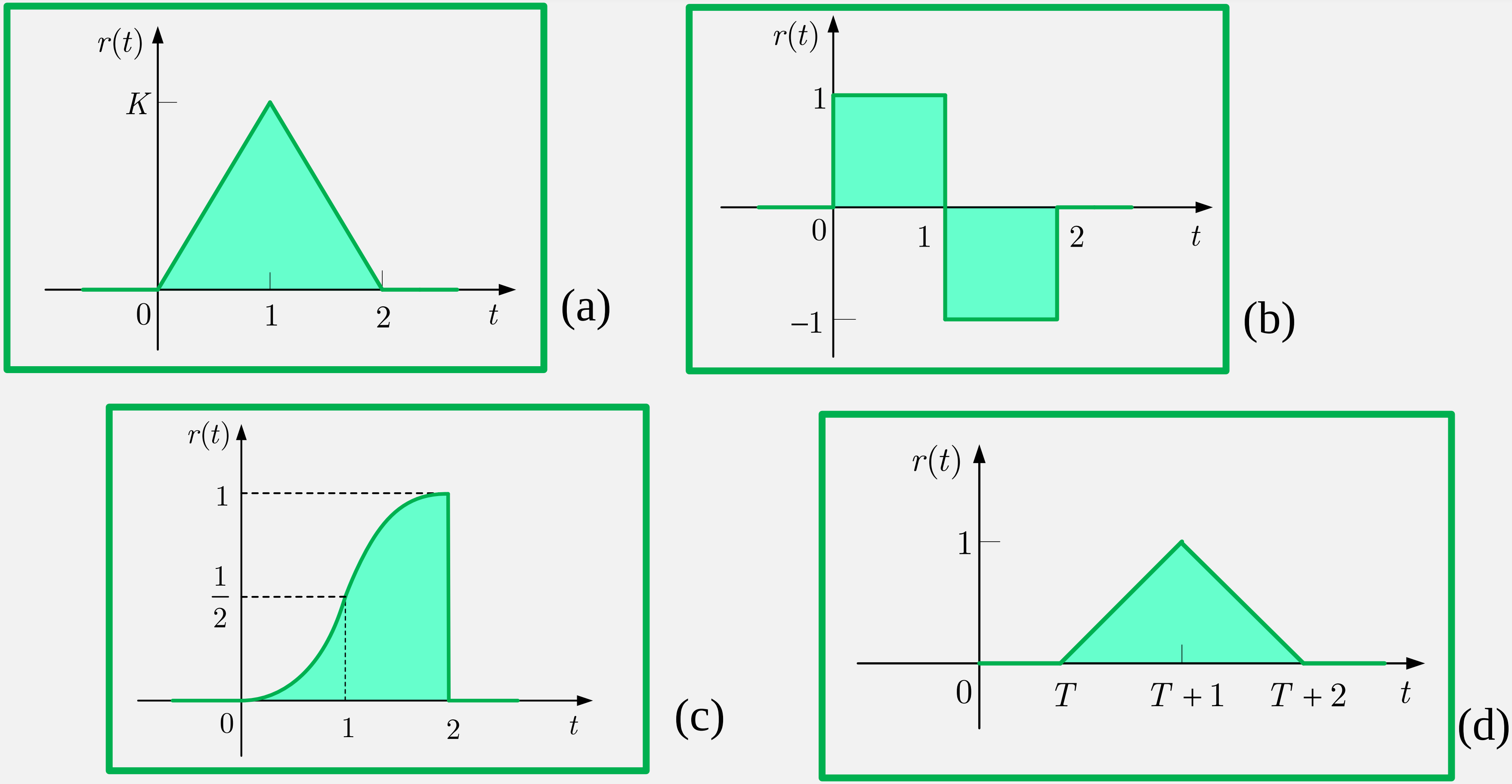

Fig. 1.17: (a) Amplifier output. (b) Differentiator output. (c) Integrator output. (d) Delayed output.

1.3 System Analysis Cont'd

Ideal elements

- Idealized linear mathematical models of physical circuits elements are used to analyse electric networks and/or systems.
- The most common elements are resistor R, [ohms], capacitor C, [farads], and inductor L, [henrys]. Any pair of two terminals into which energy is supplied or withdrawn is know as a port.

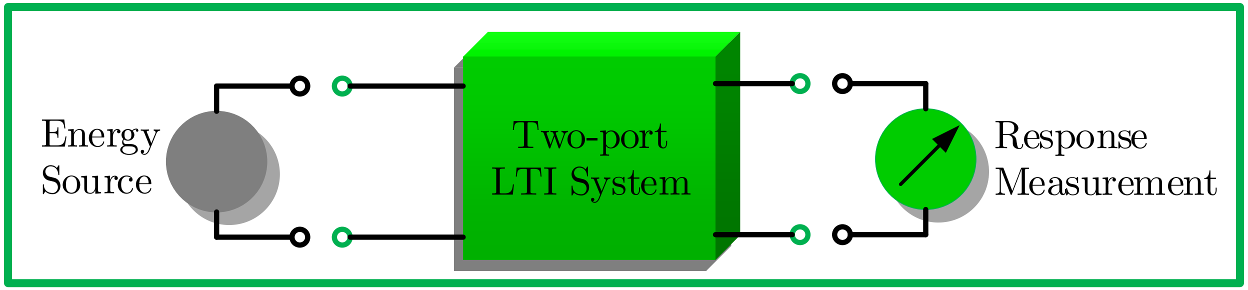

Fig 1.18: Two-port System.

- Fig. 1.18 shows an example of a two-port System. The energy sources for excitation functions are ideal current and voltage sources, see Figs. 1.19a and b.

1.3 System Analysis Cont'd

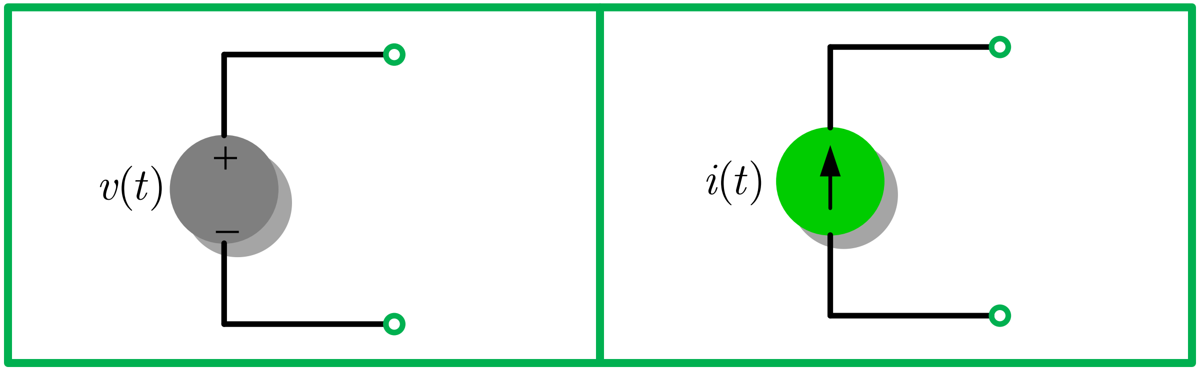

Fig 1.19a: Voltage source.   **Fig 1.19b**: Current source.

- An ideal voltage source is an energy source that provides, at a given port, a voltage signal independent of the current at that port. Interchanging the words "current" and "voltage" in the last definition, defines an ideal current source.
- The key problem in system analysis, is to find the relationships that exist between the currents and voltages at the ports of the system.

- Consider the R, L and C elements shown in Fig.1.20. Since the currents and voltages are expressed as functions of time, the equations defining them are of the form

1.3 System Analysis Cont'd

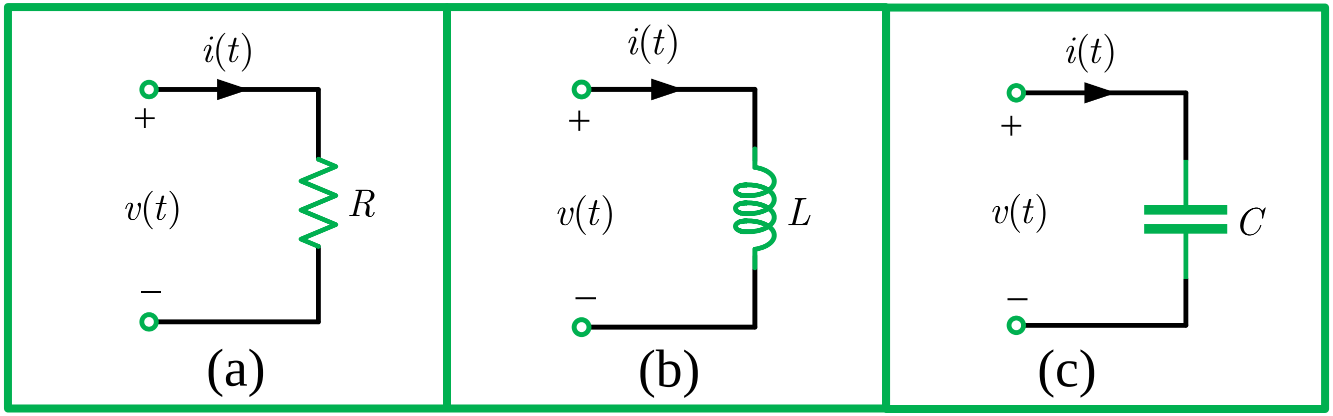

Fig 1.20: (a) Resistor. (b) Inductor. (c) Capacitor.

$$v(t) = Ri(t) \quad \text{or} \quad i(t) = \frac{1}{R}v(t)$$

$$v(t) = L\frac{di(t)}{dt} \quad \text{or} \quad i(t) = \frac{1}{L}\int_0^t v(x)\,dx + i(0) \qquad \textbf{(1.14)}$$

$$v(t) = \frac{1}{C}\int_0^t i(x)\,dx + v(0) \quad \text{or} \quad i(t) = C\frac{dv(t)}{dt}$$

1.3 System Analysis Cont'd

here, the constants of integration $i(0)$ and $v(0)$ are initial conditions.

- In complex frequency domain, using the variable s, ignoring initial conditions for now, the above equations are of the form

$$V(s) = RI(s) \quad \text{or} \quad I(s) = \frac{1}{R}V(s)$$
$$V(s) = sLI(s) \quad \text{or} \quad I(s) = \frac{1}{sL}V(s) \qquad \textbf{(1.15)}$$
$$V(s) = \frac{1}{sC}I(s) \quad \text{or} \quad I(s) = sCV(s)$$

- Notice that in the time-domain, the voltage-current relationships are given in terms of differential equations. However, in the complex-frequency domain, they are expressed in algebraic equations, much simpler to solve.

1.3 System Analysis Cont'd

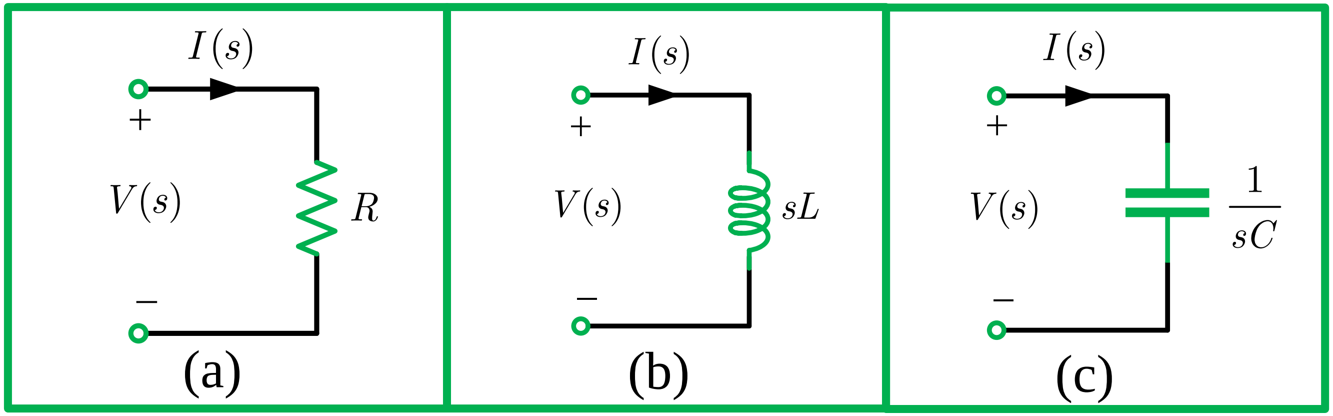

Fig 1.21: (a) Resistor. (b) Inductor. (c) Capacitor.

- If a System consists of an interconnection of linear circuit elements, it is described by a transfer function $H(s)$. Thus the response $R(s)$ and excitation $E(s)$ are related by the equation of the form

$$R(s) = H(s)E(s) \tag{1.16}$$

- In System analysis, $E(s)$ is given, $H(s)$ is directly obtained from the System. Thus, the task is to determine $R(s)$.

1.4 System Synthesis

- In System synthesis, the response $R(s)$ and excitation $E(s)$ are given, thus we are required to synthesize the System from the system function

$$H(s) = \frac{R(s)}{E(s)} \tag{1.17}$$

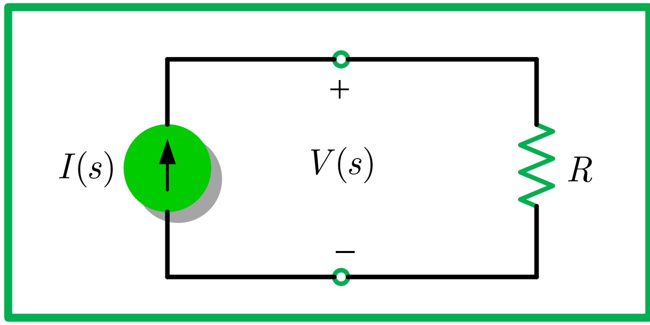

Fig 1.22: Driving-point impedance

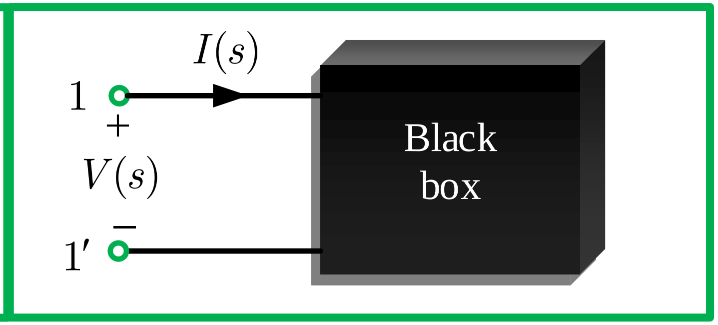

Fig 1.23: Black box.

- A driving-point immittance is defined to be a function for which the variables are measured at the same port. Thus a driving-point impedance $Z(s)$ is of the form

$$Z(s) = \frac{V(s)}{I(s)} \tag{1.18}$$

1.4 System Synthesis Cont'd

here, the excitation is a current $I(s)$ and the response is a voltage $V(s)$, as shown in Fig. 1.22.

- In Fig. 1.22 driving-point impedance is obtained as

$$Z(s) = \frac{V(s)}{I(s)} = R \quad \textbf{(1.19)}$$

- Let the resistor in Fig. 1.22 be enclosed in a "black box." Assume we have no access to this box, except at the terminals $1\text{-}1'$ in Fig.1.23. There is need to determine the System in the black box.
- Let excitation $I(s)$, the voltage response $V(s)$ is proportional to $I(s)$ by the equation

$$V(s) = KI(s) \quad \textbf{(1.20)}$$

- The trivial solution, though not unique, is that the black box consists of a resistor of value $R = K\left[\Omega\right]$.

1.4 System Synthesis Cont'd

- Suppose the excitation is a voltage $V(s)$, the response is a current $I(s)$, and that

$$Y(s) = \frac{I(s)}{V(s)} = 3 + 3s \qquad \textbf{(1.21)}$$

- Our task is to synthesize a System equivalent to the System in the black box. It follows that a possible solution is shown in Fig. 1.24.

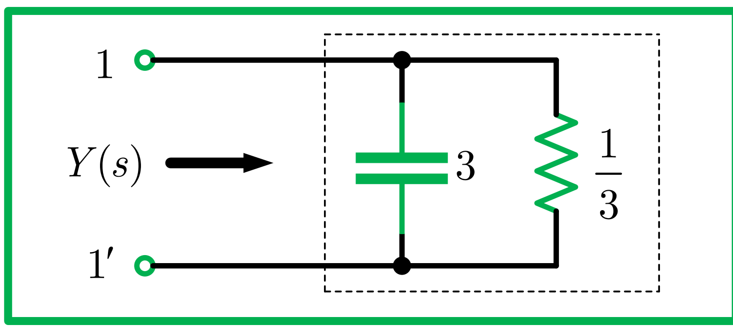

Fig 1.24: System realization for $Y(s)$.

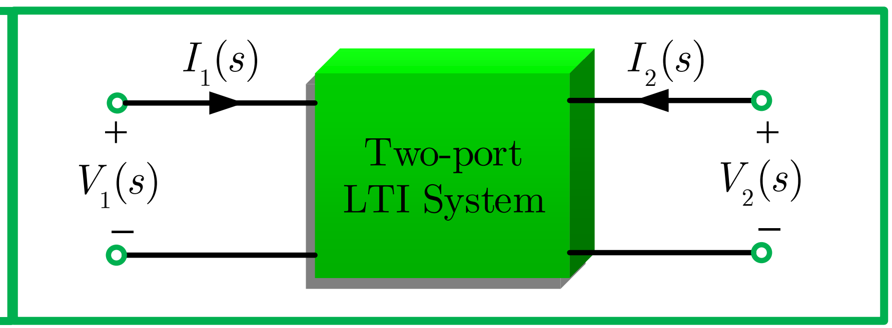

Fig 1.25: Two-port System.

1.4 System Synthesis Cont'd

- The previous example has shown that the problem of driving-point synthesis, consists of decomposing a given immittance function into basic recognizable parts (such as $3 + 3s$).
- Realizable driving-point immittances belong to a class of functions called positive real functions. Properties of p.r. functions can be used to test a given driving-point function for realizability.
- A transfer function or transmittance takes many different forms. For example, consider the two-port System in Fig. 1.25. For excitation $I_1(s)$ and response $V_2(s)$, the transfer function is a transfer impedance

$$Z_{21}(s) = \frac{V_2(s)}{I_1(s)} \quad \textbf{(1.22)}$$

- Similarly, if $V_1(s)$ were the excitation and $V_2(s)$ the response, then a voltage-ratio transfer function is obtained.

1.4 System Synthesis Cont'd

$$H(s) = \frac{V_2(s)}{V_1(s)} \tag{1.23}$$

- As for driving-point functions, there are certain properties which a transfer function must satisfy in order to be realizable.
- The filter design is critical in transfer function synthesis. A filter is defined as a System which passes a certain portion of a frequency spectrum and blocks the rest of the spectrum.

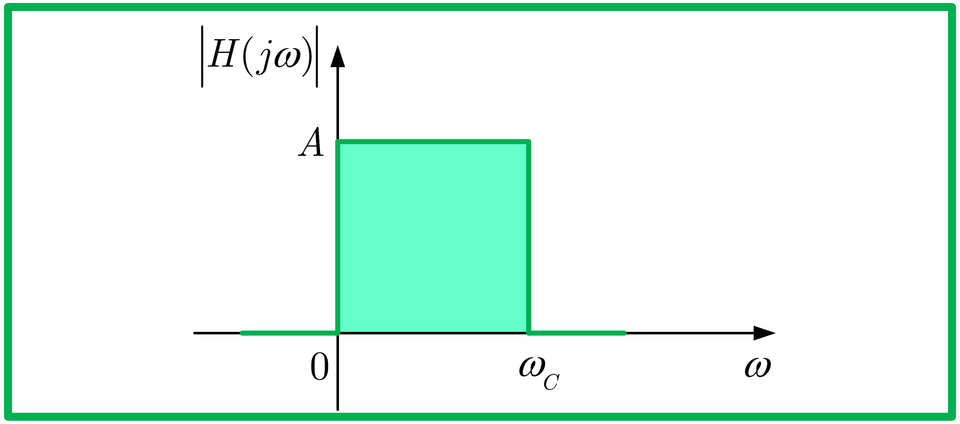

Fig 1.26: Ideal amplitude spectrum for low-pass filter.

1.4 System Synthesis Cont'd

- One aspect of filter design is to synthesize the System from the transfer function $H(s)$. The other aspect deals with the problem of obtaining a realizable transmittance $H(s)$ given the specification of, for example, the magnitude characteristic in Fig. 1.26. This part of synthesis is generally referred to as the approximation problem.
- It is approximate because the frequency response characteristics of the R, L and C elements are continuous, except at resonance points. As such a System with these elements cannot be made to cut off abruptly at the cutoff frequency ω_C.

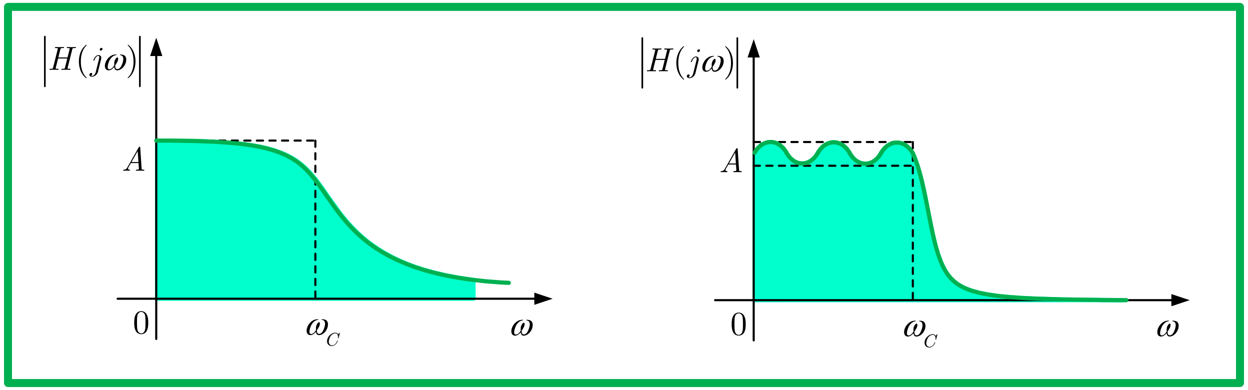

Fig 1.27: Realizable low-pass filter characteristics.

1.4 System Synthesis Cont'd

- Fig. 1.27 shows the magnitude characteristics of low-pass filters that can practically be realized.
- In filter design problems, certain problems in magnitude and frequency normalization will be discussed. This allows us to deal with element values such as $\mathrm{R} = 0.5\mathrm{ohm}$ and $\mathrm{C} = 2\mathrm{farads}$ instead of "practical" element values of, for example, $\mathrm{R} = 500{,}000\ \mathrm{ohms}$ and $\mathrm{C} = 2\ \mathrm{picofarads}\ (\mathrm{pico}{=}10^{-12})$.

End of Lecture 1

Thank you for your attention!