EEE 3121 - Signals & Systems

Lecture 8: Amplitude, Phase and Delay

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References

Our main reference text book in this course is

[1] B. P. Lathi and R. A. Green, Linear Systems and Signals, 3rd Ed., 2018, Oxford University Press, New York. ISBN 978-0-19-020017-6

[2] Kuo Franklin, F., Network Analysis and Synthesis, 3rd Ed., 1986, J. Wiley (SE), ISBN 0-471-51118-8.

[3] Sundararanjan, D., A Practical Approach to Signals and Systems, 2008, John Wiley & Sons (Asia) Pte Ltd, ISBN 978-0-470-82353-8.

However, feel free to use pretty much any additional text which you might find relevant to our course.

8.1 Amplitude and Phase Response

- In this lecture we will study the relationship between the poles and zeros of a system function and its steady-state sinusoidal response.
- Simply put, we will investigate the effect of pole and zero position upon the behavior of $H(s)$ along the $j\omega$ axis.
- The steady-state response of a system function is given by

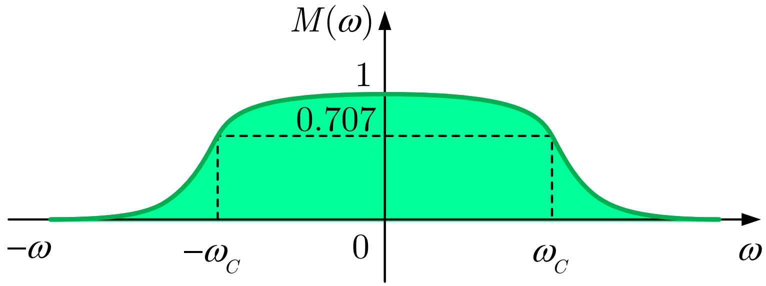
$$\boxed{H(j\omega) = M(\omega)e^{j\phi(\omega)}} \tag{8.1}$$

- Where $M(\omega)$ is the amplitude or magnitude response function, and is an even function in ω. $\phi(\omega)$ represents the phase response, and is an odd function in ω.

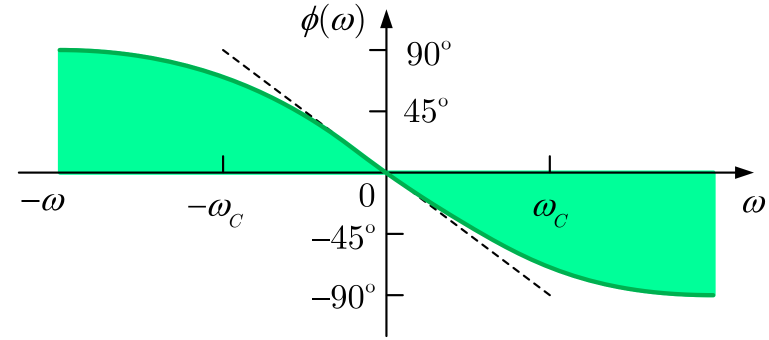
> The amplitude and phase response of a system provides valuable information in the analysis and design of transmission circuits.

8.1 Amplitude and Phase Response

- Consider the amplitude and phase characteristics of a low pass filter shown below.



(a) Amplitude response



(b) Phase response

Fig. 8.1

- ω_c is the cut off frequency. It is the half power frequency, i.e.,

$$|H(j\omega_c)| = 0.707\,|H(j\omega_{\max})| \quad \text{OR} \quad 20\log|H(j\omega_{\max})| - 20\log|H(j\omega_c)| = 3\text{dB}. \qquad \textbf{(8.2)}$$

- The system will not pass harmonics above ω_c .
- If we pass a pulse train whose amplitude spectrum contains significant harmonics above ω_c , it will result in pulse train being distorted, because many higher harmonic terms will be missing. [This is important for sending of Digital signals e.g Ethernet cables].
- It is shown in [1],[2] that when phase response $\phi(\omega)$ is linear, then minimum pulse distortion will result.
- If all significant harmonics are less than ω_c, then the system will produce minimum phase distortion.

8.1 Amplitude and Phase Response

Obtaining the amplitude and phase response analytically

Determine $H(s)$ of the LTIC system and/or network, and let $s = j\omega$ in the system function and then express $H(j\omega)$ in polar form.

$$H(s) = \frac{V_2(s)}{V_1(s)} = \frac{1/RC}{s + 1/RC} \tag{8.3}$$

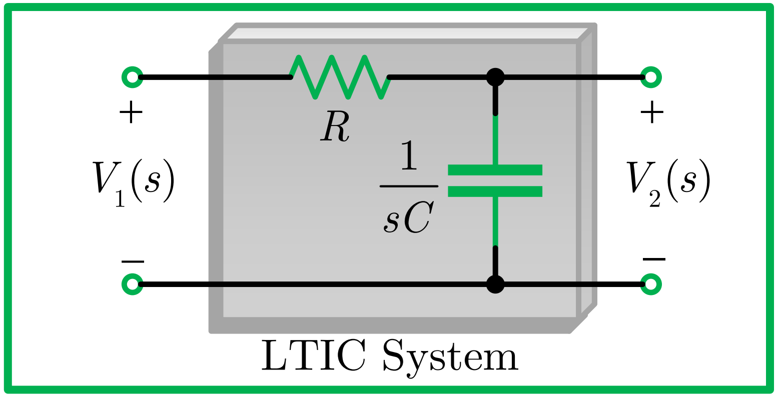


FIG. 8.2

- let $s = j\omega$.

$$H(j\omega) = \frac{1/RC}{j\omega + 1/RC} \tag{8.4}$$

- Vividly, in Polar form $H(j\omega)$ is of the form:

$$H(j\omega) = \frac{1/RC}{\left[\omega^2 + 1/R^2C^2\right]^{1/2}} e^{-j\tan^{-1}\omega RC} = M(\omega)e^{j\phi(\omega)} \tag{8.5}$$

[Uses include radio's turner]

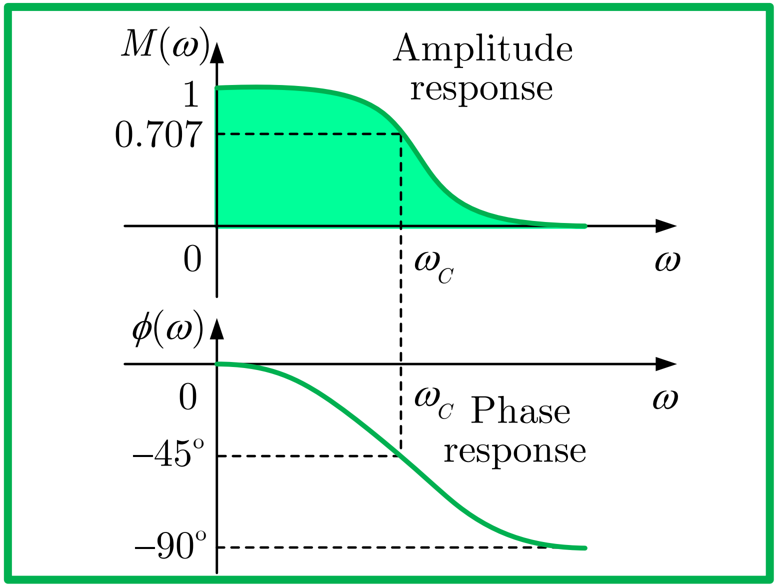


FIG. 8.3

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8.1 Amplitude and Phase Response

Obtaining the amplitude and phase response Graphically

Lets consider a method of obtaining the amplitude and phase response from the pole-zero diagram of a system function. Suppose we have the system function,

$$H(s) = \frac{A_0(s-z_0)(s-z_1)}{(s-p_0)(s-p_1)(s-p_2)} \quad \textbf{(8.6)}$$

- Vividly, $H(j\omega)$ can be written in the form

$$H(j\omega) = \frac{A_0(j\omega-z_0)(j\omega-z_1)}{(j\omega-p_0)(j\omega-p_1)(j\omega-p_2)} \quad \textbf{(8.7)}$$

- Each of the factors $j\omega - z_i$ or $j\omega - p_k$ corresponds to a vector from the zero z_i or pole p_k directed to any point $j\omega$ on the imaginary axis. Thus, in polar form,

$$j\omega - z_i = \mathbf{N}_i e^{j\psi_i}; \qquad j\omega - p_k = \mathbf{M}_k e^{j\theta_k} \quad \textbf{(8.8)}$$

- Thus, $H(j\omega)$ is of the form

$$H(j\omega) = \frac{A_0\mathbf{N}_0\mathbf{N}_1}{\mathbf{M}_0\mathbf{M}_1\mathbf{M}_2} e^{j(\psi_0+\psi_1-\theta_0-\theta_1-\theta_2)} \quad \textbf{(8.9)}$$

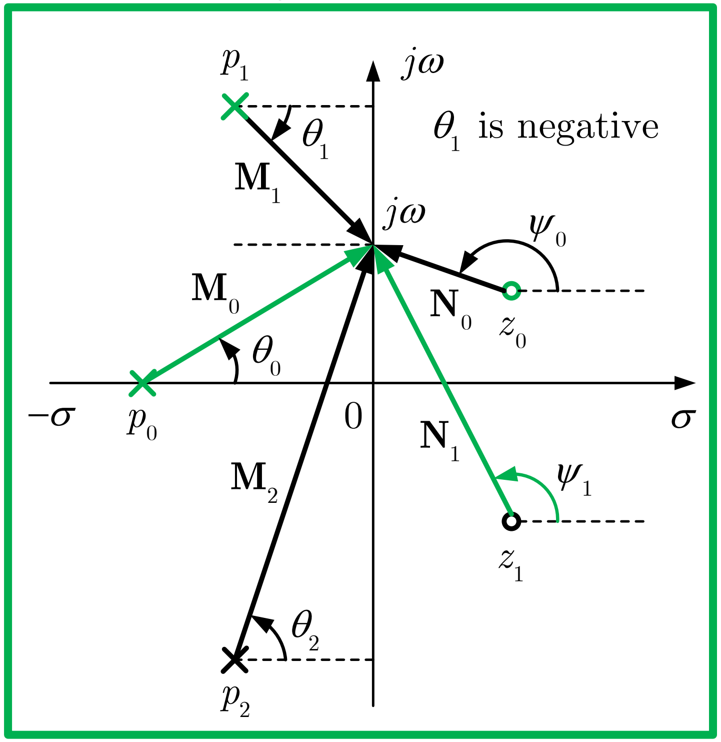

FIG. 8.4. Evaluation of amplitude and phase from pole-zero diagram.

8.1 Amplitude and Phase Response

Obtaining the amplitude and phase response Graphically

In general, we express the amplitude response $M(\omega)$ in terms of the following equation.

$$M(\omega) = \frac{\prod_{i=0}^{n} \text{vector magnitudes from zeros to the point on the } j\omega \text{ axis}}{\prod_{j=0}^{m} \text{vector magnitudes from poles to the point on the } j\omega \text{ axis}} \quad \textbf{(8.10)}$$

Similarly, the phase response is given as,

$$\begin{aligned}\phi(\omega) &= \sum_{i=0}^{n} \text{angles of the vectors from zeros to the } j\omega \text{ axis} \\ &\quad - \sum_{j=0}^{m} \text{angles of the vectors from poles to the } j\omega \text{ axis}\end{aligned} \quad \textbf{(8.11)}$$

☯ It is important to note that these relationships for amplitude and phase are point-by-point relationships only. In other words, we must draw vectors from the poles and zeros to every point on the $j\omega$ axis for which we wish to determine amplitude and phase.

8.1 Amplitude and Phase Response

Obtaining the amplitude and phase response Graphically

As an illustration, consider the LTIC system function of the form,

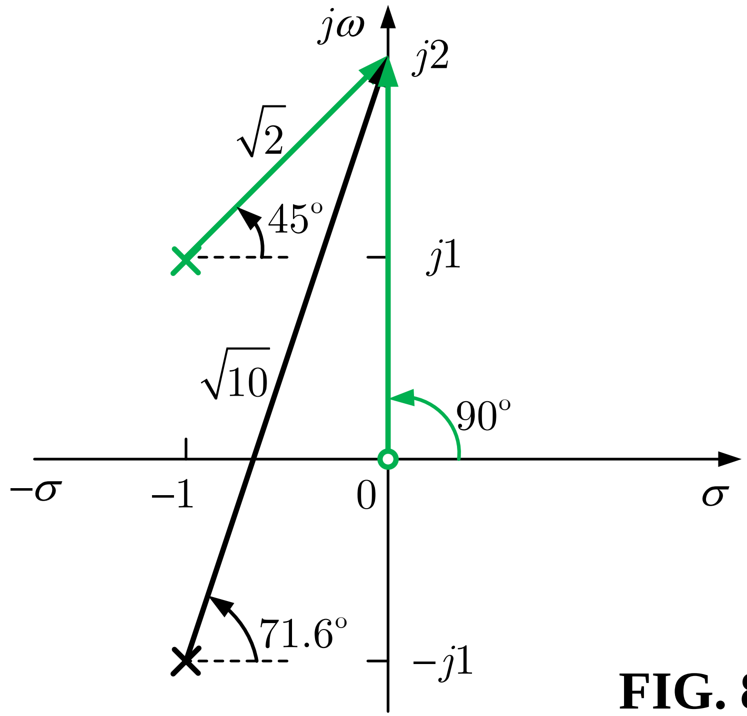

FIG. 8.5

$$F(s) = \frac{4s}{s^2 + 2s + 2} = \frac{4s}{(s+1+j)(s+1-j)} \tag{8.12}$$

Let us find the amplitude and phase for $F(j2)$.

$$M(j2) = 4\left\{\frac{2}{\sqrt{2}\times\sqrt{10}}\right\} = 1.78 \tag{8.13}$$

$$\phi(j2) = 90^{\circ} - 45^{\circ} - 71.6^{\circ} = -26.6^{\circ} \tag{8.14}$$

With these values and the amplitude and phase at three or four additional points, we have enough information for a rough estimate of the amplitude and phase response.

8.1 Amplitude and Phase Response

Obtaining the amplitude and phase response Graphically

- So let us consider at $\omega = 0$, for

$$F(j\omega) = \frac{4(j\omega)}{(j\omega)^2 + 2(j\omega) + 2} = \frac{4(j\omega)}{[(j\omega)+1+j][(j\omega)+1-j]} \quad \textbf{(8.15)}$$

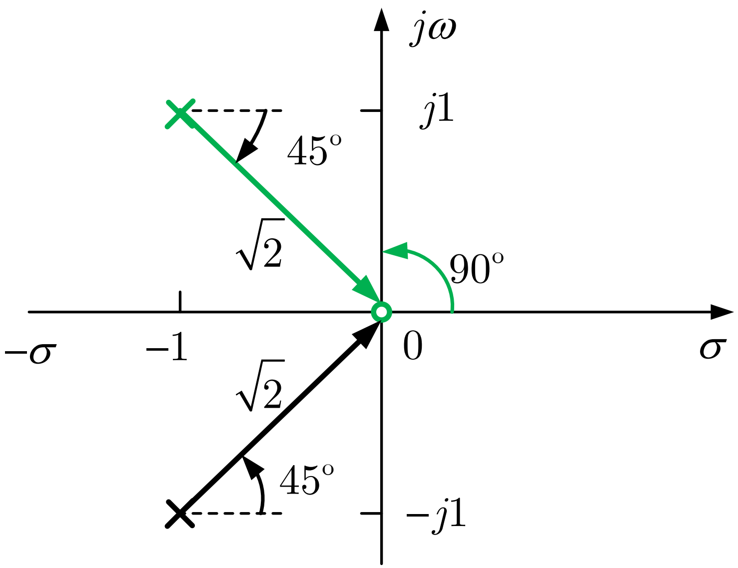

FIG. 8.6

- It follows that,

$$\lim_{\substack{\omega\to 0\\ \omega>0}} F(j\omega) = \frac{4(j0)}{(1+j)(1-j)} \quad \textbf{(8.16)}$$

- Thus, $M(j0) = 0$ **(8.17)**

- It is worth noting that the zero at the origin still contributes a 90 degree phase shift, even though the vector magnitude is zero.

- Furthermore, the net phase at $\omega = 0$ is of the form

$$\phi(j0) = 90^\circ - 45^\circ + 45^\circ = 90^\circ \quad \textbf{(8.18)}$$

8.1 Amplitude and Phase Response

Obtaining the amplitude and phase response Graphically

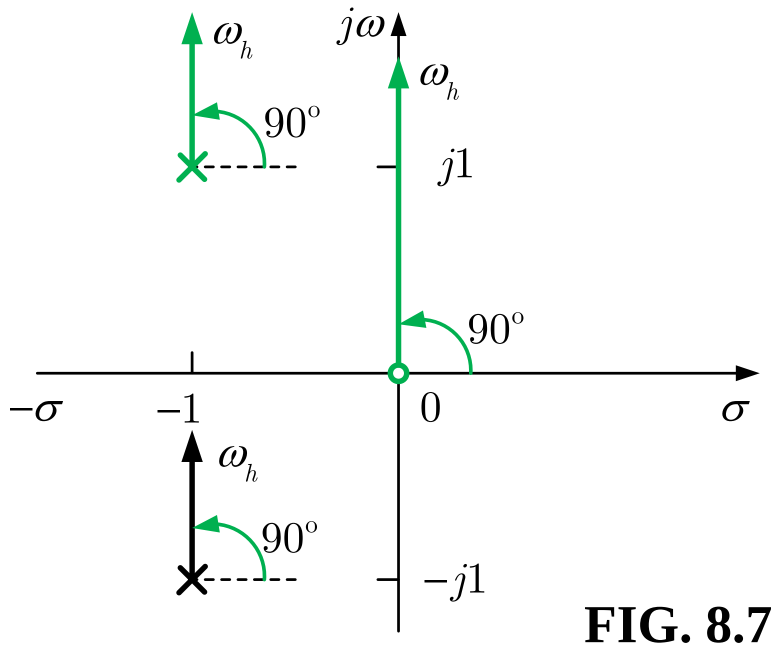

FIG. 8.7

- Next lets consider at very high frequency ω_h , where $\omega_h \gg 1$.
- Vividly, all vectors are approximately equal to $\omega_h e^{j90^o}$.
- It follows that,

$$M(\omega_h) \simeq \frac{4\omega_h}{\omega_h^2} = \frac{4}{\omega_h} \qquad \textbf{(8.19)}$$

$$\phi(\omega_h) \simeq 90^o - 90^o - 90^o = -90^o \qquad \textbf{(8.20)}$$

- Expanding this Analysis for the frequencies listed in Table 8.1, we obtain values for amplitude, $M(\omega)$ and phase, $\phi(\omega)$.

8.1 Amplitude and Phase Response

Obtaining the amplitude and phase response Graphically

Table 8.1

Frequency, ω	Amplitude, $M(\omega)$	Phase, $\phi(\omega)$ [degrees]
1.0	1.8	26.6
1.5	2.0	–4.8
3.0	1.3	–49.4
5.0	0.8	–66.5
10.0	0.4	–78.5

- Thus, from Table 8.1, we can sketch the amplitude and phase curves and/or frequency spectra shown in Fig 8.8 for the LTIC system function of equation 8.21.

$$F(s) = \frac{4s}{s^2 + 2s + 2} = \frac{4s}{(s+1+j)(s+1-j)};\ i.e.,\quad F(j\omega) = \frac{4(j\omega)}{[1+j(\omega+1)][1+j(\omega-1)]} \quad \textbf{(8.21)}$$

- Implying that,

$$F(j\omega) = \left\{ \frac{4\omega}{\sqrt{[1+(\omega+1)^2]}\cdot\sqrt{[1+(\omega-1)^2]}} \right\} e^{j[90^\circ - \tan^{-1}(\omega+1) - \tan^{-1}(\omega-1)]} \quad \textbf{(8.22)}$$

8.1 Amplitude and Phase Response

Obtaining the amplitude and phase response Graphically

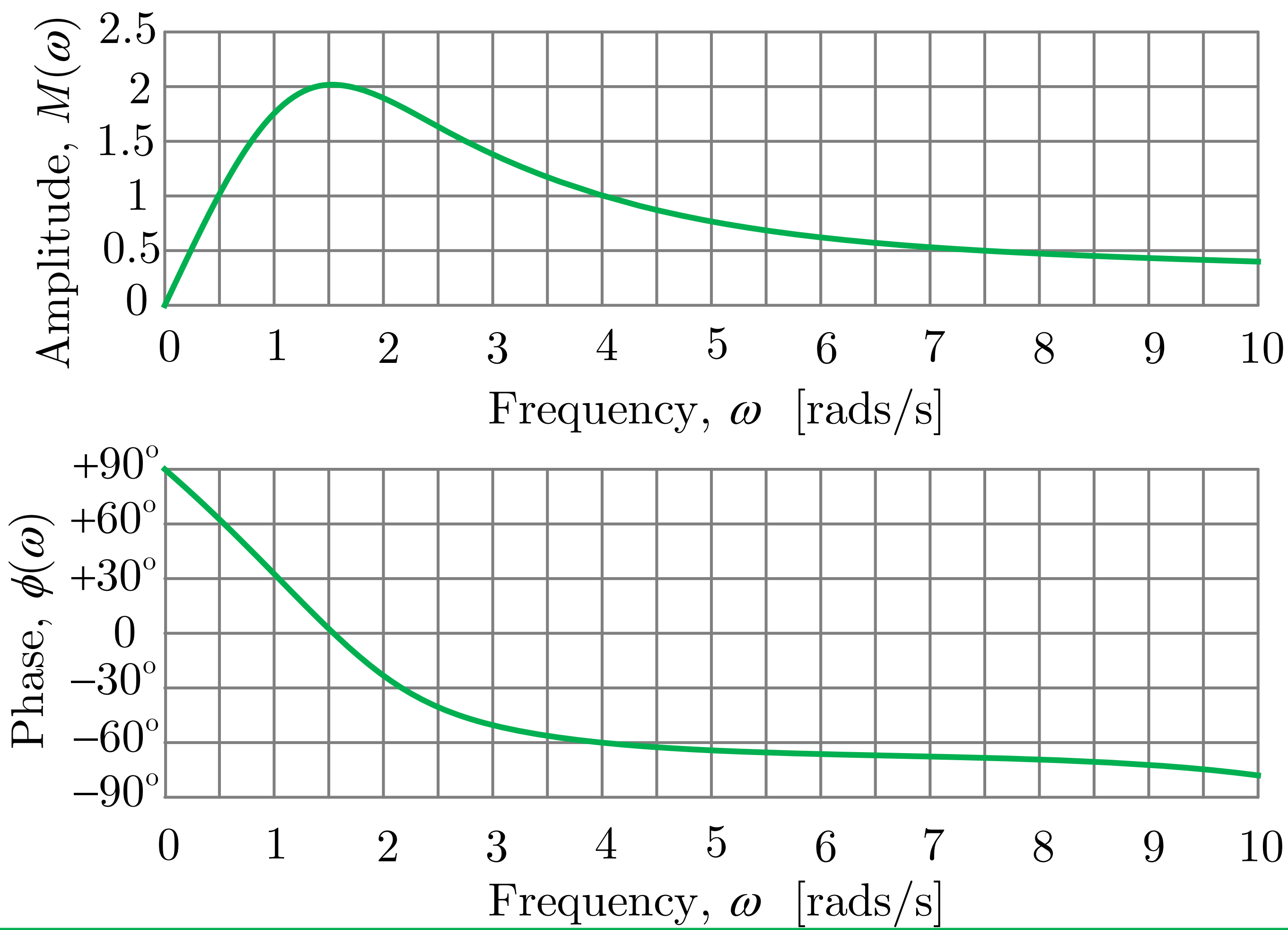

FIG. 8.8. Amplitude and phase response for $F(s)$ in Eq. (8.21).

8.1 Amplitude and Phase Response

Effect of poles and zeros on the $j\omega$ axis upon frequency response

Consider the LTIC system function of the form,

$$F(s) = \frac{s^2 + 1.03}{s^2 + 1.23} = \frac{(s + j1.015)(s - j1.015)}{(s + j1.109)(s - j1.109)} \qquad \textbf{(8.23)}$$

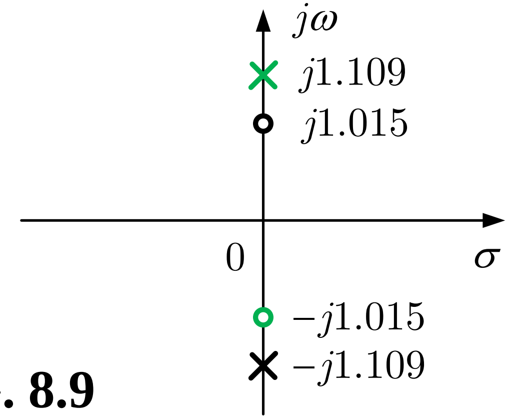

FIG. 8.9

Amplitude:

- At $\omega = 1.015$, vector from zero to that frequency is of zero magnitude.
- Therefore at a zero on the $j\omega$ axis, the amplitude response is zero.
- At $\omega = 1.109$ the vector from the pole to that frequency is zero magnitude.
- The amplitude is infinite at a pole.

Phase:

- when $\omega < 1.015$, phase is zero
- when $\omega > 1.015$ and $\omega < 1.109$ the vector from the zero is now pointing upward, while the vectors from the other poles and zeros are oriented in the same direction as for $\omega < 1.015$ resulting in $+180$ for increasing frequency and the opposite for reducing frequency.

8.1 Amplitude and Phase Response

Effect of poles and zeros on the $j\omega$ axis upon frequency response

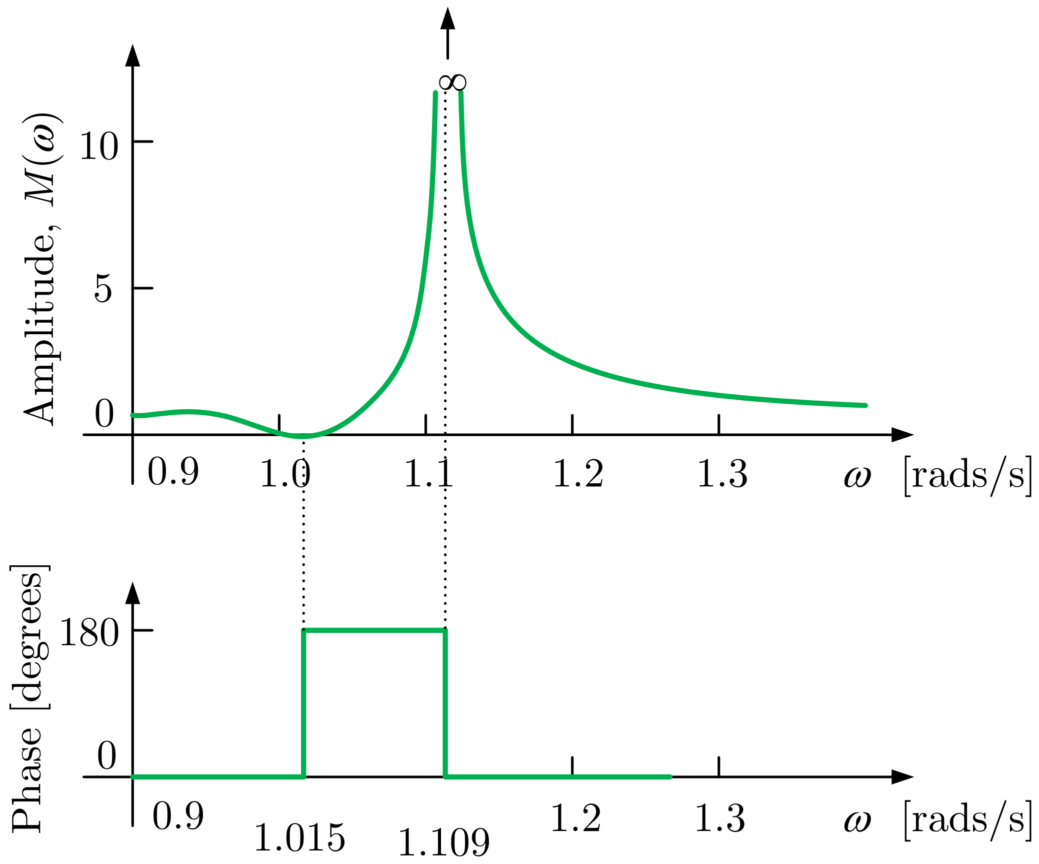

FIG. 8.10. Amplitude and phase response for $F(s)$ in Fig. 8.9.

8.1 Amplitude and Phase Response

Effect of poles and zeros on the very near $j\omega$ axis upon frequency response

- For $z = -\sigma \pm j\omega_i$, where σ is very small compared to ω_i, then we will have a dip in the amplitude, and a rapid change of phase near $\omega = \omega_i$.
- Similarly for $p = -\sigma \pm j\omega_i$, where σ is very small compared to ω_j, then amplitude will peak, and a phase will decrease rapidly near $\omega = \omega_j$.

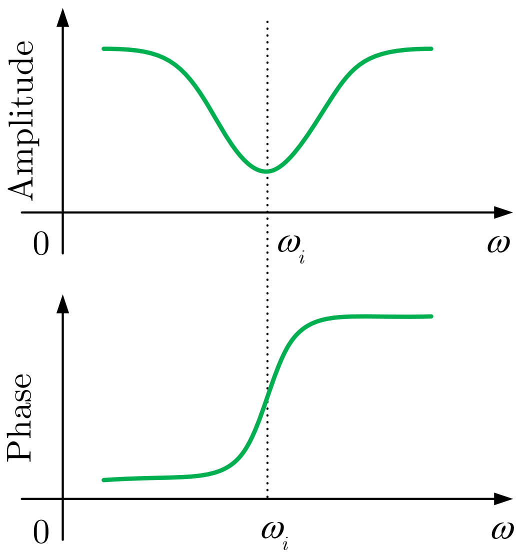

FIG. 8.11. Effect of zero very near the $j\omega$ axis.

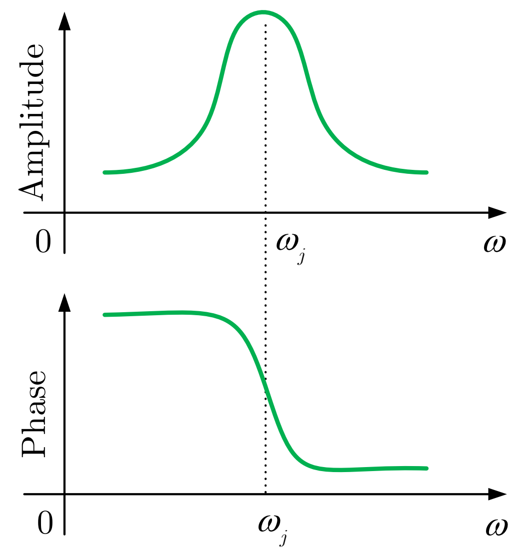

FIG. 8.12. Effect of pole very near the $j\omega$ axis.

- For poles or zeros very far from the $j\omega$ axis, i.e. σ is large when compared to frequency of interest, then these contribute less to shaping of the amplitude and phase. Their only effect is to scale up or down overall amplitude response.

8.1 Amplitude and Phase Response

Minimum and Nonminimum phase functions

- For stability we know that there must be no poles on the right half of the *s* plane.
- However, transfer function may have zeros on the right half of the plane.

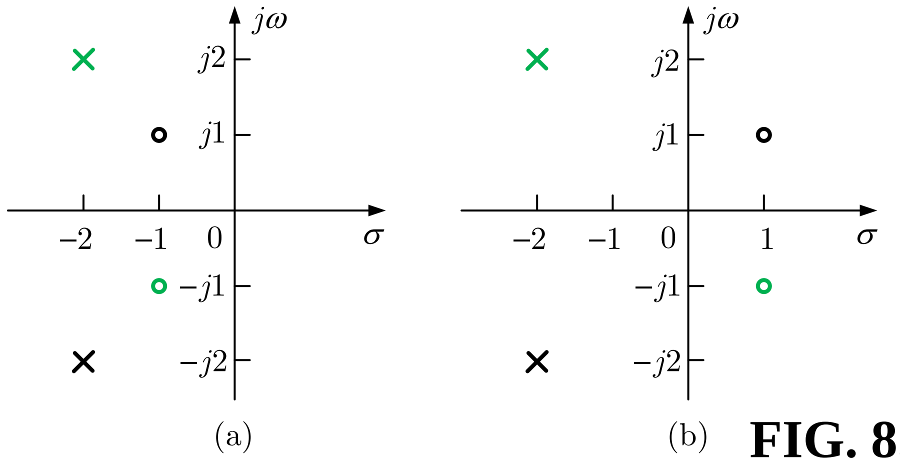

FIG. 8.13

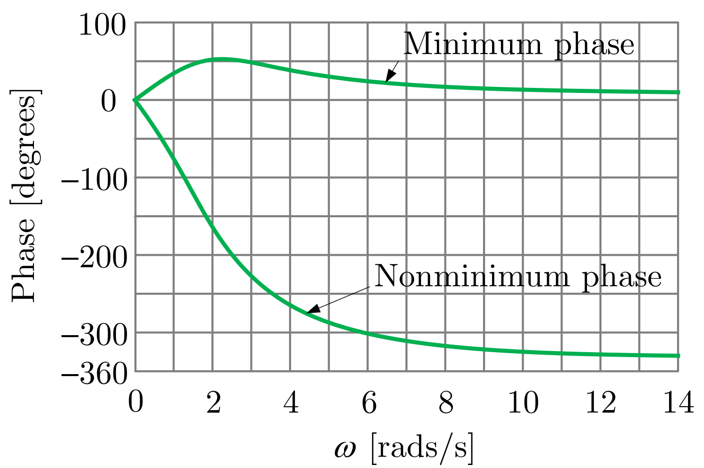

FIG. 8.14

- Consider Fig 8.13
- phase of (b) is greater than the phase of (a) for all frequencies. This is because the zeros in the right-half plane contribute more phase shift (on absolute magnitude basis) than their counterparts in the left-half plane
- Minimum phase function: system with zeros in the left-half plane or $j\omega$ axis.
- Nonminimum phase function: function with one, or more zeros in the right-half plane.

8.1 Amplitude and Phase Response

All-pass function

- A system whose poles are only in the left-half plane and whose zeros are mirror images of the poles about the $j\omega$ axis.
- These networks are often used to correct for phase distortion in a transmission system.

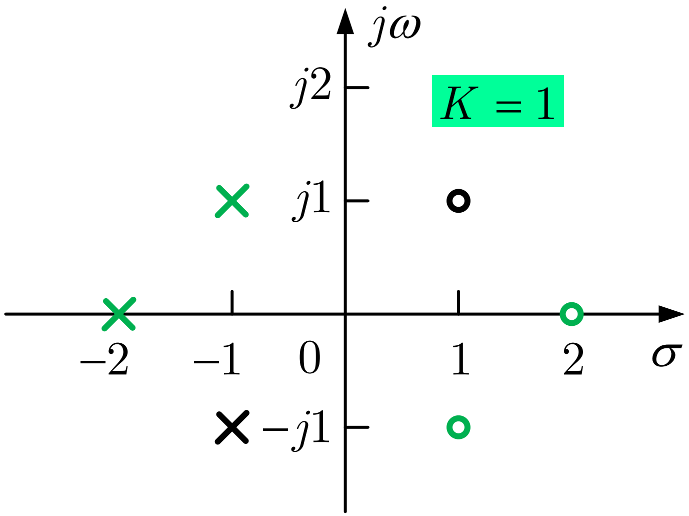

FIG. 8.15. All-pass LTIC system.

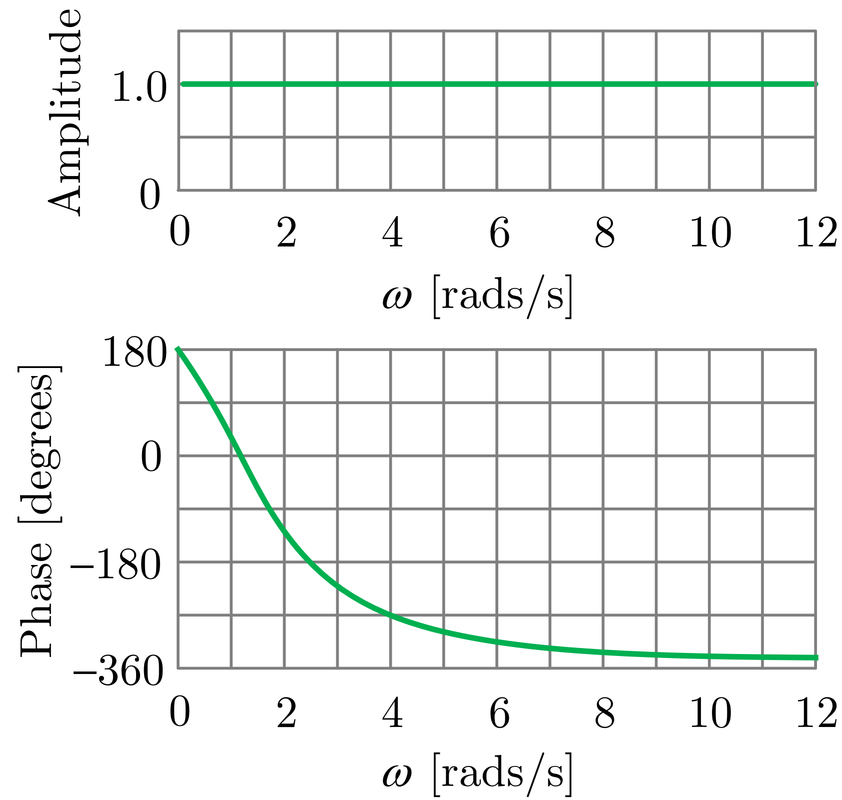

FIG. 8.16.
Amplitude and phase of all-pass LTIC system in FIG. 8.15.

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8.2 Bode Plots

In this section we turn our attention to semi logarithmic plots of amplitude and phase versus frequency, commonly known as Bode plots.

Consider the system function

$$H(s) = \frac{N(s)}{D(s)} \tag{8.24}$$

We know that the amplitude is

$$M(\omega) = |H(j\omega)| = \frac{|N(j\omega)|}{|D(j\omega)|} \tag{8.25}$$

If we express the amplitude in terms of decibels, we have

$$20\log M(\omega) = 20\log|N(j\omega)| - 20\log|D(j\omega)| \tag{8.26}$$

Significance of Bode plots:

- Datasheets for transistor amps, etc
- Does transistor have L/C ?
- Does capacitor have L?

8.2 Bode Plots

In factored form both $N(s)$ and $D(s)$ are made up of four kinds of factors:

a) A constant, K
b) A root at the origin, s
c) A simple real root, $s + \alpha$
d) A complex pair of roots, $s^2 + 2\alpha s + \alpha^2 + \beta^2$

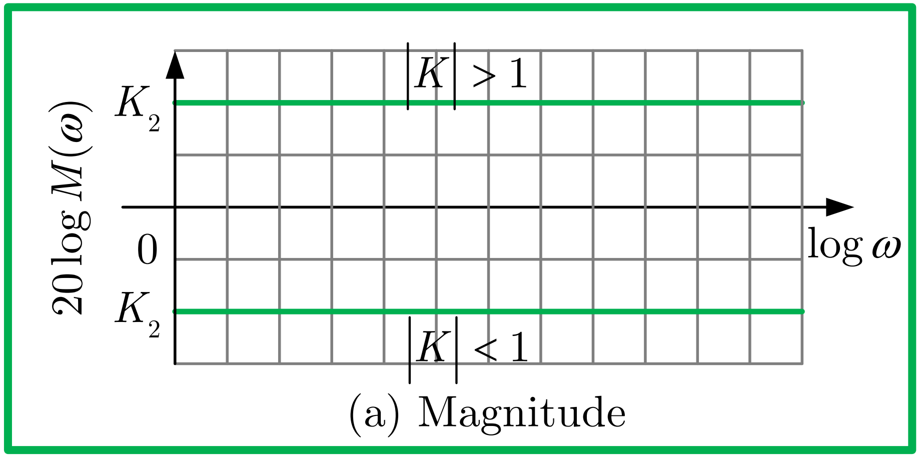
To understand the nature of log-amplitude plots, we need only examine the amplitude response of the four kinds of factors just cited.

- ☯ if these factors are in $N(s)$ → their magnitudes in decibels carry positive sign
- ☯ if these factors are in $D(s)$ → their magnitudes in decibels carry negative sign

8.2 Bode Plots

The constant, *K*

- The dB loss (or gain) is $K_2 = 20\log K$.
- The constant K_2 is either negative if $|K|<1$, or positive if $|K|>1$.
- The phase response is either zero or 180° depending on whether K is positive or negative.



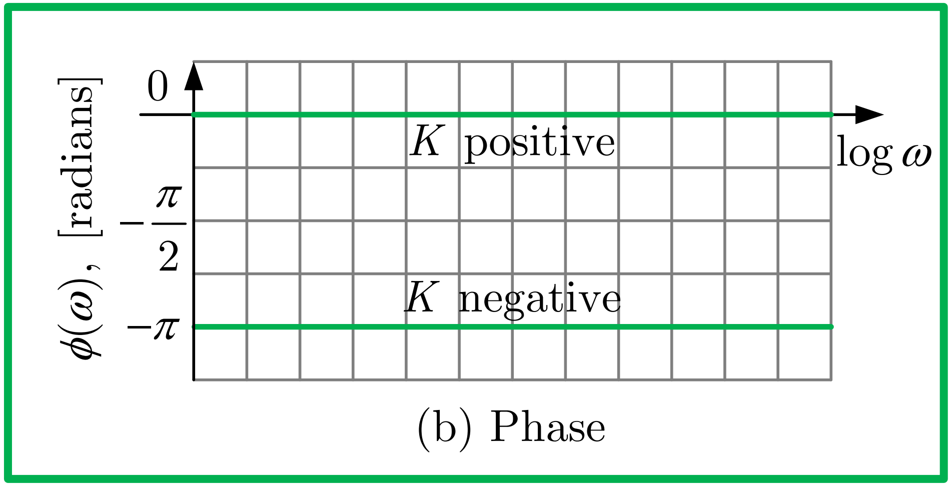


FIG. 8.17. Magnitude and phase of constant.

8.2 Bode Plots

The factor s

- The dB loss (or gain) associated with a pole (zero) at the origin is $\pm 20 \log \omega$.
- Thus the plot of magnitude in decibels versus frequency in semilog coordinates is a straight line with slope of ± 20 dB/decade or ± 6 dB/octave.
- From the bode plot we see that the zero loss point (in decibels) is at $\omega = 1$, and the phase is constant for all ω.

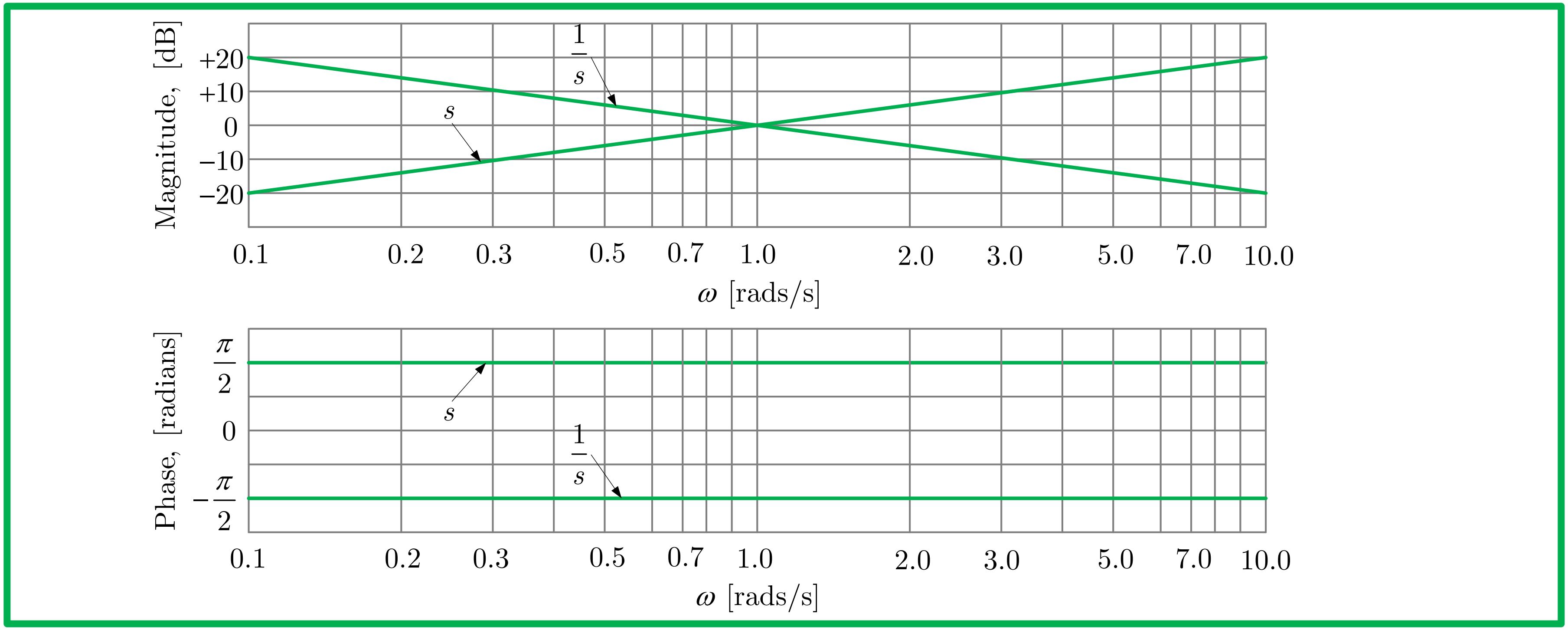


FIG. 8.18. Magnitude and phase of pole or zero at $s = 0$.

8.2 Bode Plots

The factor $s + \alpha$

- For convenience, let us set $\alpha = 1$,
- Thus, a straight-line approximation of the actual magnitude against frequency curve can be obtained from examining the asymptotic behavior of the factor $j\omega + 1$.
- For $\omega \ll 1$, the low-frequency asymptote is

$$20\log\left|j\omega+1\right|_{\omega\ll 1} \cong 20\log 1 = 0\,\mathrm{dB} \qquad \textbf{(8.27)}$$

- For $\omega \gg 1$, the high-frequency asymptote is

$$20\log\left|j\omega+1\right|_{\omega\gg 1} \cong 20\log\omega\ \mathrm{dB} \qquad \textbf{(8.28)}$$

- Vividly, the latter line has a slope of $20\mathrm{dB}/\mathrm{decade}$ or $6\mathrm{dB}/\mathrm{Octave}$.
- The low- and high frequency asymptotes meet at $\omega = 1$, which we designate as the break frequency or cutoff frequency.
- The maximum error is at the break frequency $\omega = 1$, or in unnormalized form: $\omega = \alpha$.
- For quick estimates of magnitude response, the straight-line approximation is an invaluable visual aid.
- An important example of it's use is in the design of linear control systems.

8.2 Bode Plots

The factor $s + \alpha$

Table 8.2 brings to the fore the comparison between the actual magnitude response values and the quick estimate of magnitude response values obtained by straight-line approximation.

Table 8.2

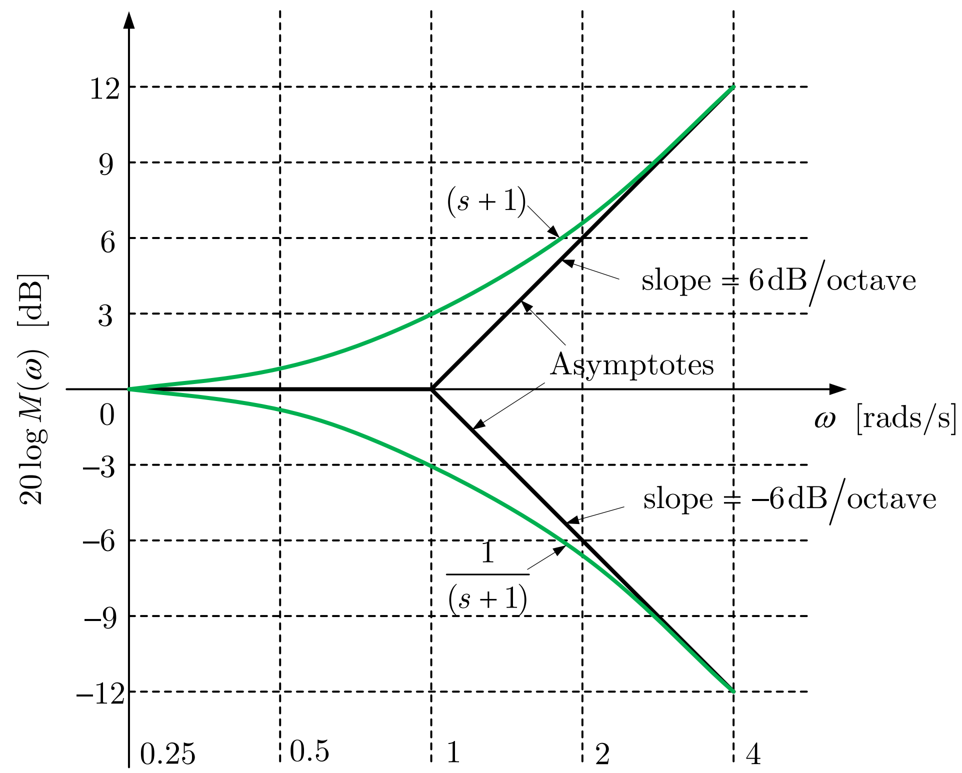
Frequency, [radians]	Actual Magnitude, [dB]	Straight-Line Approximation [dB]	Error, [dB]
$\omega = \frac{1}{4}$ 2 octaves below	±0.3	0	±0.3
$\omega = \frac{1}{2}$ octave below	±1	0	±1
$\omega = 1$ break frequency	±3	0	±3
$\omega = 2$ octave above	±7	±6	±1
$\omega = 4$ 2 octaves above	±12.3	±12	±0.3

8.2 Bode Plots

The factor $s + \alpha$

For convenience, let us set $\alpha = 1$, then the magnitude is

$\pm 20\log|j\omega + 1| = \pm 20\log(\omega^2 + 1)^{1/2}$, and the phase is $\phi(\omega) = \mathrm{Arg}(j\omega + 1)^{\pm 1} = \pm\tan^{-1}\omega$



(a) Magnitude Bode plot

FIG. 8.19(a). Magnitude of a simple real pole or zero $(s + \alpha)$.

8.2 Bode Plots

The factor $s+\alpha$

Vividly, since the phase for $\alpha = 1$ is $\phi(\omega) = \mathrm{Arg}(j\omega+1)^{\pm 1} = \pm\tan^{-1}\omega$, it follows that the curve of a real pole or zero is of the form illustrated in FIG. 8.19(b).

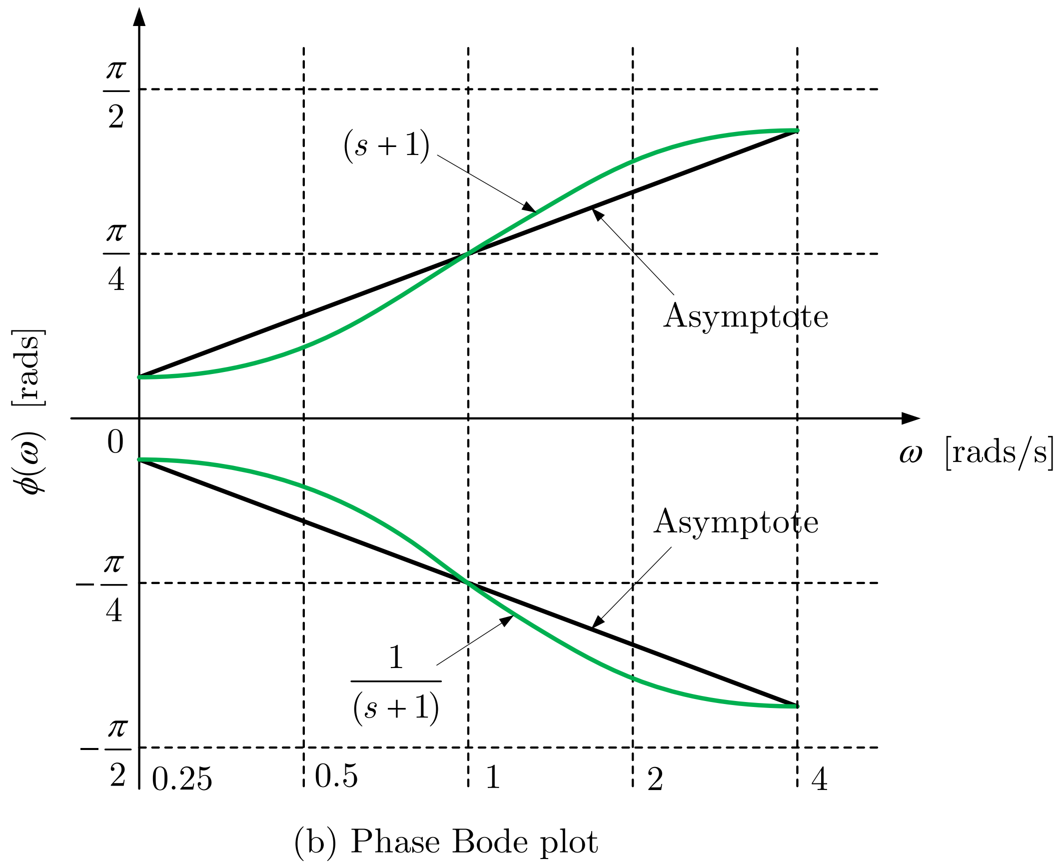


FIG. 8.19(b). Phase of a simple real pole or zero $(s+\alpha)$.

8.2 Bode Plots

Complex conjugate roots

- For complex conjugate roots, it is convenient to adopt standard symbols so that we can use the universal curves that result therefrom.
- We describe the conjugate pole(zero) pair in terms of a magnitude ω_0 and an angle θ measured from the negative real axis, as shown in Fig 8.20.
- Explicitly, the parameters that describe the pole (zero) positions are ω_0 which is the undamped frequency of oscillation, and $\zeta = \cos\theta$ known as the damping factor.

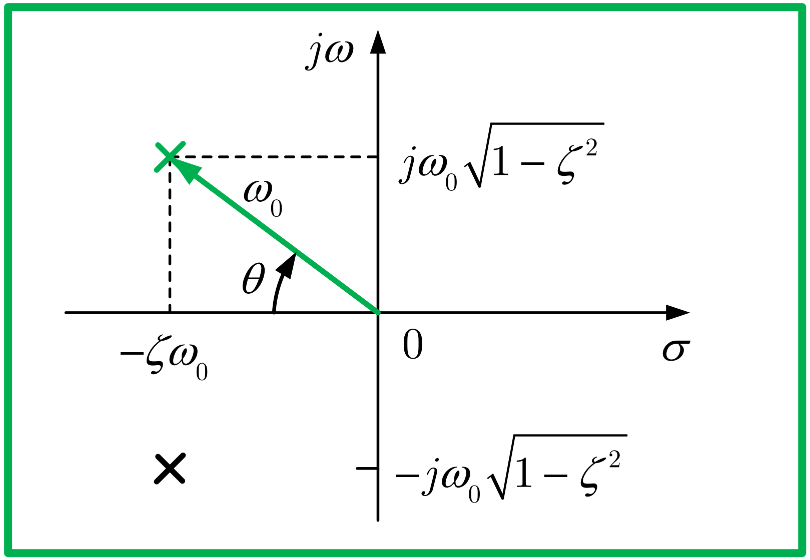


FIG. 8.20. Pole location in terms of ζ and ω_0.

$$p_{1,2} = -\alpha \pm j\beta \qquad \textbf{(8.29)}$$

$$\begin{aligned} \alpha &= \omega_0 \cos\theta = \omega_0\zeta \\ \beta &= \omega_0 \sin\theta = \omega_0\sqrt{1-\zeta^2} \end{aligned} \qquad \textbf{(8.30)}$$

$$G(s) = \frac{1}{\left(s/\omega_0\right)^2 + 2\zeta\left(s/\omega_0\right) + 1} \qquad \textbf{(8.31)}$$

- Being a frequency normalized LTIC system function.

8.2 Bode Plots

Complex conjugate roots

- The closer the angle θ is to $\pi/2$, the smaller is the damping factor $(\zeta = \cos\theta)$.
- When the angle θ is nearly zero degrees, the damping factor is nearly unity.
- To examine the Bode plots for the conjugate pole (zero) pair, let us set $\omega_0 = 1$ for convenience.
- Thus, the magnitude is of the form

$$20\log M(\omega) = \pm 20\log\left|1 - \omega^2 + j2\zeta\omega\right| = \pm 20\log[(1-\omega^2)^2 + 4\zeta^2\omega^2]^{1/2} \qquad \textbf{(8.32)}$$

- It follows that the phase is of the form

$$\phi(\omega) = \mathrm{Arg}[(1-\omega^2) + j2\zeta\omega] = \tan^{-1}\left\{\frac{2\zeta\omega}{1-\omega^2}\right\} \qquad \textbf{(8.33)}$$

- If we examine the low and high-frequency asymptotes of the magnitude, we see that the low-frequency asymptote is 0 dB.
- The high-frequency asymptote for $\omega >> 1$ is at $\pm 40\log\omega$ dB, which is a straight line of slope $40\text{dB}/\text{decade}$ or $12\text{dB}/\text{octave}$.

8.2 Bode Plots

Complex conjugate roots

- The damping factor always plays a significant part in the closeness of the straight-line approximation. (Straight-line approximation close only when $\zeta = \ 0.6$).

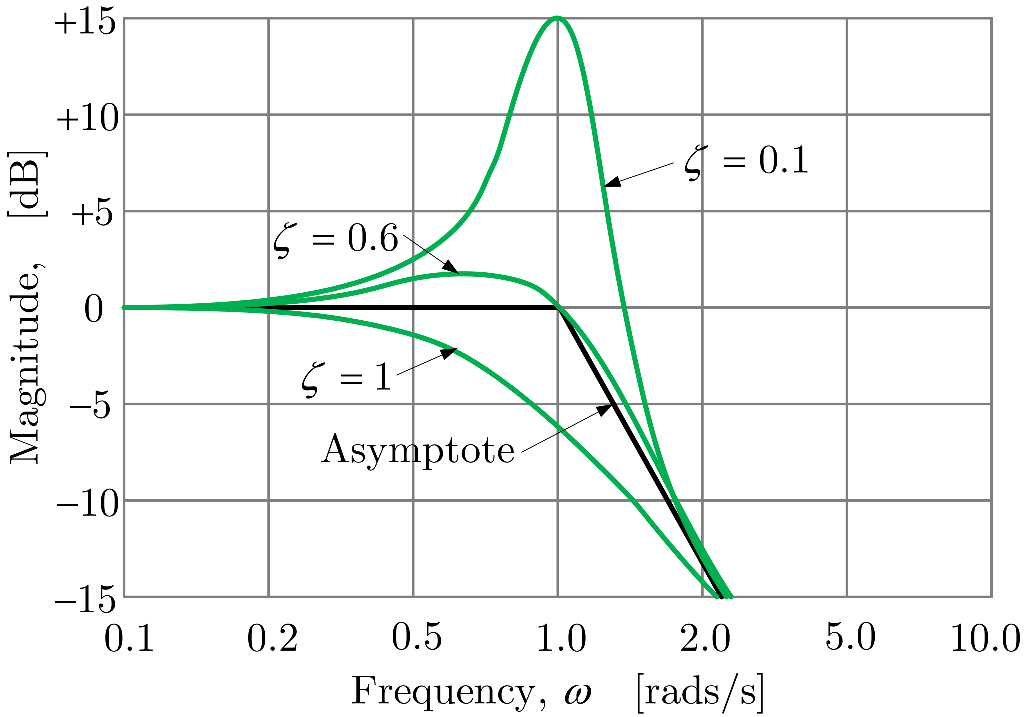


FIG. 8.21. Magnitude versus frequency for second-order pole.

8.2 Bode Plots

Complex conjugate roots

- Universal curves for magnitude and phase are plotted in Figs. 8.22 and 8.23 for the normalized function.

$$G(s) = \frac{1}{(s/\omega_0)^2 + 2\zeta(s/\omega_0) + 1} \quad \textbf{(8.33)}$$

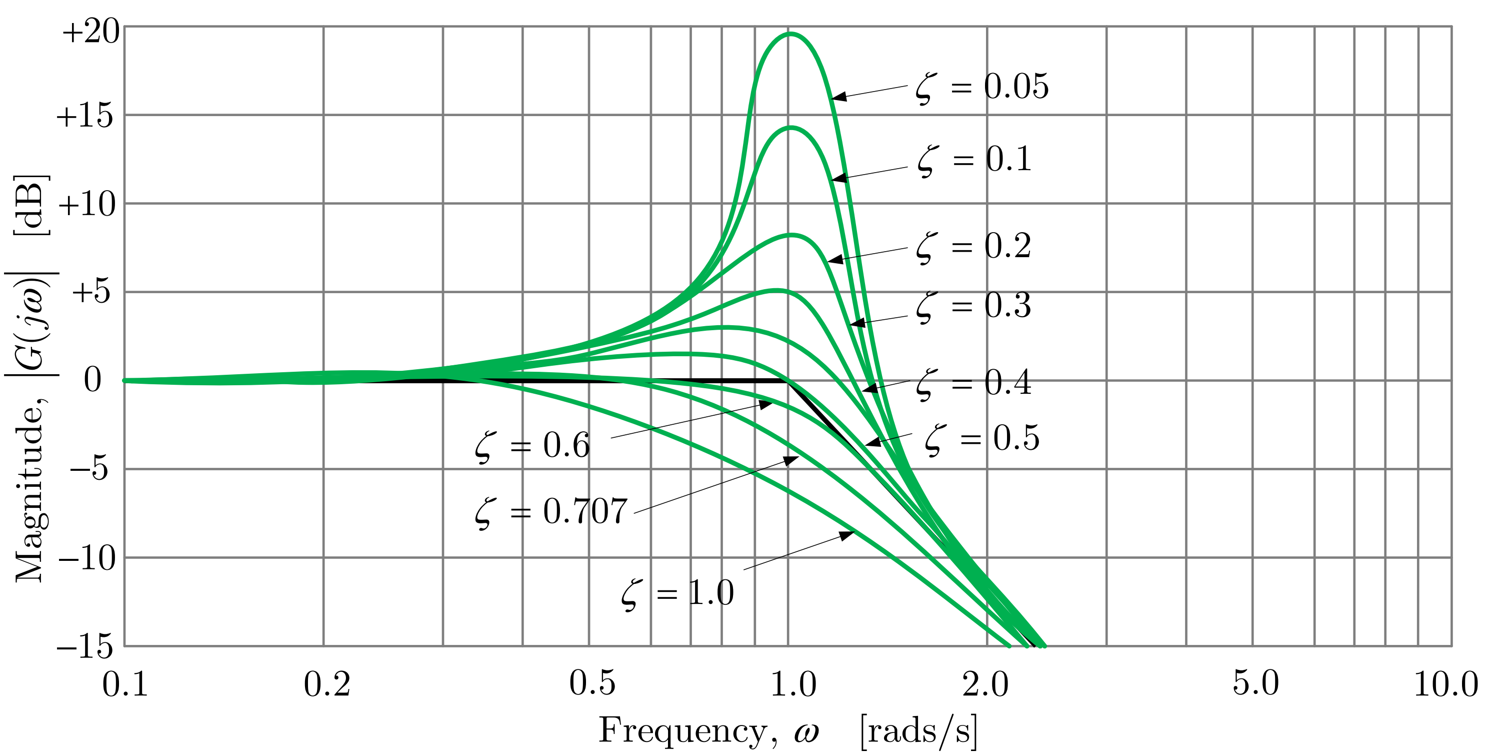


FIG. 8.22. Magnitude versus frequency for second-order pole.

8.2 Bode Plots

Complex conjugate roots

Vividly, we see that the phase response, as viewed from the semilog scale, ia an odd function about $\omega/\omega_0 = 1$. The phase at $\omega = \omega_0$ is -90° or $-\pi/2$ radians.

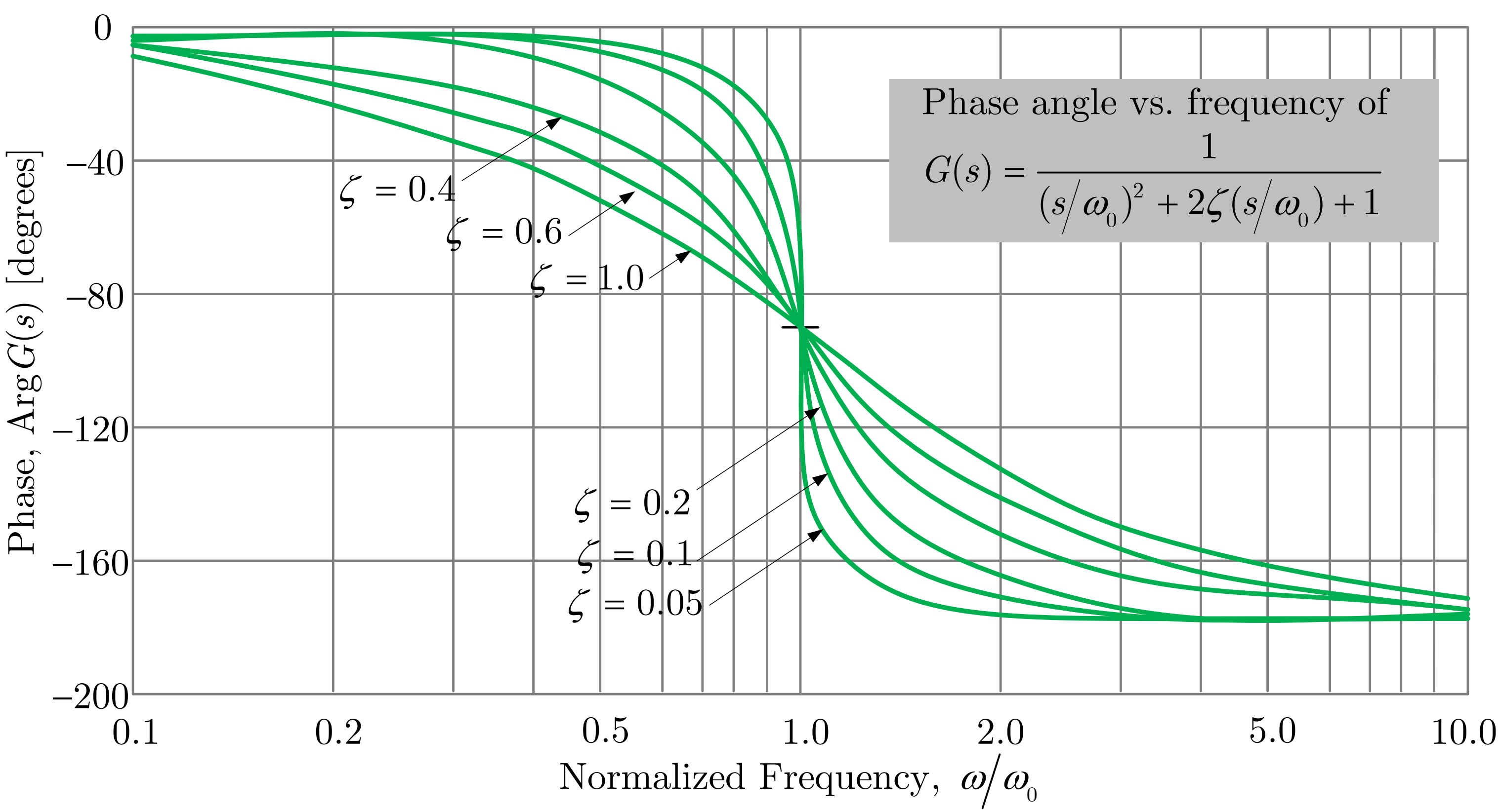


FIG. 8.23. Phase versus frequency for second-order pole.

8.2 Bode Plots

[Example 8.1]

- Using Bode plot asymptotes, construct the magnitude versus frequency curve for LTIC system of the form

$$G(s) = \frac{0.1s}{\left(\frac{s}{50}+1\right)\left(\frac{s^2}{16\times 10^4}+\frac{s}{10^3}+1\right)} \quad \mathbf{(8.34)}$$

[SOLUTION]

- Vividly, there are two first-order break frequencies at $\omega = 0$ and $\omega = 50$.
- In addition, there is a second-order break frequency at $\omega = 400$.
- With a quick calculation we find that $\zeta = 0.2$ for the second-order factor.
- It follows that the asymptotes are as shown in Fig. 8.24.
- The magnitude and phase plots are given in Fig. 8.25 through computer simulation using a MATLAB program and/or script.

8.2 Bode Plots

[Example 8.1] [SOLUTION]

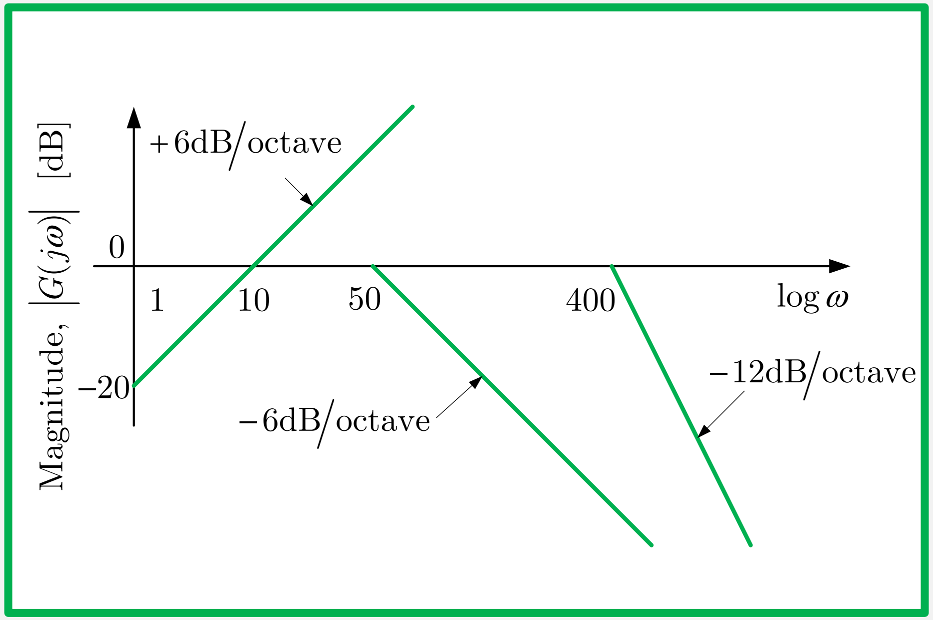
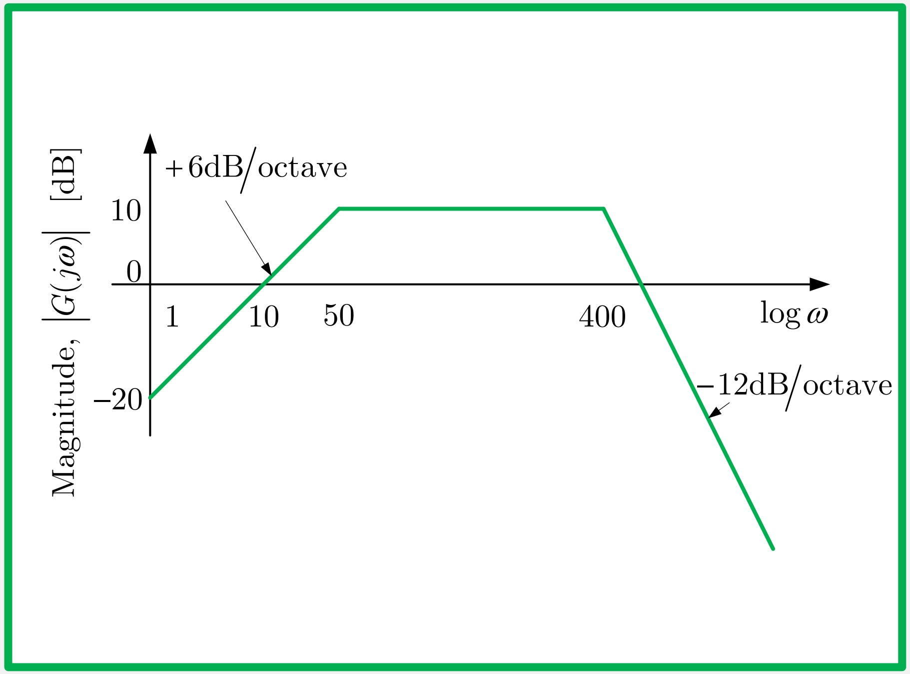
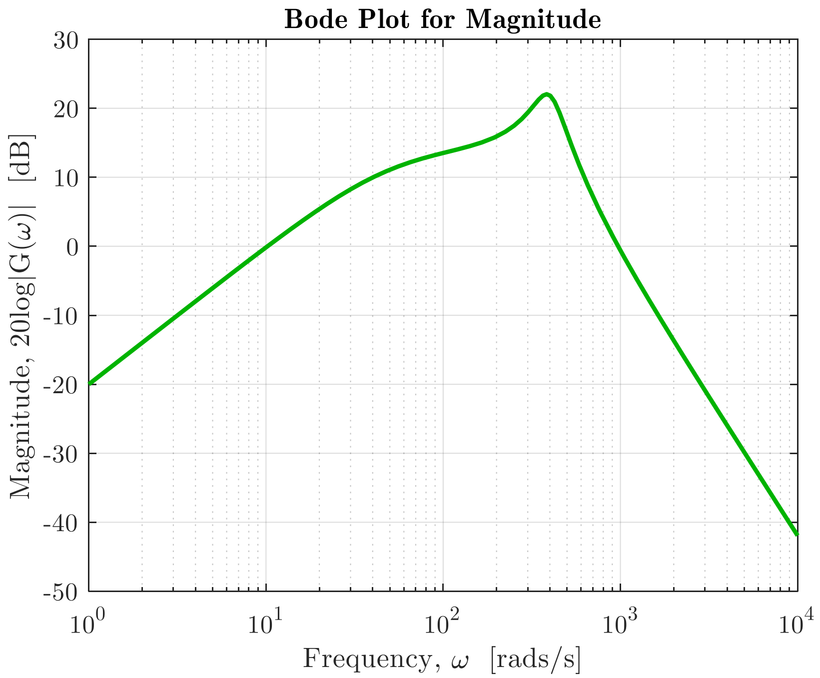


FIG. 8.24. Asymptotes for $G(s)$ in Eq. (8.34).



8.2 Bode Plots
[Example 8.1] [SOLUTION]



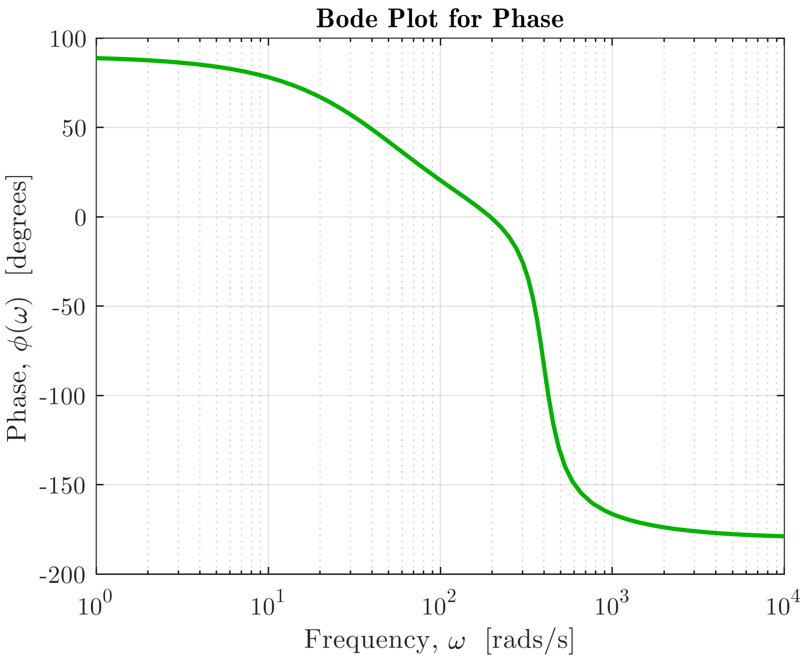


FIG. 8.25. (a) Magnitude and **(b)** phase for $G(s)$ in Eq. (8.34).

8.2 Bode Plots

[Example 8.1] [SOLUTION]

```
%------------------------------------------------------------------------
% lecture8_BodePlot.m
% Program to plot a Bode Diagram for the LTIC System Function
% G(s) = 0.1s/[{s/50+1}{(s^2/(16*10^4))+(s/10^3)+1}]
% Programmer: Eng. Jerry Muwamba
% University of Zambia, School of Engineering
% Department of Electrical and Electronic Engineering
% Date: 26 May 2021
%------------------------------------------------------------------------
clc;
close all;
clf;
num = [0.1 0];
den1 = [(1/50) 1];
den2 = [(1/400)^2 (1/(10^3)) 1];
den = conv(den1,den2);
H_sys = tf(num,den);
[M_w,Phi_w,w] = bode(H_sys);
M_W = zeros(1,length(M_w));
Phi_W = zeros(1,length(Phi_w));
for i=1:length(M_w)
    M_W(:,i) = M_w(:,:,i);
    Phi_W(:,i) = Phi_w(:,:,i);
end
M_dB = 20*log10(M_W);
Phi_deg = Phi_W;
W = w';
```

[Example 8.1] [SOLUTION]

```
%-------------------------------------------------------------------------
% Plotting of Magnitude Bode Diagram
%-------------------------------------------------------------------------
semilogx(W,M_dB, 'Color',[0 0.7 0],'LineWidth',2);
xlabel('Frequency, \omega  [rads/s]');
ylabel('Magnitude, 20log|G(\omega)|  [dB]');
title('Bode Plot for Magnitude')
set(gca, 'FontName','Euclid', 'FontSize',12);
grid on; figure;
%-------------------------------------------------------------------------
%Plotting of Phase Bode Diagram
%-------------------------------------------------------------------------
semilogx(W,Phi_deg, 'Color',[0 0.7 0],'LineWidth',2);
xlabel('Frequency, \omega  [rads/s]');
ylabel('Phase, \phi(\omega)  [degrees]');
title('Bode Plot for Phase')
set(gca, 'FontName','Euclid', 'FontSize',12);
grid on;
%------------------------- End of Program --------------------------------
```

FIG. 8.26. MATLAB program to plot the magnitude and phase for $G(s)$ in Eq. (8.34) depicted in Fig. 8.25.

End of Lecture 8

Thank you for your attention!