EEE 3121 - Signals & Systems

Lecture 9: Systems Analysis II

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References

Our main reference text book in this course is

[1] B. P. Lathi and R. A. Green, Linear Systems and Signals, 3rd Ed., 2018, Oxford University Press, New York. ISBN 978-0-19-020017-6

[2] Kuo Franklin, F., Network Analysis and Synthesis, 3rd Ed., 1986, J. Wiley (SE), ISBN 0-471-51118-8.

[3] Sundararanjan, D., A Practical Approach to Signals and Systems, 2008, John Wiley & Sons (Asia) Pte Ltd, ISBN 978-0-470-82353-8.

However, feel free to use pretty much any additional text which you might find relevant to our course.

9.1 System and Network Functions

- In electric system theory, the word port has a special meaning.
- A port may be regarded as a pair of terminals in which the current into one terminal equals the current out of the other.
- Thus, for the one-port LTIC System shown in Fig. 9.1, $I = I'$.
- A one-port LTIC System is completely specified when the voltage-current relationship at the terminals of the port is given.
- Whether the one-port is actually a single 5-ohm resistor, two 2.5-ohm resistors in series, or two 10-ohm resistors in parallel, is of little importance because the primary concern is the current-voltage relationship at the port.
- Consider the example in which $I = 2s + 3$ and $V = 1$; then the input admittance of the one-port is of the form of Fig. 9.2.

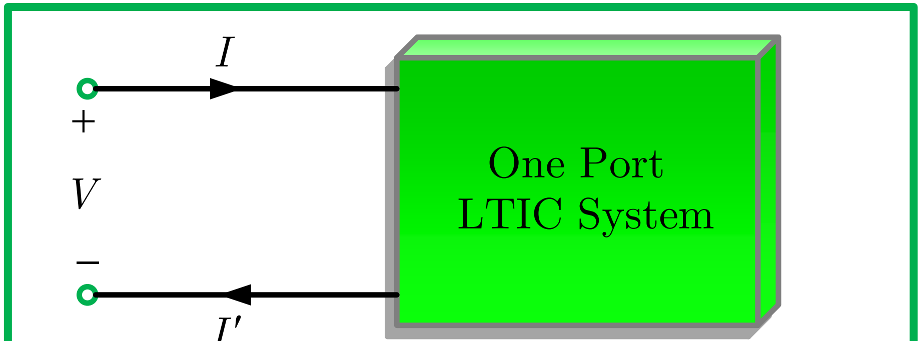

FIG. 9.1

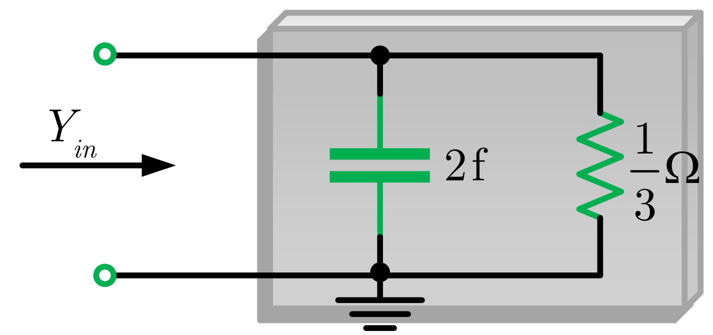

FIG. 9.2

9.1 System and Network Functions

- General two-port LTIC Systems, as shown in Fig 9.3 have two pairs of voltage current relationships consisting of the variables V_1, V_2, I_1, and I_2.
- Two of these are dependent variables; the other two are independent variables.
- The number of possible combinations generated by four variables taken two at a time is six.
- Thus there are six possible sets of equations describing a two-port network. We will discuss the most useful descriptions here.

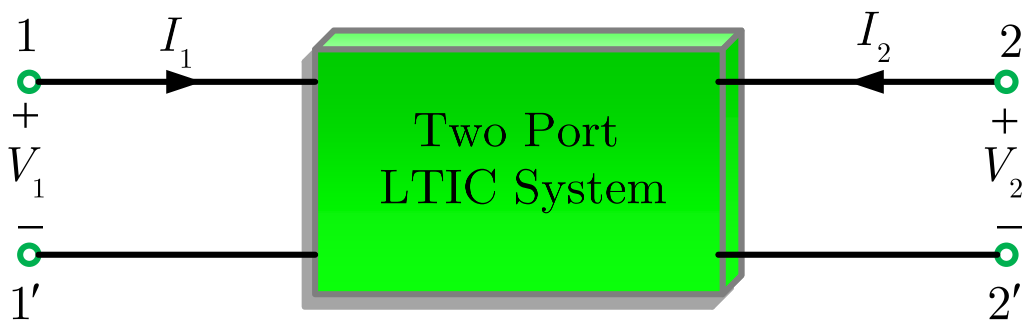

FIG. 9.3

9.1 System and Network Functions

The z-parameters

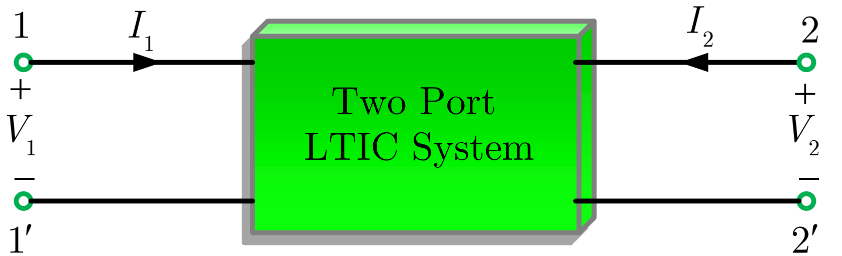

FIG. 9.4

$$\begin{aligned} V_1 &= z_{11} I_1 + z_{12} I_2 \\ V_2 &= z_{21} I_1 + z_{22} I_2 \end{aligned} \qquad \textbf{(9.1)}$$

- In these equations the variables V_1, and V_2 are the dependent variables, and I_1, I_2 are independent.
- The individual z-parameters are defined by

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \; ; \qquad z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} \; ; \qquad \textbf{(9.2)}$$

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} \; ; \qquad z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} \; ; \qquad \textbf{(9.3)}$$

- z_{11} and z_{22} give voltage current relationship for the port as such they are called open circuit driving point impedance.
- z_{12} and z_{21} relate voltage in one port to current in the other port thus they are called open circuit transfer impedance.

9.1 System and Network Functions

The z-parameters

[Example 9.1]

Obtain the z-parameters for the two-port LTIC System depicted in Fig. 9.5

[SOLUTION]

$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} = Z_a + Z_b$$

$$z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0} = Z_b + Z_c$$

$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} = Z_b$$

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} = Z_b$$

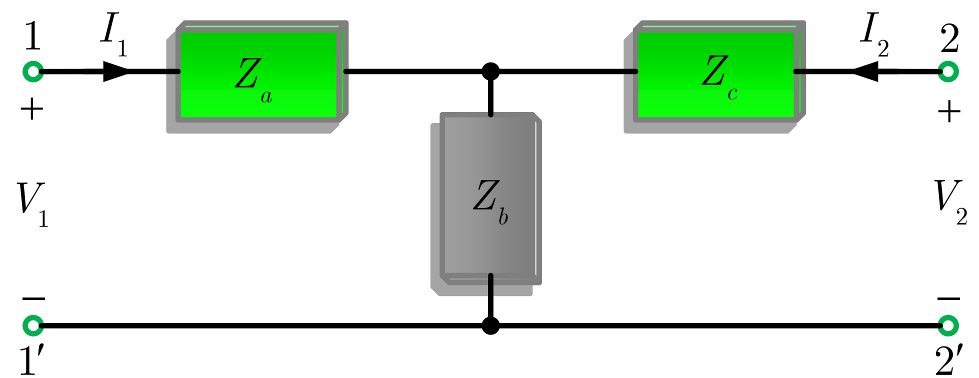

FIG. 9.5

Observe that in this case $z_{12} = z_{21}$. Thus we say that the network is reciprocal.

9.1 System and Network Functions

The y-parameters (also known as short circuit admittance parameters)

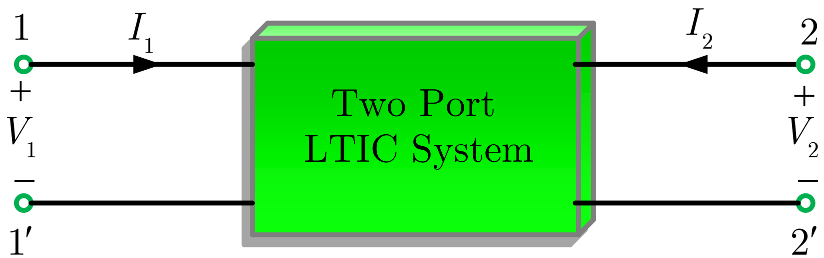

FIG. 9.6

$$I_1 = y_{11}V_1 + y_{12}V_2$$
$$I_2 = y_{21}V_1 + y_{22}V_2 \qquad \textbf{(9.4)}$$

- In these equations the variables I_1 and I_2 are the dependent variables, and certainly, V_1, and V_2 are independent variables.
- The individual y parameters are defined by

$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} ; \qquad y_{12} = \left.\frac{I_1}{V_2}\right|_{V_1=0} ; \qquad \textbf{(9.5)}$$

$$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0} ; \qquad y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0} ; \qquad \textbf{(9.6)}$$

- To obtain y_{11} and y_{21} we short circuit terminals 2- 2′.
- To obtain y_{21} and y_{22} we short circuit terminals 1- 1′.

9.1 System and Network Functions

The h-parameters

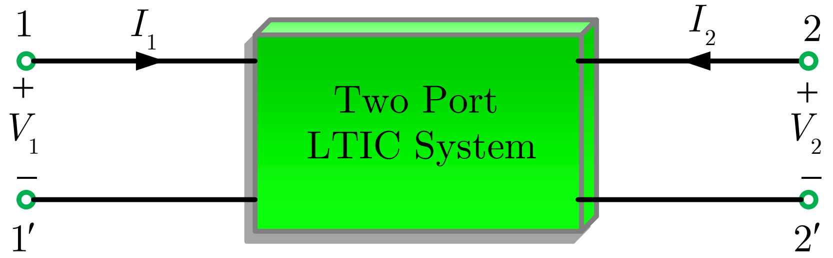

FIG. 9.7

$$V_1 = h_{11} I_1 + h_{12} V_2$$
$$I_2 = h_{21} I_1 + h_{22} V_2 \qquad \textbf{(9.7)}$$

- These parameters are extremely useful in describing transistor circuits
- By definition the individual parameters are of the form

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} ; \qquad h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} ; \qquad \textbf{(9.8)}$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} ; \qquad h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0} ; \qquad \textbf{(9.9)}$$

- h_{11} and h_{21} are short-circuit type parameters.
- where as h_{12} and h_{22} are open-circuit type parameters.
- h_{11} is the input impedance of port 1 i.e., $h_{11} = 1/y_{11}$.
- h_{22} is an open-circuit admittance , i.e., $h_{22} = 1/z_{22}$.
- h_{21} and h_{12} are transfer functions; h_{21} is a short circuit current ratio, and h_{12} is an open-circuit voltage ratio.

9.1 System and Network Functions

The ABCD parameters

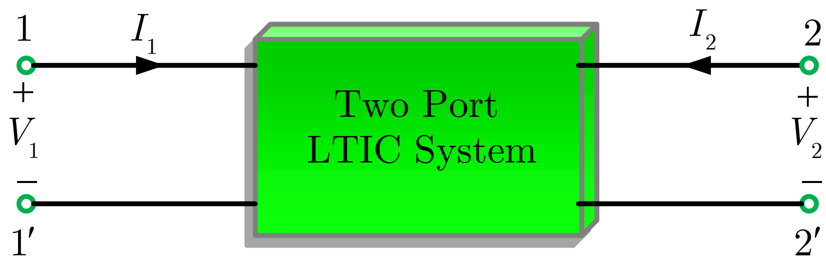

FIG. 9.8

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \qquad \textbf{(9.10)}$$

- Port 1 voltage and current are the dependent variables
- matrix with ABCD is known as the transmission matrix

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} \quad ; \qquad B = -\left.\frac{V_1}{I_2}\right|_{V_2=0} \quad ; \qquad \textbf{(9.11)}$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} \quad ; \qquad D = -\left.\frac{I_1}{I_2}\right|_{V_2=0} \quad ; \qquad \textbf{(9.12)}$$

- A is open circuit transfer voltage ratio.
- B is short circuit transfer impedance.
- C is open circuit transfer admittance.
- D is the short circuit current ratio.

- The relationship between two-port parameters are quite easy to obtain because of the simple algebraic nature of the two-port equations.
- Depicted in Table 9.1 is the conversion table.

Matrix Conversion Table $\quad \Delta_x = x_{11}x_{22} - x_{12}x_{21}$

	$[z]$	$[y]$	$[h]$	$[T]$
$[z]$	$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}$	$\begin{matrix} \frac{y_{22}}{\Delta_y} & -\frac{y_{12}}{\Delta_y} \\ -\frac{y_{21}}{\Delta_y} & \frac{y_{11}}{\Delta_y} \end{matrix}$	$\begin{matrix} \frac{\Delta_h}{h_{22}} & \frac{h_{12}}{h_{22}} \\ -\frac{h_{21}}{h_{22}} & \frac{1}{h_{22}} \end{matrix}$	$\begin{matrix} \frac{A}{C} & \frac{\Delta_T}{C} \\ \frac{1}{C} & \frac{D}{C} \end{matrix}$
$[y]$	$\begin{matrix} \frac{z_{22}}{\Delta_z} & -\frac{z_{12}}{\Delta_z} \\ -\frac{z_{21}}{\Delta_z} & \frac{z_{11}}{\Delta_z} \end{matrix}$	$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$	$\begin{matrix} \frac{1}{h_{11}} & -\frac{h_{12}}{h_{11}} \\ \frac{h_{21}}{h_{11}} & \frac{\Delta_h}{h_{11}} \end{matrix}$	$\begin{matrix} \frac{D}{B} & -\frac{\Delta_T}{B} \\ -\frac{1}{B} & \frac{A}{B} \end{matrix}$
$[h]$	$\begin{matrix} \frac{\Delta_z}{z_{22}} & \frac{z_{12}}{z_{22}} \\ -\frac{z_{21}}{z_{22}} & \frac{1}{z_{22}} \end{matrix}$	$\begin{matrix} \frac{1}{y_{11}} & -\frac{y_{12}}{y_{11}} \\ \frac{y_{21}}{y_{11}} & \frac{\Delta_y}{y_{11}} \end{matrix}$	$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$	$\begin{matrix} \frac{B}{D} & -\frac{\Delta_T}{D} \\ -\frac{1}{D} & \frac{C}{D} \end{matrix}$
$[T]$	$\begin{matrix} \frac{z_{11}}{z_{21}} & \frac{\Delta_z}{z_{21}} \\ \frac{1}{z_{21}} & \frac{z_{22}}{z_{21}} \end{matrix}$	$\begin{matrix} -\frac{y_{22}}{y_{21}} & -\frac{1}{y_{21}} \\ -\frac{\Delta_y}{y_{21}} & -\frac{y_{11}}{y_{21}} \end{matrix}$	$\begin{matrix} -\frac{\Delta_h}{h_{21}} & -\frac{h_{11}}{h_{21}} \\ -\frac{h_{22}}{h_{21}} & -\frac{1}{h_{21}} \end{matrix}$	$\begin{bmatrix} A & B \\ C & D \end{bmatrix}$

where $\Delta_Z = z_{11}z_{22} - z_{12}z_{21}$; and $\Delta_T = AD - BC$

9.4 Interconnection of Two-Ports

Cascaded two-ports

- When a pair of two-ports are cascaded, it is easier to use the transmission matrix of both to obtain the overall transmission matrix of the configuration.

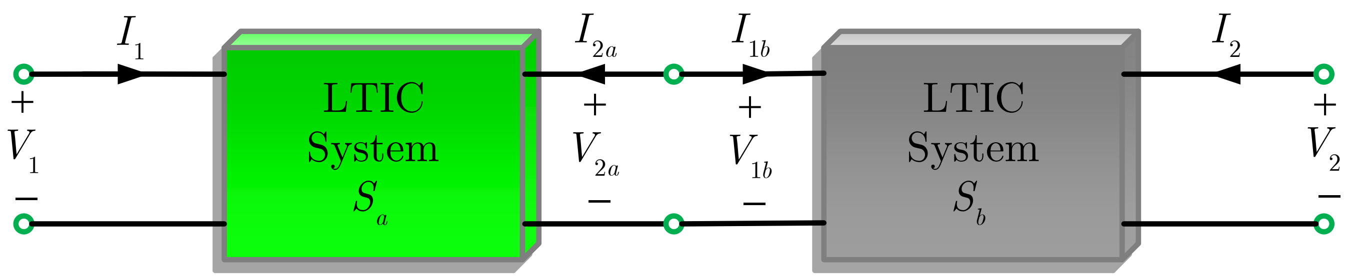

FIG. 9.9

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_a & B_a \\ C_a & D_a \end{bmatrix} \cdot \begin{bmatrix} A_b & B_b \\ C_b & D_b \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \qquad \textbf{(9.13)}$$

- The transmission matrix of the configuration is the product of the individual transmission matrices.

9.4 Interconnection of Two-Ports

Parallel connection of two-ports

- When a pair of two-ports are connected in parallel, it is easier to use the *y* parameters of both to obtain the overall matrix equation of the configuration.

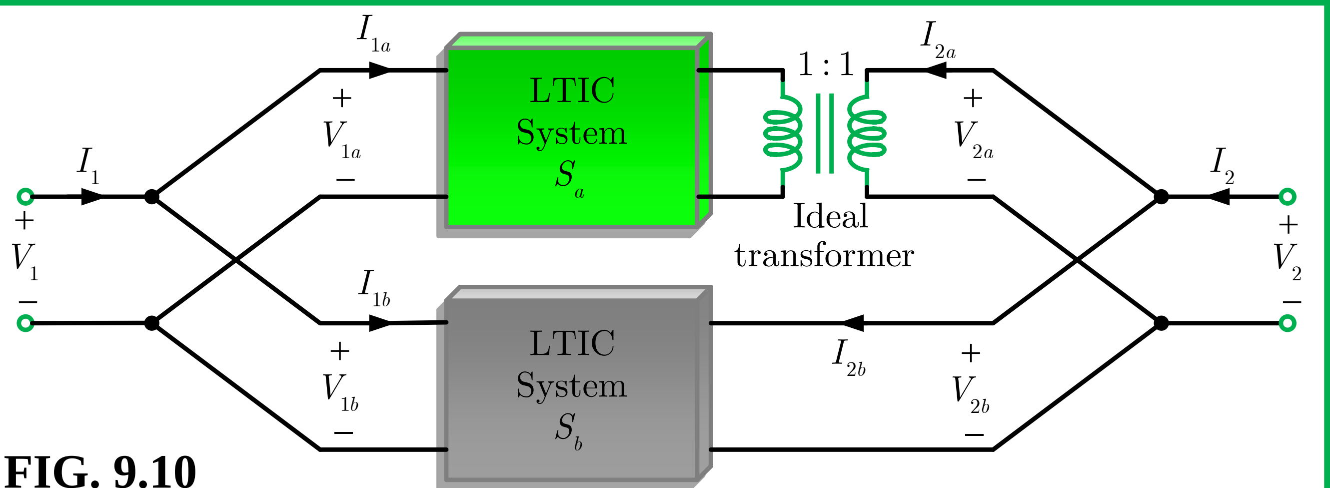

FIG. 9.10

$$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} = \begin{bmatrix} y_{11a} + y_{11b} & y_{12a} + y_{12b} \\ y_{21a} + y_{21b} & y_{22a} + y_{22b} \end{bmatrix} \quad \textbf{(9.14)}$$

- The y parameters of the overall two-port network can be obtained by summing the y parameters of the individual-ports

9.4 Interconnection of Two-Ports

Series connection of two-ports

- For series connected two-ports it is usually easier to find the z-parameters.

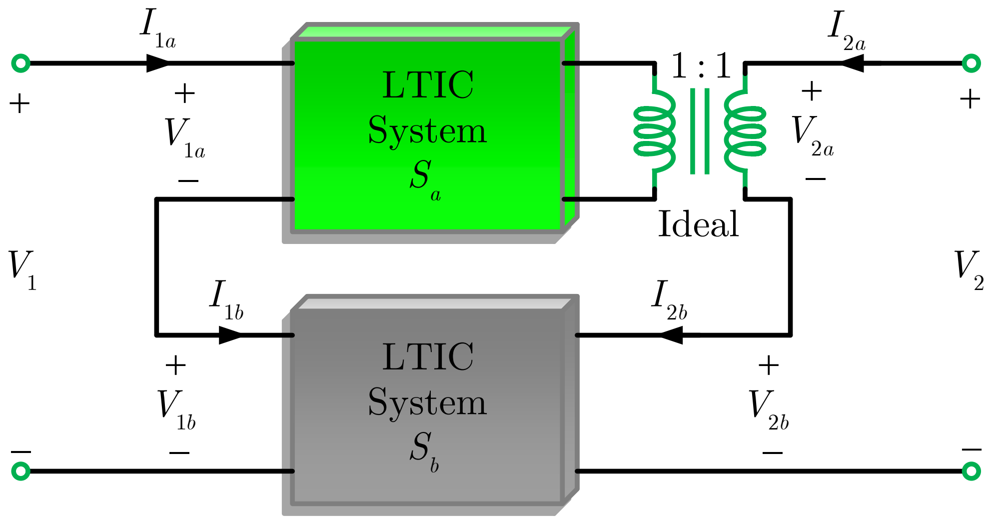

FIG. 9.11

$$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} = \begin{bmatrix} z_{11a} + z_{11b} & z_{12a} + z_{12b} \\ z_{21a} + z_{21b} & z_{22a} + z_{22b} \end{bmatrix} \quad \textbf{(9.15)}$$

- The z parameters of the overall two-port network can be obtained by summing the z parameters of the individual-ports

End of Lecture 9

Thank you for your attention!