EEE 3112 – ELECTRICAL ENGINEERING PRACTICE

MECHANICAL COMPONENT

TUTORIAL 1 – TORSION

On completion of this tutorial you should be able to do the following:

(1) Derive the torsion equation
(2) Derive polar second moment of area
(3) Solve problems involving torque, shear stress and angle of twist
(4) Derive formula for the power transmitted by a shaft
(5) Relate power transmission to torsion
(6) Outline the method of solution for circular cross sections
(7) Solve problems with shafts of circular cross section

It is assumed that students doing this tutorial are already familiar with the concepts of moments of area and shear stress

TORSION OF SHAFTS

Torsion occurs when any shaft is subjected to a torque. This is true whether the shaft is rotating (such as drive shafts on engines, motors and turbines) or stationary (such as with a bolt or screw). The torque makes the shaft twist and one end rotates relative to the other inducing shear stress on any cross section. Failure might occur due to shear alone or because the shear is accompanied by stretching or bending.

1.1 TORSION EQUATION

The equation was derived in class and will not be derived here

1.2 POLAR SECOND MOMENT OF AREA

This tutorial only covers circular sections. The formula for *J* is found by carrying out integration or may be found in standard tables.

For a shaft of diameter D the formula is $J = \frac{\pi D^4}{32}$

This is not to be confused with the second moment of area about diameter, used in bending of beams (*I*) but should be noted that *J = 2I*

WORKED EXAMPLE No. 1

A shaft 50mm diameter and 0.7m long is subjected to a torque of 1200Nm. Calculate the shear stress and the angle of twist. Take G = 90GPa.

SOLUTION

Important values to use are D = 0.05m, L = 0.7m, T = 1200Nm, G = 90x10^9Pa

$J = \frac{\pi D^4}{32} = \frac{\pi 0.05^4}{32} = 613.59x10^{-9}m^4$

$\tau_{max} = \frac{TR}{J} = \frac{1200x0.025}{613.59x10^{-9}} = 48.89x10^6 Pa$ or 48.89Mpa

$\theta = \frac{TL}{GJ} = \frac{1200x0.7}{90x10^9x613.59x10^{-9}}$ = 0.0152 radian

Alternatively $\theta = \frac{\tau L}{GR} = \frac{48.89x10^6x0.7}{90x10^9x0.025}$ = 0.0152 radian

Converting to degrees $\theta = 0.0152x\frac{180}{\pi} = 0.871^0$

1.3 HOLLOW SHAFTS

Since the shear stress is small near the middle, then if there are no other stress considerations other than torsion, a hollow shaft maybe used to reduce the weight.

The formula for the polar second moment of area is $J = \frac{\pi(D^4 - d^4)}{32}$

WORKED EXAMPLE No. 2

Repeat the previous problem but this time the shaft is hollow with an internal diameter of 30 mm.

SOLUTION

56.2MPa

WORKED EXAMPLE NO.3

A shaft 40mm diameter is made from steel and the maximum allowable shear stress for the material is 50MPa. Calculate the maximum torque that can be safely transmitted. Take G = 90GPa.

SOLUTION

J=2.51X10-7; 628.32NM

SELF ASSESSMENT EXERCISE No. 1

(1) A shaft is made from tube 25mm outer diameter and 20mm inner diameter. The shear stress must not exceed 150 MPa. Calculate the maximum torque that should be placed on it. (Ans. 271.69Nm)

(2) A shaft is made of solid round bar 30mm diameter and 0.5m long. The shear stress must not exceed 200MPa. Calculate the following.
   (i) The maximum torque that should be transmitted
   (ii) The angle of twist which will occur.
   Take G = 90GPa, (Ans. 1060 Nm and 4.2 degree)

1.4 MECHANICAL POWER TRANSMISSION

In this section you will derive the formula for the power transmitted by a shaft and combine it with torsion theory.

Mechanical power is defined as the as work done per second. Work done is defined as force times distance moved.

Hence

$P = Fx/t$ where *P* is the power
F is the Force
x is the distance moved
t is the time taken

Since distance moved/time taken is the velocity of the force we may write

$P = Fv$ ---------------------------------- (1), where v is the velocity

When a force rotates at radius R it travels one circumference in the time of one revolution. Hence the distance moved in one revolution is $x = 2\pi R$.

If the speed is N rev/second then the time of one revolution is 1/N seconds. The mechanical power is hence $P = F.\frac{2\pi R}{\frac{1}{N}} = 2\pi NFR$

Since *FR* is the torque produced by the force this reduces to

$P = 2\pi NT$ ----------------------(2)

Since $2\pi N$ is the angular velocity ω radians/s it further reduces to

$P = \omega T$ -------------------------(3)

Note that equation (3) is the angular equivalent of equation (1) and all three equations should be remembered.

WORKED EXAMPLE No.4

A shaft is made from a tube. The ratio of inside diameter to the outside diameter is 0.6. The material must not experience a shear stress greater than 500kPa. The shaft must transmit 1.5MW of mechanical power at 1500 rev/min. calculate the shaft diameters.

SOLUTION

SELF ASSESSMENT EXERCISE No.2

(1) A shaft is made from a tube 25 mm outer diameter and 20mm inner diameter. The shear stress must not exceed 150MPa. Calculate the maximum power that should be transmitted at 1500rev/min (Ans 14.226Kw/42.7kw)

(2) A shaft must transmit 20kW of power at 300rev/min. The shear stress must not exceed 150MPa. Calculate a suitable diameter. (Ans. 27.8mm)

(3) A steel shaft 5m long, having a diameter of 50mm, is to transmit power at a rotational speed of 600rev/min. If the maximum shear stress is limited to 60MN/m2. Determine the following

    (i) The maximum power that can be transmitted (92.5kW)

    (ii) The corresponding angle of twist (8.59degree)

    Assume the modulus of rigidity for steel is 80GN/m2.

(4) A hollow steel shaft with a diameter ratio of 0.75 and a length of 4m is required to transmit 1 MW at 120rev/min. The maximum shear stress is not to exceed 70MN/m2 nor is the overall angle of twist to exceed 1.75degree. Determine the following

    (i) The necessary outside diameter of the shaft so that both of the above limitations are satisfied (222mm)

    (ii) The actual maximum shear stress and the actual angle of twist (1.75degree)
    Assume the modulus of rigidity for steel is 80GPa