EE321

ELECTROMECHANICS & ELECTRICAL MACHINES

UNIVERSITY EXAMINATIONS

NOVEMBER/DECEMBER 2011

MODEL SOLUTIONS

1.

(a)

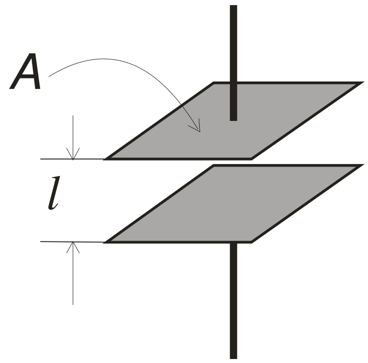
A = plate area

l = distance of separation

$$C = \frac{Q}{V}$$

$$= \frac{DA}{El}$$

$$\underline{\underline{C = \varepsilon\frac{A}{l}}}$$



(b)

l = 15 mm, V = 20 kV, ε_r = 26.

(c)

l = 1 k m, V = 10 kV, ε_r = 4, f = 50 Hz, $d_1 = 2r_1$ = 12 mm, t = 4 mm.

(i) $$C = \frac{2\pi l\varepsilon}{\ln\frac{r_2}{r_1}} = \frac{2\pi\times1\times10^3\times(4\times8.85\times10^{-12})}{\ln\left(\frac{6+4}{6}\right)} = 4.35\times10^{-7} = \underline{\underline{43.5\ \mu\text{F}}}$$

(ii) $$\hat{W} = \frac{1}{2}C\hat{V}^2 = \frac{1}{2}\times4.35\times10^{-7}\times(1\times10^3)^2 = \underline{\underline{0.218J}}$$

(iii) $$|I| = V\omega C = 2\pi fVC = 2\pi\times50\times(1\times10^3)\times4.35\times10^{-7} = \underline{\underline{0.137\ \text{A}}}$$

2.

(a)

$$\Lambda = \frac{\phi}{F} = \frac{BA}{Hl} = \mu\frac{A}{l}$$

$$S = \frac{1}{\Lambda} = \frac{l}{\mu A}$$

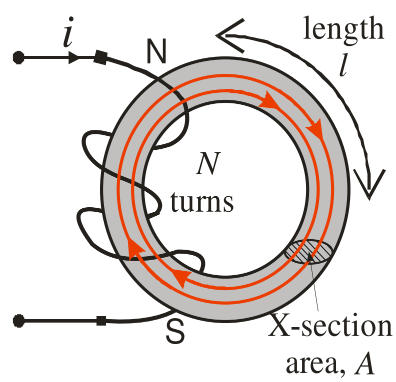
Circuit: $v = L\dfrac{di}{dt}$ and Faraday's law; $v = N\dfrac{d\phi}{dt}$

$$L\frac{di}{dt} = N\frac{d\phi}{dt}$$

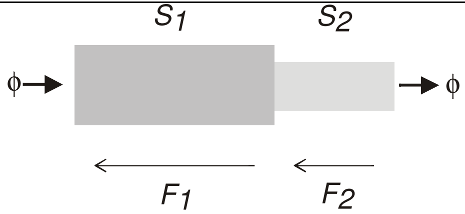
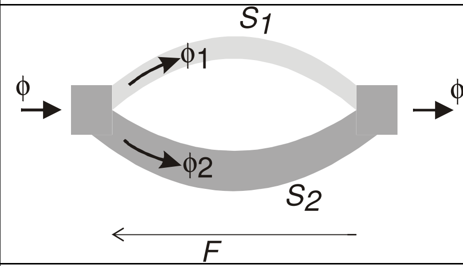
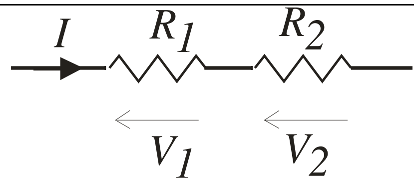
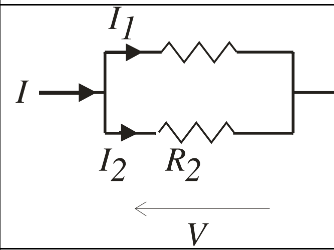
$$L = N\frac{d\phi}{di}$$

Uniform case: $L = N\dfrac{\phi}{i}$ and $\phi = \dfrac{F}{S} = \dfrac{Ni}{S}$

$$\therefore \underline{\underline{L = \frac{N^2}{S}}}$$



(b)

<table>
<tr><td></td><td>Series combination</td><td>Parallel combination</td></tr>
<tr><td>Magnetic</td><td>
</td><td>
</td></tr>
<tr><td></td><td>$F = F_1 + F_2$
$\frac{F}{\phi} = \frac{F_1}{\phi} + \frac{F_2}{\phi}$
$S_{tot} = S_1 + S_2$</td><td>$\phi = \phi_1 + \phi_2$
$\frac{\phi}{F} = \frac{\phi_1}{F} + \frac{\phi_2}{F}$
$\frac{1}{S_{tot}} = \frac{1}{S_1} + \frac{1}{S_2}$</td></tr>
<tr><td></td><td>
</td><td>
</td></tr>
<tr><td></td><td>$V = V_1 + V_2$
$\frac{V}{I} = \frac{V_1}{I} + \frac{V_2}{I}$
$R_{tot} = R_1 + S_2$</td><td>$I = I_1 + I_2$
$\frac{I}{V} = \frac{I_1}{V} + \frac{I_2}{V}$
$\frac{1}{R_{tot}} = \frac{1}{R_1} + \frac{1}{R_2}$</td></tr>
</table>

(c)

B = 1.2 T, l_g = 1 mm, l_{Fe} = 80 cm, A = 20 cm^2.

(i) $S_g = \frac{l_g}{\mu A} = \frac{1\times10^{-3}}{4\pi\times10^{-7}\times(20\times10^{-4})} = 397{,}887 \text{ A/Wb}$

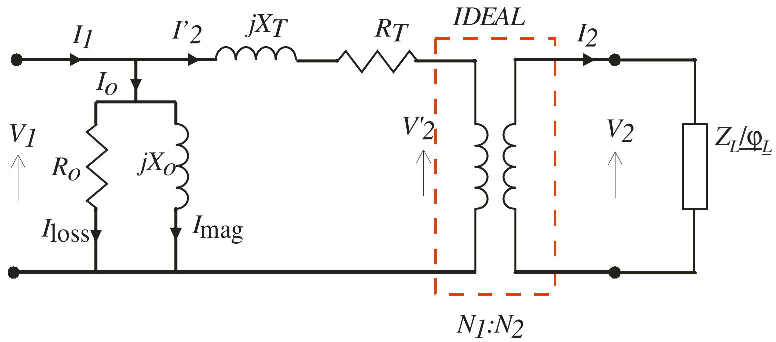
$$S_{Fe} = \frac{l_{Fe}}{\mu A} = \frac{80\times10^{-2}}{10^{-3}\times(20\times10^{-4})} = 400{,}000 \text{ A/Wb}$$

$$S_{tot} = 2\times S_g + S_{Fe} = 2\times397887 + 400000 = 1{,}195{,}774 \text{ A/Wb}$$

$$F = \phi S_{tot} = BAS_{tot} = 1.2\times(20\times10^{-4})\times1195774 = 2869 \approx \underline{\underline{2870 \text{ A}}}$$

(ii) $F = \text{"}PA\text{"} = \frac{1}{2}\frac{B^2}{\mu}(2A) = \frac{1}{2}\times\frac{1.2^2\times(2\times20\times10^{-4})}{4\pi\times10^{-7}} = \underline{\underline{2292 \text{ N}}}$

3.
(a)



Parameters of transformer:

- Parallelelements: R_o and X_o
- Series elements: R_T and X_T

Open circuit test

- Apply V_1 = rated voltage
    - All current is in the parallel elements
- Measure V_1, I_1,and P_1

$$P_1 = \frac{V_1^2}{R_o} \rightarrow R_o$$

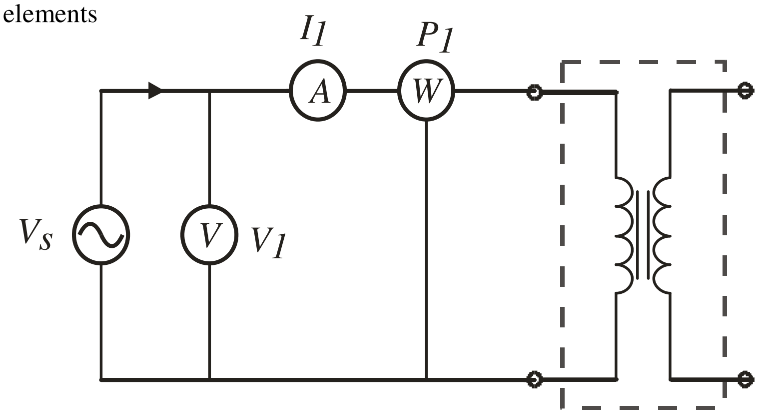
$$I_{Loss} = \frac{V_1}{R_o}$$

$$I_o = \frac{P_1}{V_1}$$

$$I_{mag}^2 + I_{Loss}^2 = I_o^2$$

$$X_o = \frac{V_1}{I_{mag}}$$

Gives Xo



Short circuit test

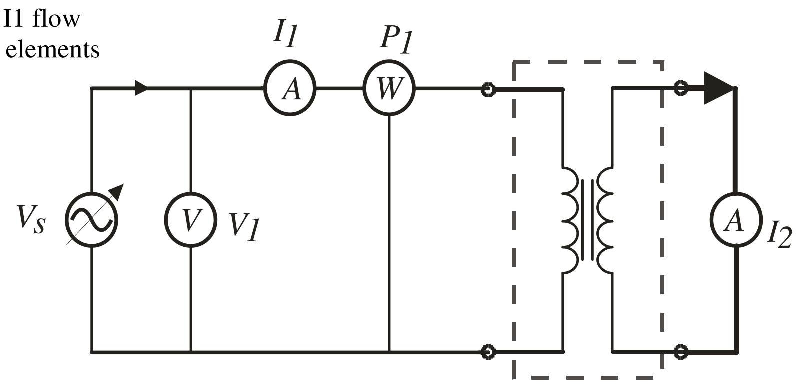
- Apply low V_1 gradually until rated I1 flow
    - All current is in the series elements
- Measure V_1, I_1,and P_1

$$I_1^2 R_T = P_1$$

$$Z_T = \frac{V_1}{I_1}$$

$$Z_T^2 = R_T^2 + X_T^2$$

Gives R_T and X_T



(b)

(i) $R_T = \frac{P_1}{I_1^2} = \frac{202}{4.17^2} = \underline{\underline{11.6\ \Omega}}$

$$Z_T = \frac{V_1}{I_1} = \frac{138}{4.17} = 33.1\ \Omega$$

$$X_T = \sqrt{Z_T^2 - R_T^2} = \sqrt{33.1^2 - 11.6^2} = \underline{\underline{31\ \Omega}}$$

$$\varphi_T = \tan^{-1}\frac{X_T}{R_T} = \tan^{-1}\frac{31}{11.6} = \tan^{-1} 2.67 = 69.5^\circ$$

$$\varphi_L = \cos^{-1} 0.866 = 30^\circ$$

(ii)

$$I'_{2FL} = I_{1FL} = \frac{S_r}{V_1} = \frac{10\times10^3}{2400} = 4.17\ \text{A}$$

$$\operatorname{Re}g = \frac{I'_2 Z_T}{V_1}\cos(\varphi_T - \varphi_L) = \frac{(0.8\times4.17)\times33.1}{2400}\cos(30^\circ - 69.5^\circ) = 0.035 = \underline{\underline{3.5\%}}$$

4.

(a)

- In three phase power is constant from from the prime to the load, while in signle pphase it fluctuates at twice the supply frequency
- There is a saving in transmission conductors from 6 in sigle phase to 3 or 4 in three phase
- The specific output (power unit volume) of three phase machinesis higher thatn for single phase machines

(b)

$$P_1 = (v_{ac}i_a)\big|_{AV} = (v_a - v_b)i_a\big|_{AV}$$

$$P_2 = (v_{bc}i_b)\big|_{AV} = (v_b - v_c)i_b\big|_{AV}$$

$$P_1 + P_2 = (v_a i_a)\big|_{AV} + (v_b i_b)\big|_{AV} - v_c(i_a + i_b)\big|_{AV}$$

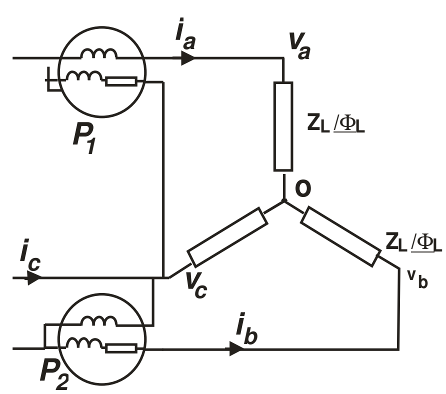
if there is no neutral

$$i_a + i_b + i_c = 0$$

And therefore

$$P_1 + P_2 = (v_a i_a)\big|_{AV} + (v_b i_b)\big|_{AV} + (v_c i_c)\big|_{AV}$$

$$P_1 + P_2 = P_a + P_b + P_c$$



(c)

V_L= 400 V, I_L = 20 A, P_T = 20 kW.

(i) $p.f. = \cos\varphi_L = \frac{P_T}{\sqrt{3}V_L I_L} = \frac{12\times10^3}{\sqrt{3}\times400\times20} = \underline{\underline{0.866}}$

$$Z_L = \frac{V_p}{I_p} = \frac{V_L/\sqrt{3}}{I_L} = \frac{400/\sqrt{3}}{20} = 11.55\ \Omega$$

$$\cos^{-1}\varphi_L = 30^\circ$$

$$Z_L = \underline{\underline{11.55\angle 30^\circ\ \Omega = 10 + j5.775\ \Omega}}$$

(ii)

$$P_1 + P_2 = 12\times10^3$$

$$P_1 - P_2 = V_L I_L \sin\varphi_L = 400\times20\times\sin 30^0 = 4\times10^3$$

$$2P_1 = 16\times10^3 \Rightarrow P_1 = \underline{\underline{8\times10^3\ \text{W}}}$$

$$P_2 = (8-4)\times10^3 = \underline{\underline{4\times10^3\ \text{W}}}$$

5.

(a)

DC Shunt

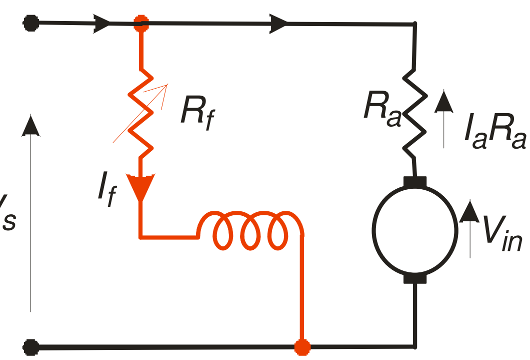
$$V_s = V_{in} + I_a R_a$$

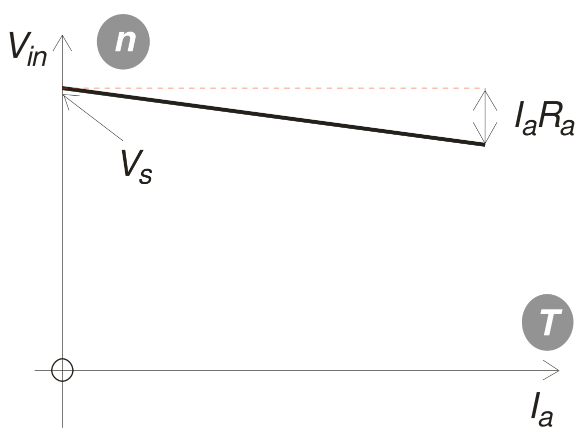
$$T = \frac{1}{2\pi}\left(\frac{2pZ}{c}\right)\phi I_a$$

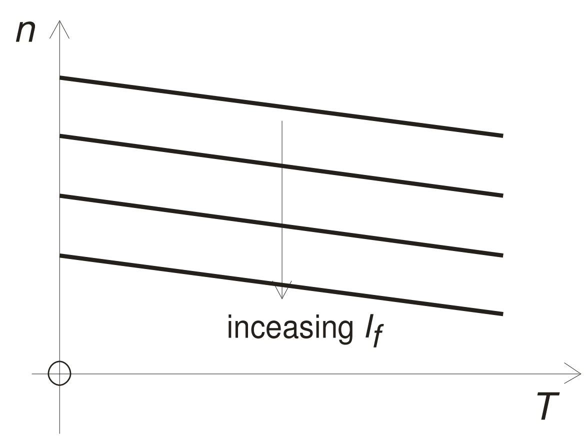
At constant field current (and therefore constant flux)

$$V_{in} \propto n$$

$$T \propto I_a$$





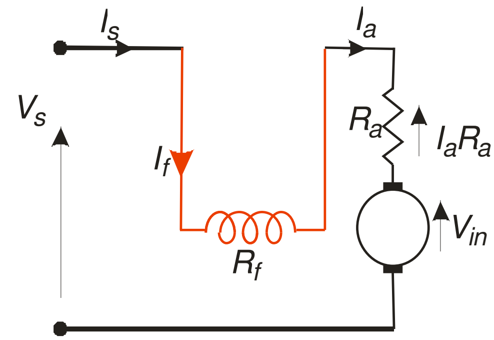


DC Series

$I_s = I_a = I_f = I$

Two assumptions:

- Neglect IR drops
- Neglect saturation



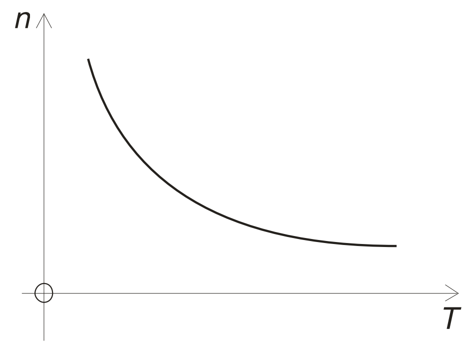
$$V_s = V_{in} = \left(\frac{2pZ}{c}\right) n\phi$$

$$\therefore n \propto \frac{1}{\phi} \propto \frac{1}{I}$$

$$T = \frac{1}{2\pi}\left(\frac{2pZ}{c}\right)\phi I_a$$

$$T \propto I^2$$

$$T \propto \frac{1}{n^2}$$



(b)

V_s = 240 V, n_1 = 800 r/.min, I_{a1} = 0, R_f = 160 Ω, R_a = 0.4 Ω; n_2 = 950 r/min, I_{a2} = 20 A.

$$V_{in1} = kn_1\phi_1 = 240$$

$$k\phi_1 = \frac{240}{n_1} = \frac{240}{800} \propto I_{f1}$$

$$V_{in2} = V_s - I_aR_a = 240 - 20\times 0.4 = 232$$

$$kn_2\phi_2 = 232$$

$$k\phi_2 = \frac{240}{n_2} = \frac{232}{950} \propto I_{f2}$$

$$I_{f2} = \frac{232}{950}\cdot\frac{800}{240} I_f = \frac{232}{950}\cdot\frac{800}{240}\frac{V_s}{R_{f1}} = \frac{232}{950}\cdot\frac{800}{240}\cdot\frac{240}{160} = 0.814 \text{ A}$$

$$R_{f2} = \frac{V_s}{I_{f2}} = \frac{240}{0.814} = 295\ \Omega$$

$$R_{ex} = R_{f2} - 160 = 295 - 160 = \underline{\underline{135\ \Omega}}$$

6.

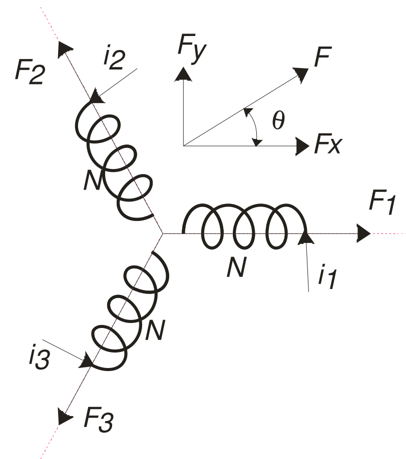
(a)

$$F_y = \frac{\sqrt{3}}{2}(F_2 - F_3)$$

$$= \frac{\sqrt{3}}{2} NI_m \left[\cos\overline{\omega t - 120^\circ} - \cos\overline{\omega t + 120^\circ}\right]$$

$$= \frac{\sqrt{3}}{2} NI_m 2\sin\omega t \sin 120^\circ$$

$$F_y = \frac{3}{2} NI_m \sin\omega t$$



$$i_1 = I_m \cos\omega t$$

$$i_2 = I_m \cos\overline{\omega t - 120^\circ}$$

$$i_3 = I_m \cos\overline{\omega t + 120^\circ}$$

$$F_1 = NI_m \cos\omega t$$

$$F_2 = NI_m \cos\overline{\omega t - 120^\circ}$$

$$F_3 = NI_m \cos\overline{\omega t + 120^\circ}$$

$$F_x = F_1 - \frac{1}{2}(F_2 + F_3)$$

$$= NI_m \left[\cos\omega t - \frac{1}{2}(\cos\overline{\omega t - 120^\circ} + \cos\overline{\omega t + 120^\circ})\right]$$

$$= NI_m \left[\cos\omega t - \frac{1}{2} 2\cos\omega t \cos 120^\circ\right]$$

$$F_x = \frac{3}{2} NI_m \cos\omega t$$

$$|F| = \sqrt{F_x^2 + F_y^2} = \underline{\underline{\frac{3}{2} NI_m}}$$

$$\tan\theta = \frac{F_y}{F_x} = \tan\omega t$$

$$\underline{\underline{\theta = \omega t}}$$

(b)

Synchronous machine:

Rtor has field winding excited by dc current; rotor speed is constant and equal to speed of rotating flux

Induction machine:

Rotor has windings short-circuited on themselves and are unexcited; rotor slips so its speed is a little less than the speed of the rotating magnetic flux

(c)
F = 50 Hz, p = 3, VS = 220 V, NS= 2NR

(i) $s = \frac{f_R}{f} = \frac{2}{50} = \underline{\underline{0.04}}$

(ii) $n = n_s(1-s) = \frac{f}{p}(1-s) = \frac{50}{3}(1-0.04) = 16 \text{ r/s} = \underline{\underline{960 \text{ r/min}}}$

(iii) $E_R = \frac{N_R}{N_S} sE_S = \frac{1}{2} \times 0.04 \times 220 = \underline{\underline{4.4\ V}}$

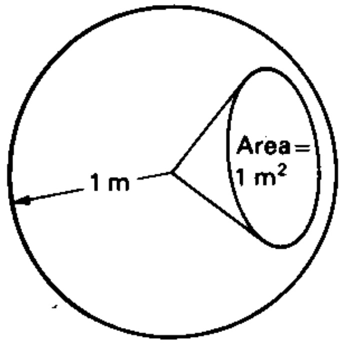
7.

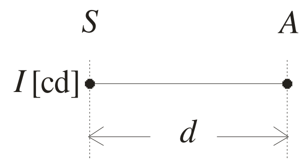
(a)

Inverse square law:

$$\frac{E_{\text{at } r}}{E_{\text{at 1 m radius}}} = \frac{\phi/4\pi r^2}{\phi/4\pi} = \frac{4\pi}{4\pi r^2} = \frac{1}{r^2}$$

$$E_A = \frac{I}{d^2}$$

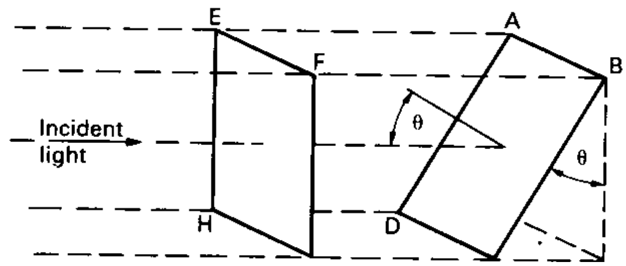


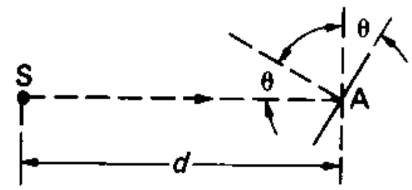


Cosine law of illumination:

$$\frac{E_{\text{ABCD}}}{E_{\text{EFGH}}} = \frac{\phi/S_{ABCD}}{\phi/S_{EFGH}} = \frac{S_{EFGH}}{S_{ABCD}} = \cos\theta$$

$$E_A = \frac{I}{d^2}\cos\theta$$





(b)

EA = 20 lx, IA = 500 dc, d = 1 m, d1= 9m, d2 = ?

$$E_A = \frac{I_A}{d_1^2} + \frac{I_B}{d_2^2}\cos\theta$$

$$d_2^2 = 9^2 + 1^2 = 82$$

$$\cos\theta = \frac{9}{\sqrt{9^2 + 1^2}} = 0.99$$

$$I_B = \left(E_A - \frac{I_A}{d_1^2}\right)\frac{d_2^2}{\cos\theta} = \left(20 - \frac{500}{9^2}\right)\frac{82}{0.99} = \underline{\underline{1145 \text{ cd}}}$$

(c) A = 14 x 12 = 144 m2, E = 80 lx, UF = 0.5 , MF =0.8, Eff. = 14.75 lm/W, P =100 W.

$$\phi_{total} = \frac{EA}{UF \times UF} = \frac{12\times 12\times 80}{0.5\times 0.8} = 28{,}800 \text{ lm}$$

$$P_{total} = \frac{\phi_{total}}{Eff.} = \frac{28800}{14.75} = 1955 \text{ W}$$

$$N = \frac{1955}{100} = 19.55 \approx \underline{\underline{20 \text{ lamps}}}$$