EE321

ELECTROMECHANICS & ELECTRICAL MACHINES

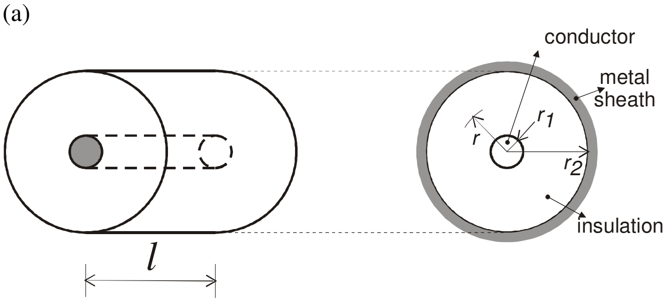
MID-SEMESTER TESTS

JANUARY 2013

MODEL SOLUTIONS

1.

(a)



Total electric flux $\psi = q$

At radius r

$$D = \frac{\psi}{A} = \frac{q}{2\pi r l}$$

$$E = \frac{D}{\varepsilon} = \frac{q}{2\pi r l \varepsilon}$$

$$V = \int_{r_1}^{r_2} E dr = \int_{r_1}^{r_2} \frac{q}{2\pi r l \varepsilon} dr$$

$$= \frac{q}{2\pi l \varepsilon} \ln \frac{r_2}{r_1}$$

$$C = "\frac{Q}{V}"$$

$$C = \frac{2\pi l \varepsilon}{\ln \frac{r_2}{r_1}}$$

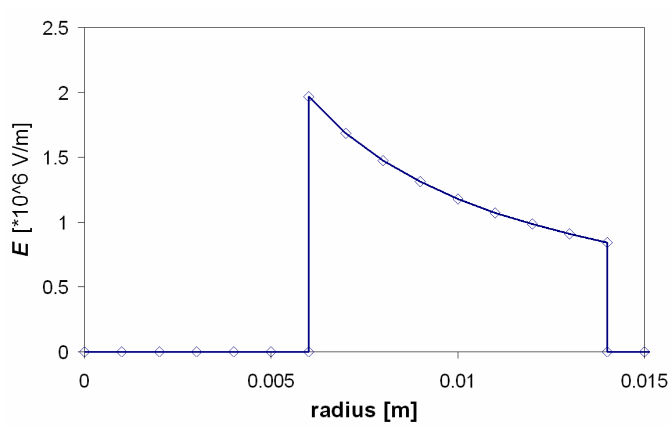
(b) l =1 km, V = 10 kV, f = 50 Hz, r_1 = 6 mm, r_2 = 14 mm

(i)

$$E = \frac{V}{\ln \frac{r_2}{r_1}} \frac{1}{r} = \frac{k}{r} = \frac{11802}{r}$$

$$E_{\max} = \frac{V}{r_1 \ln \frac{r_2}{r_1}} = \frac{10000}{6 \times 10^{-3} \ln(\frac{14}{6})} = 1.97 \times 10^6 \text{ V/m}$$

$$E_{\min} = \frac{V}{r_2 \ln \frac{r_2}{r_1}} = \frac{10000}{14 \times 10^{-3} \ln(\frac{14}{6})} = 8.43 \times 10^5 \text{ V/m}$$



(ii)

$$1)\ C = \frac{2\pi l \varepsilon}{\ln \frac{r_2}{r_1}} = \frac{2\pi \times 10 \times 10^3 \times 4 \times 8.85 \times 10^{-12}}{\ln \frac{14}{6}} = 9.5 \times 10^{-7} \ F = \underline{\underline{2.63 \ \mu\text{F}}}$$

$$2)\ W = \frac{1}{2} C V^2 = \frac{1}{2} \times 9.5 \times 10^{-7} \times (\sqrt{2} \times 10 \times 10^3)^2 = \underline{\underline{263}} \text{ J}$$

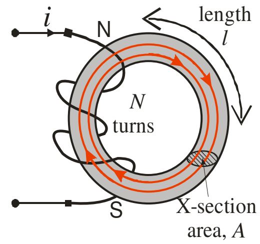
$$3)\ |I| = V \omega C = 2\pi f V C = 2\pi \times 50 \times (10 \times 10^3) \times 9.5 \times 10^{-7} = \underline{\underline{8.26 \text{ A}}}$$

2.

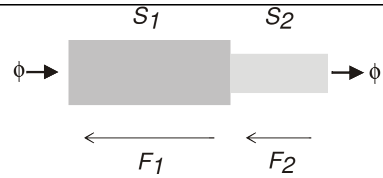
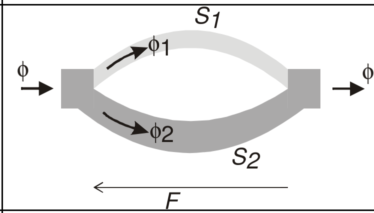
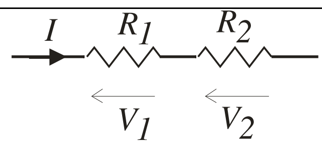
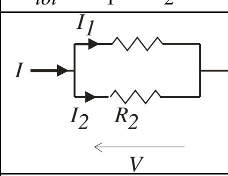
(a)

$$\Lambda = \frac{\phi}{F} = \frac{BA}{Hl} = \mu \frac{A}{l}$$

$$S = \frac{1}{\Lambda} = \frac{l}{\mu A}$$



(b)

	Series combination	Parallel combination
Magnetic		
	$F = F_1 + F_2$ $\frac{F}{\phi} = \frac{F_1}{\phi} + \frac{F_2}{\phi}$ $S_{tot} = S_1 + S_2$	$\phi = \phi_1 + \phi_2$ $\frac{\phi}{F} = \frac{\phi_1}{F} + \frac{\phi_2}{F}$ $\frac{1}{S_{tot}} = \frac{1}{S_1} + \frac{1}{S_2}$
Electric		
	$V = V_1 + V_2$ $\frac{V}{I} = \frac{V_1}{I} + \frac{V_2}{I}$ $R_{tot} = R_1 + S_2$	$I = I_1 + I_2$ $\frac{I}{V} = \frac{I_1}{V} + \frac{I_2}{V}$ $\frac{1}{R_{tot}} = \frac{1}{R_1} + \frac{1}{R_2}$

(c)

$B = 1.2$ T, $l_g = 1$ mm, $l_{Fe} = 80$ cm, $A = 20$ cm^2.

(i) $$S_g = \frac{l_g}{\mu A} = \frac{1 \times 10^{-3}}{4\pi \times 10^{-7} \times (20 \times 10^{-4})} = 397{,}887 \quad A/Wb$$

$$S_{Fe} = \frac{l_{Fe}}{\mu A} = \frac{80 \times 10^{-2}}{10^{-3} \times (20 \times 10^{-4})} = 400{,}000 \quad A/Wb$$

$$S_{tot} = 2 \times S_g + S_{Fe} = 2 \times 397887 + 400000 = 1{,}195{,}774 \quad A/Wb$$

$$F = \phi S_{tot} = BAS_{tot} = 1.2 \times (20 \times 10^{-4}) \times 1195774 = 2869 \approx \underline{\underline{2870 \quad A}}$$

(ii) $$F = \text{"}PA\text{"} = \frac{1}{2}\frac{B^2}{\mu}(2A) = \frac{1}{2} \times \frac{1.2^2 \times (2 \times 20 \times 10^{-4})}{4\pi \times 10^{-7}} = \underline{\underline{2292 \quad N}}$$

3.

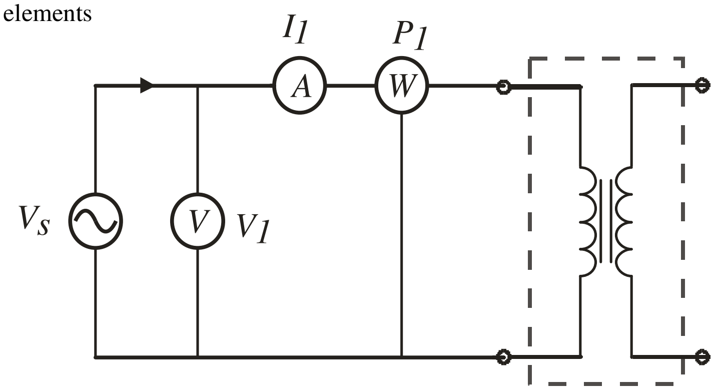
(a)

Parameters of transformer:

- Parallelelements: R_o and X_o
- Series elements: R_T and X_T

Open circuit test

- Apply V_1 = rated voltage
  - All current is in the parallel elements
- Measure V_1, I_1,and P_1



$$P_1 = \frac{V_1^2}{R_o} \rightarrow R_o$$

$$I_{Loss} = \frac{V_1}{R_o}$$

$$I_o = \frac{P_1}{V_1}$$

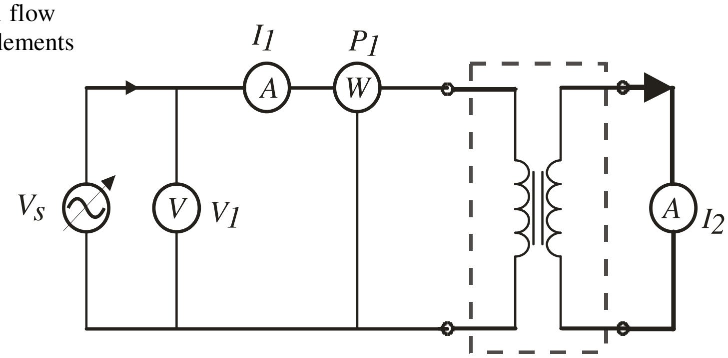
$$I_{mag}^2 + I_{Loss}^2 = I_o^2$$

$$X_o = \frac{V_1}{I_{mag}}$$

Gives Xo

Short circuit test

- Apply low V_1 gradually until rated I1 flow
  - All current is in the series elements
- Measure V_1, I_1,and P_1



$$I_1^2 R_T = P_1$$

$$Z_T = \frac{V_1}{I_1}$$

$$Z_T^2 = R_T^2 + X_T^2$$

Gives R_T and X_T

(b)

(i) $R_T = \frac{P_1}{I_1^2} = \frac{202}{4.17^2} = 11.6 \quad \Omega$

$$Z_T = \frac{V_1}{I_1} = \frac{138}{4.17} = 33.1 \quad \Omega$$

$$X_T = \sqrt{Z_T^2 - R_T^2} = \sqrt{33.1^2 - 11.6^2} = 31 \quad \Omega$$

(ii)

$$\varphi_T = \tan^{-1}\frac{X_T}{R_T} = \tan^{-1}\frac{31}{11.6} = \tan^{-1} 2.67 = 69.5^\circ$$

$$\varphi_L = \cos^{-1} 0.866 = 30^\circ$$

$$I_{2FL}^{'} = I_{1FL} = \frac{S_r}{V_1} = \frac{10\times10^3}{2400} = 4.17 \quad A$$

$$\mathrm{Re}\,g = \frac{I_2^{'} Z_T}{V_1}\cos(\varphi_T - \varphi_L) = \frac{(0.8\times4.17)\times33.1}{2400}\cos(30^\circ - 69.5^\circ) = 0.035 = 3.5\%$$

4.

(a) For one conductor, Faraday's law:

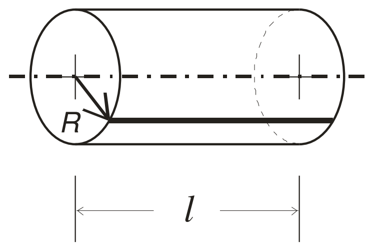
$v_{in} = Blu$

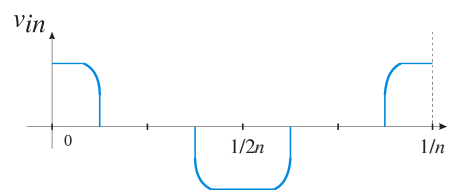
Speed: $u = 2\pi Rn$

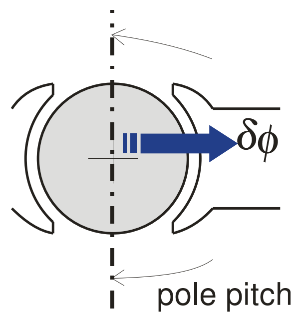
$v_{in} = (2\pi Rnl)B$

Given flux per pole

$\int \delta\phi = \phi$







Over one pole pitch:

$$v_{in}\big|_{av} = \frac{\int_0^{\frac{T}{2}} Blu\,dt}{\frac{T}{2}} = \frac{\int_0^{\frac{1}{2n}} Blu\,dt}{\frac{1}{2n}} = \frac{\int d\phi}{\frac{1}{2n}} = 2n\phi$$

$$= \frac{1}{2p}\frac{1}{n}$$

$$V_{in}\big|_{av} = \frac{\phi}{1/2np}$$

$$V_{in}\big|_{av} = 2np\phi$$

For ac output with slip rings:

$V_{av} = 2pn\phi Z_s$

Taking into account spread of conductors and rms value

$V = 2pn\phi.Z_s.k.(1.11)$

$\quad = 2.22\,pnZ_s\phi k$

$V = 2.22Z_s f\phi k$

(b)

$p = 6$, $n = 3$ r/s, $N = 1200$, $k = 0.8$, A = A cm^2, $B = 0.4$ T

(i)

$V = 2.22Z_s\,pnBAk$

$= 2.22\times(2\times1200)\times3\times6\times(0.4\times4\times10^{-4})\times0.8$

$= 12.27$ V

(ii)

$f = np$

$= 3\times6 = 18$ Hz