THE UNIVERSITY OF ZAMBIA

SCHOOL OF ENGINEERING

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING

TERM TEST

TERM III – July 2017

MODEL SOLUTIONS

EEE 3352

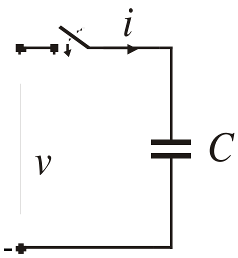
ELECTROMECHANICS & ELECTRICAL MACHINES

TIME	:	Two (2) hours
INSTRUCTIONS	:	Answer all questions
ADDITIONAL INFORMATION	:	$\mu_0 = 4\pi \times 10^{-7}$ H/m and $\varepsilon_0 = 8.85\times 10^{-12}$ F/m

Question 1.

(a) Prove, from first principles, that the energy stored in an electric field is $\frac{1}{2}\varepsilon E^2$, defining all terms.

[10 marks]



Parallel plate capacitor:

C	= capacitance
A	= area of plate
l	= separation of plates
v	= voltage across the plates
i	= current charging/discharging the capacitor

Voltage-current relation on capacitance: $i = C\dfrac{dv}{dt}$

Power: $P = vi = Cv\dfrac{dv}{dt}$;

Energy stored: $W = \int P dt = \int_0^V cv dv = C\dfrac{V^2}{2} = \dfrac{1}{2}qV$;

Energy per unit volume: $w = \dfrac{\frac{1}{2}qV}{Al} = \dfrac{1}{2}\dfrac{q}{A}\dfrac{V}{l}$; $w = \dfrac{1}{2}DE$;

Knowing $D = \varepsilon E$; where D is electric flux density, E is electric field intensity and ε is permittivity of medium between the plates;

$w = \frac{1}{2}\varepsilon E^2$

(b) A parallel plate capacitor has its plates separated by 1.5 mm of air. The area of the plates is 1 m^2 and they are charged to a potential difference of 1 kV. The plates are pulled apart until they are separated by 2 mm of air. Assuming the potential difference remains the same, what is the average mechanical force experienced in separating the plates?

[10 marks]

Energy stored before pulling apart:

$$W_1 = \frac{1}{2}\varepsilon E_1^2 \times \text{Volume}_1 = \frac{1}{2}\times 8.85\times 10^{-12}\times\left(\frac{1000}{1.5\times 10^{-3}}\right)^2 \times 1\times(1.5\times 10^{-3}) = 0.00295\text{ J}$$

Energy stored after pulling apart:

$$W_2 = \frac{1}{2}\varepsilon E_2^2 \times \text{Volume}_2 = \frac{1}{2}\times 8.85\times 10^{-12}\times\left(\frac{1000}{1.5\times 10^{-3}}\right)^2 \times 1\times(2\times 10^{-3}) = 0.0022125\text{ J}$$

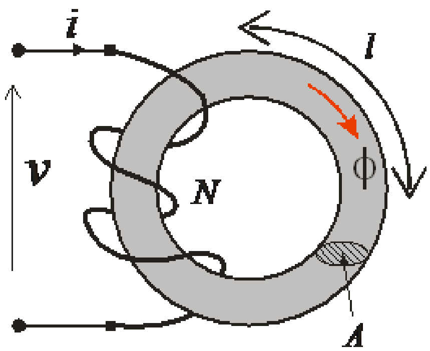
Change is energy or work done: $\Delta W = W_1 - W_2 == 0.00295 - 0022125 = 0.0007375\text{ J}$

Force as associated with work done: $F_{\text{mech}} = \dfrac{\Delta W}{\Delta l} = \dfrac{0.0007375}{(2-1.5)\times 10^{-3}} = 1.475\text{ N}$

Question 2.

(a) Derive the expression of the inductance $\boldsymbol{L}$ of a coil of $\boldsymbol{N}$ turns wound on a uniform magnetic core of constant cross-sectional area $\boldsymbol{A}$, relative permeability $\boldsymbol{\mu_r}$, and length $\boldsymbol{l}$.

[**10 marks**]



l	= capacitance
A	= cross-sectional area
μ	= permeability
F	= mmf
ϕ	= magnetic flux
B	= magnetic flux density
H	= magnetic field intensity
S	= reluctance

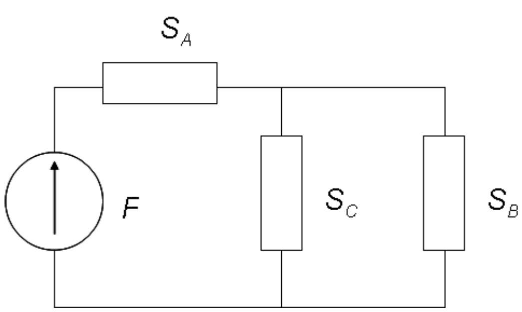
$S = \dfrac{F}{\phi} = \dfrac{Hl}{BA}$; knowing $B = \mu H$, then $S = \dfrac{l}{\mu A}$

By definition: $L = \dfrac{\text{Flux linkage}}{\text{current}} = \dfrac{N\phi}{i} = \dfrac{NF}{iS} = \dfrac{N^2}{S}$ with $F = Ni$; then $L = \dfrac{N^2}{\frac{l}{\mu A}} = \dfrac{\mu N^2 A}{l}$

(b) The magnetic circuit in Figure Q2 has reluctances of the three portions $S_A = S_B = 3 \times 10^7$ At/Wb and $S_C = 2 \times 10^7$ At/Wb. Coil 1 with 100 turns and negligible resistance has an applied voltage of 10 V at 1 kHz, while coil 2 with 1000 turns is on open circuit. Calculate

(i) the self inductance and rms current of coil 1;

[**10 marks**]



Total Reluctance $S_T = S_A + S_C // S_B = (3 + 1.2) \times 10^7 = 4.2 \times 10^7$ A/Wb

$$L = \frac{N^2}{S} = \frac{100^2}{4.2 \times 10^7} = 2.38 \times 10^{-4} \text{ H}$$

(ii) the peak magnetic flux in portion A;

[10 marks]

$V = 4.44 fNB_m A = 4.44 fN\phi_m$;

$$\phi_m = \frac{V}{4.44 fN} = \frac{10}{4.44 \times 1000 \times 100} = 2.25 \times 10^{-5} \text{ Wb}$$

(iii) the rms voltage induced in coil 2.

[10 marks]

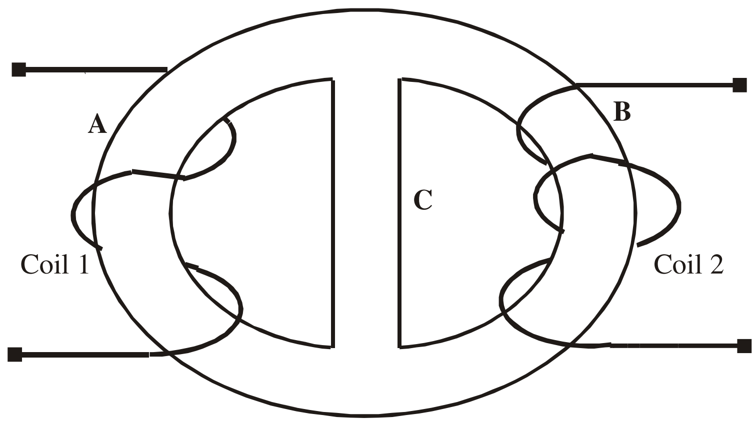


Figure Q2.

Like current divider:

$$\phi_{mB} = \phi_{mA}\left(\frac{S_C}{S_B + S_C}\right) = 2.25 \times 10^{-5} \times \left(\frac{2}{2+3}\right)$$

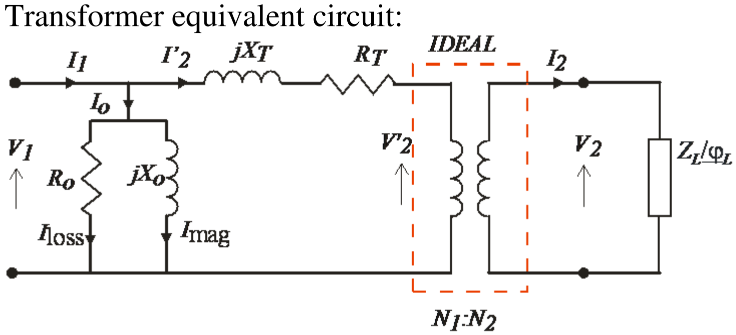
$$V_B = 4.44 fN\phi_{mB} = 4.44 \times 1000 \times 100 \times 2.25 \times 10^{-5} \times \frac{2}{5} = 40 \text{ V}$$

Question 3.

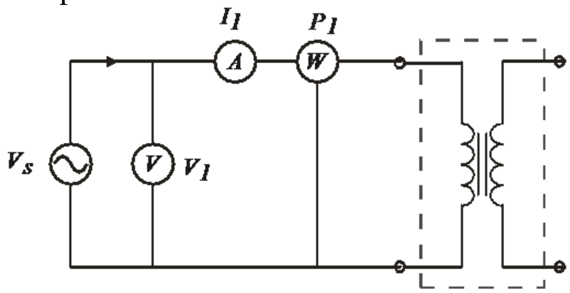
(a) Describe and justify the methods used to determine the transformer parameters from experiments.

[10 marks]

Transformer equivalent circuit:

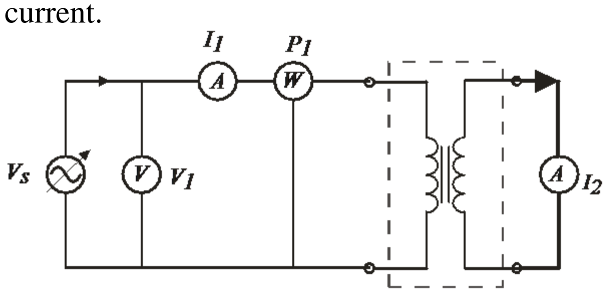


1. Open circuit test: Connect circuit as below, and apply rated voltage



Measure V_1, I_1, and P_1.
Justification: on open-circuit, all current appears as I_o in the equivalent circuit. Thus, find R_o, X_o, from P_1, V_1, and I_1 (showing how).

1. Short circuit test: Connect circuit as below, and apply low voltage that allows full-load current.

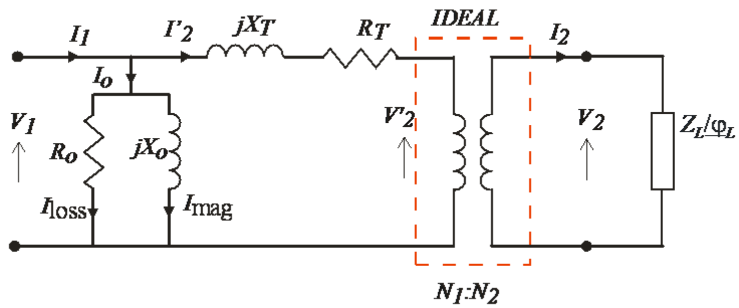


Measure V_1, I_1, and P_1.
Justification: on short-circuit, all current passes through as R_T and X_T in the equivalent circuit. Thus, find R_T , X_T, from P_1, V_1, and I_1 (showing how).

(b) A 240/12-V, 50-Hz transformer has no-load current of 25 mA at 60^{o} lag, a full-load secondary current of 2.0 A, and full-load copper loss of 4 W. At full-load and power factor of 0.5 lag, calculate

(i) the efficiency;

[**5 marks**]



$$P_{Fe} = V_o I_o \cos\theta_o = 240 \times (25 \times 10^{-3}) \times \cos 60^\circ = 3 \text{ W}$$

$$\eta = \frac{\text{output}}{\text{output} + \text{losses}} = \frac{\text{output}}{\text{output} + (P_{Fe} + P_{Cu})} = \frac{xS_r \cos\varphi}{xS_r \cos\varphi + P_{Fe} + x^2 P_{CuFL}}$$

$$= \frac{12 \times 2 \times 0.5}{12 \times 2 \times 0.5 + 3 + 4} = 0.631$$

(ii) the supply current.

[15 marks]

$$I_s = I_1 = I_o + I_2^{'} = 25 \times 10^{-3} \angle -60^\circ + 2 \times \frac{12}{240} \angle -60^\circ = 0.125 \angle -60^\circ \text{ A}$$

If a capacitor is now placed in parallel with the load to make the power factor unity at the secondary terminals, determine

(iii) the new efficiency.

[10 marks]

$$\eta_{new} = \frac{12 \times 2 \times 0.5}{12 + 3 + \left(\frac{1}{2}\right)^2 \times 4} = 0.75$$

END OF EEE 3352 TEST