EEE 3352

ELECTROMECHANICS & ELECTRICAL MACHINES

TERM II TEST

SEPTEMBER 2018

MODEL SOLUTIONS

1.

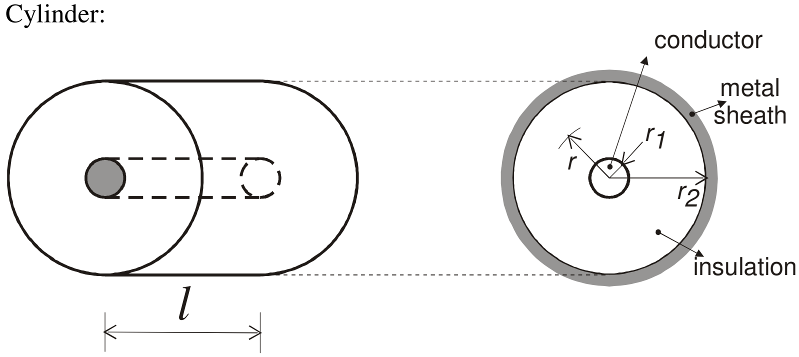
(a)

r = radius from centre of electrode

l = axial length of cable

q = charge on electrode

Cylinder:



Total electric flux $\psi = q$

At radius r, electric flux density D and electric field intensity E are

$$D = \frac{\psi}{A} = \frac{q}{2\pi rl}$$
$$E = \frac{D}{\varepsilon} = \frac{q}{2\pi rl\varepsilon}$$
; where ε is the permittivity of insulation medium

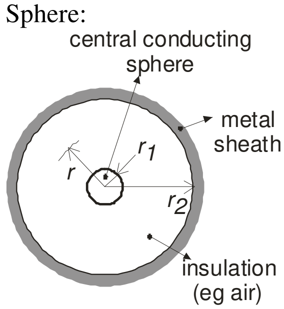
Potential difference V at r is

$$V = \int_{r_1}^{r_2} E dr = \int_{r_1}^{r_2} \frac{q}{2\pi rl\varepsilon} dr = \frac{q}{2\pi l\varepsilon} \ln \frac{r_2}{r_1}$$

$$C = "\frac{Q}{V}"$$

$$C = \frac{2\pi l\varepsilon}{\ln \frac{r_2}{r_1}}$$

Sphere:



$$D = \frac{\psi}{A} = \frac{q}{4\pi r^2}; \quad E = \frac{D}{\varepsilon} = \frac{q}{4\pi r^2 \varepsilon}$$

$$V = \int E dr = \frac{q}{4\pi\varepsilon}\int_{r_1}^{r_2}\frac{dr}{r^2} = \frac{q}{4\pi\varepsilon}\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

Capacitance: $C = \frac{q}{V} = \frac{4\pi\varepsilon}{\left(\frac{1}{r_1}-\frac{1}{r_2}\right)}$

(b)

(i)

$E = \left(\frac{q}{2\pi l\varepsilon}\right)\frac{1}{r} = \left(\frac{V}{\ln\frac{r_2}{r_1}}\right)\frac{1}{r}$; for maximum E , r is minimum, i.e., $r = r_1$, and $E_{\max} = \frac{V}{r_1 \ln\frac{r_2}{r_1}}$

$\frac{dE_{\max}}{dr_1} = 0 = \frac{-V}{\left(r_1\ln\frac{r_2}{r_1}\right)^2}\left[\ln\frac{r_2}{r_1} + r_1(\frac{r_1}{r_2})(\frac{-r_2}{r_1^2})\right]$, $\qquad \ln\frac{r_2}{r_1} - 1 = 0; \therefore \frac{r_2}{r_1} = e$

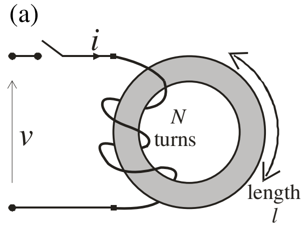
(ii)

V_{rms} = 10 kV, ε_r = 2.5, , $d_2 = 2r_2$ = 10 cm.

$$E_{\max} = \frac{V}{r_1\ln\frac{r_2}{r_1}} = \frac{V}{\frac{r_2}{e}\ln e} = \frac{eV}{r_2} = \frac{2.718\times\sqrt{2}\times(10\times10^3)}{5\times10^{-2}} = \underline{\underline{7.7\times10^5\ \text{V/m}}}$$

2.

(a)



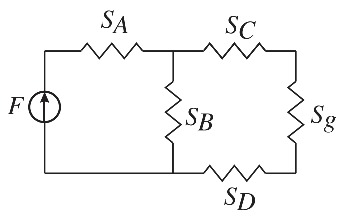
$Li = N\phi$

$v = L\frac{di}{dt}; P = vi = Li\frac{di}{dt}$

$W = \int Pdt = \int_0^I Lidi = L\frac{I^2}{2} = \frac{1}{2}N\phi I$; $w = \frac{\frac{1}{2}N\phi i}{Al} = \frac{1}{2}\frac{\phi}{A}\frac{NI}{l}$; $w = \frac{1}{2}BH = \frac{B^2}{2\mu}$

(b)

S_A = 3 x10^6 A/Wb; S_B = 9 x10^6 A/Wb; S_C = 2 x10^6 A/Wb; S_D = 2 x10^6 A/Wb; ; A = 159 mm^2, l_g =1 mm.;



(i)

$S_g = \frac{l_g}{\mu A} = \frac{1\times10^{-3}}{1\times4\pi\times10^{-7}\times(159\times10^{-6})} = \underline{\underline{5\times10^6 \text{ A/Wb}}}$

(ii)

$S_T = [3+9//(2+2+5)]10^6 == 7.5\times10^6$ A/Wb

$L = \frac{N^2}{S_T} = \frac{100^2}{7.5\times10^6} = 1.3\times10^{-3}\text{ H} = \underline{\underline{1.3\text{ mH}}}$

(iii)

$\phi = \frac{F}{S} = \frac{NI}{S} = \frac{20\times10}{7.5\times10^6} = 2.67\times10^{-4} = \underline{\underline{267\ \ \mu\text{Wb}}}$

(iii)

$\phi_g = \frac{1}{2}\phi = \frac{1}{2}\times2.67\times10^{-4} = 1.33\times10^{-4}$ Wb; $B_g = \frac{\phi_g}{A} = \frac{133\times10^{-6}}{159\times10^{-6}} = \underline{\underline{0.84\ \ \text{T}}}$

(iv)

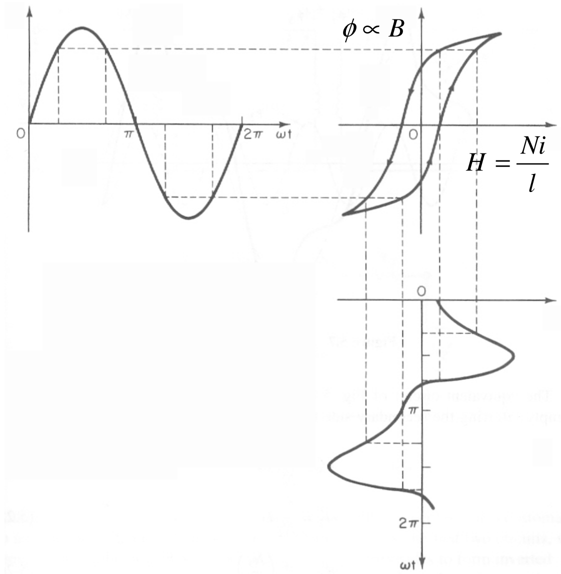
$W = \frac{1}{2}LI^2 = \frac{1}{2}\times0.84\times20^2 = \underline{\underline{0.26\ \ \text{J}}}$

(v)

$W_g = \frac{1}{2}\frac{B^2}{\mu}\times(Volume) = \frac{1}{2}\times\frac{0.84^2}{4\pi\times10^{-7}}\times(1\times10_{-3})\times(159\times10^{-6}) = \underline{\underline{0.089\ \ \text{J}}} \approx \underline{\underline{0.09\ \ \text{J}}}$

3.

(a)



$v = V_m \cos\omega t \;\; v = V_m \cos\omega t$;

$v = N\dfrac{d\phi}{dt}; \phi = \dfrac{1}{N}\int v dt; \phi = \dfrac{1}{N}\int V_m \cos\omega t$

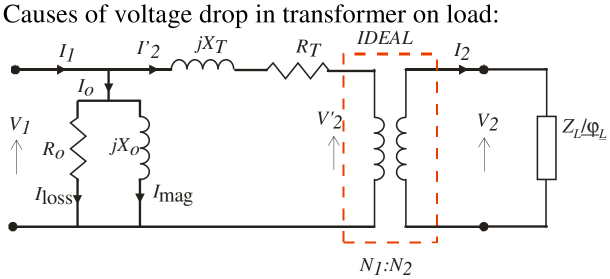
$\phi = \dfrac{V_m}{\omega N}\sin\omega t$

Shape of no-load waveform:

- nonsinusoidal due to hysteresis;
- "peaky" waveform due to saturation".

(b)

Causes of voltage drop in transformer on load:



As in the equivalent circuit, the series elements R_T and X_T cause voltage drop when load current I_2 flows.

(c)

$S_r = 75$ kVA, $V_1/V_2 = N_1/N_2 = 11000/240$; $V_{INL} = 310$V;

$$I'_{2FL} = \frac{S_r}{V_1} = \frac{75\times10^3}{11000} = 6.82 \quad \text{A}$$

(i)

$$I'^2_2 R_T = 1600 \rightarrow R_T = 34.4 \quad \Omega$$

$$Z_T = \frac{V_1}{I'_2} = \frac{310}{6.82} = 45.5\ \Omega\,;\ \cos\phi_T = \frac{R_T}{Z_T} = \frac{34.4}{45.5} = 0.756 \rightarrow \phi_T = 40.9^\circ \text{ and } \ p.f = 0.8 \rightarrow \phi_L = 36.9^\circ$$

$$\text{Reg} = \frac{I'_2 Z_T}{V_1}\cos(\varphi_T - \varphi_L) = \frac{6.82\times45.5}{11000}\times\cos(40.9^\circ - 36.9^\circ) = 0.028 = \underline{\underline{2.8\%}}$$

(ii)

$$V'_2 = V_1 - I'_2 Z_T = 11000\angle0^\circ - 6.82\angle-36.9^\circ \times 45.5\angle40.9^\circ = 10690 - j21.6 = 10690\angle0.1^\circ$$

$$V_2 = V'_2\frac{N_2}{N_1} = 10690\times\frac{240}{11000} = \underline{\underline{233.2\ \text{V}}}$$