THE UNIVERSITY OF ZAMBIA

SCHOOL OF ENGINEERING

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING

TERM TEST

TERM II – September 2019

EEE 3352

ELECTROMECHANICS & ELECTRICAL MACHINES

Model Solutions

Question 1. [20 marks]

Select and write down the choice you think is the most appropriate response to the following.

(a) The relative permeability of vacuum is
A. $4\pi \times 10^{-7}$ H/m B. 1 H/m C. 1 D. $1/4\pi$ E. None of these.

[4 marks]

(b) A capacitor that stores a charge of 0.5 C at 10 V has a capacitance in Farads of
A. 5 B. 20 C. 10 D. 0.05 E. None of these.

[4 marks]

(c) If dielectric slab of thickness 5 mm and $\varepsilon_r = 5$ is inserted between the plates of an air capacitor increasing the plate separation from 1 mm to 5 mm, its capacitance is
A. Halved B. Doubled C. Tripled D. The Same E. None of these.

[4 marks]

(d) Which one of the following will not affect the iron loss of an iron-cored inductor?
A. Applied voltage B. Current in coil C. Frequency D. Magnetic flux density E. No correct answer.

[4 marks]

(e) Which one of the following best describes the phase relationship between the applied voltage phasor $\overline{V}$ and the magnetising flux phasor $\overline{\Phi}$ in a coil-excited magnetic circuit?
A. $\overline{V}$ leads $\overline{\Phi}$ by 90° B. $\overline{V}$ lags $\overline{\Phi}$ by 90° C. $\overline{V}$ and $\overline{\Phi}$ are in phase D. Depends on the phase angle of the coil current E. None of these.

[4 marks]

---**Solutions**---

(a) C (b) D (c) D (d) B (e) A

Question 2. [20 marks]

(a) Given $E = \left(\dfrac{V}{\ln \frac{r_2}{r_1}} \right) \dfrac{1}{r}$ for a concentric cable, where the variables have their usual meaning, discuss the conditions for maximum electric field, E_{max}. Develop the conditions for reaching the minimum value of E_{max}.

[8 marks]

(b) The radius of the copper core of a single-core rubber-insulated cable is 2.25 mm. The rubber insulation has a relative permittivity of 4 and breakdown field strength of 18×10^6 V/m. A voltage of 10 kV may be safely applied between the core and the earthed lead sheath with a safety factor of 3. Calculate, the

(i) radius of the lead sheath which covers the rubber insulation;

[6 marks]

(ii) cable capacitance per metre.

[6 marks]

--**Solutions**--

(a)

$E = \left(\dfrac{V}{\ln \frac{r_2}{r_1}} \right) \dfrac{1}{r}$: with V, r_1, & r_2 fixed, E_{max} occurs when r is minimum. Thus $r = r_1$.

[2 marks]

$$E = \left(\frac{V}{\ln \frac{r_2}{r_1}} \right) \frac{1}{r_1}$$

To determine minimum E_{max}, find minima or maxima:

$$\frac{dE_{\max}}{dr_1} = 0 = \frac{-V}{\left(r_1 \ln \frac{r_2}{r_1}\right)^2} \left[\ln \frac{r_2}{r_1} + r_1 (\frac{r_1}{r_2})(\frac{-r_2}{r_1^2}) \right]$$

$$\ln \frac{r_2}{r_1} - 1 = 0 \rightarrow \frac{r_2}{r_1} = e$$

$$r_1 = \frac{r_2}{e}$$

[6 marks]

(b)

r_1 =2.25 mm, $\varepsilon_r = 4$, $E_{max} = 18 \times 10^6$ V/m, $V =$ 10 kV, $SF = 3$;

$$V_{safe} = SF \times V = 3 \times (10 \times 10^3) = 30 \times 10^3 \text{ V}$$

[2 marks]

(i)

$$E_{\max} = \frac{V}{r_1 \ln \frac{r_2}{r_1}} \rightarrow \ln \frac{r_2}{r_1} = \frac{V}{r_1 E_{\max}} = \frac{30 \times 10^3}{(2.25 \times 10^{-3}) \times (18 \times 10^6)} = 0.7407$$

$$\frac{r_2}{r_1} = 2.0975 \rightarrow r_2 = 2.0975 \times 2.25 = \underline{\underline{4.72 \text{ mm}}}$$

[6 marks]

(ii)

$$C = \frac{2\pi\varepsilon}{\ln \frac{r_2}{r_1}} = \frac{2\pi\varepsilon_r\varepsilon_0}{\ln(2.0975)} = \frac{2\pi \times 4 \times 8.85 \times 10^{-12}}{0.7407} = 3 \times 10^{-10} \text{ F/m} = \underline{\underline{30 \text{ nF/m}}}$$

[4 marks]

Question 3. [20 marks]

(a) From first principles, derive the expression for the energy stored in an electric field.

[**10 marks**]

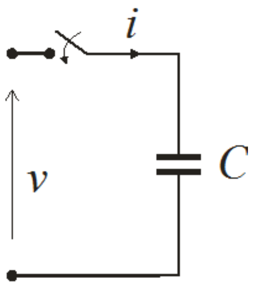
(b) Calculate the change in the stored energy of a parallel-plate capacitor if a dielectric slab of relative permittivity 5 is introduced between its two plates and the plates have been disconnected from the voltage source.

(c) [**10 marks**]

--**Solutions**--

(a)

Parallel plate capacitor:



A	=	plate area
l	=	plate separation
i	=	charging current
V	=	Plate voltage
C	=	capacitance of arrangement
q	=	charge between plates
E	=	Electric field intensity
D	=	Electric flux density

Charge: $q = Cv$

Charging current: $i = C\dfrac{dv}{dt}$

Power: $P = vi = Cv\dfrac{dv}{dt}$

Energy: $W = \int P dt = \int_0^V cvdv = C\dfrac{V^2}{2} = \dfrac{1}{2}qV$

Energy per unit volume: $w = \dfrac{\frac{1}{2}qV}{Al} = \dfrac{1}{2}\dfrac{q}{A}\dfrac{V}{l} = \dfrac{1}{2}DE$

[10 marks]

(b)

$Q = CV =$ constant , A and l fixed, all the time:

$W = \dfrac{1}{2}DE = \dfrac{1}{2}\varepsilon E$; $C = \varepsilon\dfrac{A}{l}$

Case 1: C_1, V_1, E_1,

$W_1 = \dfrac{1}{2}\varepsilon_{r1}\varepsilon_0 E_1^2 = \dfrac{1}{2}\varepsilon_0\left(\dfrac{V_1}{l}\right)^2$; $C_1 = \varepsilon_0\dfrac{A}{l}$

Case 1: C_2, V_2, E_2,

$$W_2 = \frac{1}{2}\varepsilon_2\varepsilon_0 E_2^2 = \frac{1}{2}\varepsilon_{r2}\varepsilon_0\left(\frac{V_2}{l}\right)^2 ; \; C_2 = \varepsilon_{r2}\varepsilon_0\frac{A}{l}$$

$$C_1V_1 = C_2V_2 \to \varepsilon_0\frac{A}{l}V_1 = \varepsilon_{r2}\varepsilon_0\frac{A}{l}V_2 \to V_2 = \frac{V_1}{\varepsilon_{r2}}$$

$$W_2 = \frac{1}{2}\varepsilon_{r2}\varepsilon_0\left(\frac{V_1}{\varepsilon_{r2}l}\right)^2 = \frac{1}{2}\frac{\varepsilon_{r2}\varepsilon_0}{\varepsilon_{r2}^2}\left(\frac{V_1}{l}\right)^2 = \frac{1}{2}\frac{\varepsilon_0}{\varepsilon_{r2}}\left(\frac{V_1}{l}\right)^2$$

$$\frac{W_2}{W_1} = \frac{1}{\varepsilon_{r2}} = \frac{1}{5} \to \underline{\underline{W_2 = \frac{W_1}{5}}}$$

[10 marks]

Question 4. [20 marks]

(a) Derive the expression for the resistance R that subsists between two electrolyte-filled concentric metal cylinders of diameters d_1 and d_2. The electrolyte has resistivity ρ and fills to a depth l.

[**10 marks**]

(b) A liquid resistor consists of two concentric metal cylinders of diameters d_1 = 20 and d_2 = 35 cm, respectively, and is filled with water of resistivity ρ = 8000 Ω-cm between them. The length of both cylinders is l = 60 cm.

(i) Calculate the resistance of the liquid resistor.

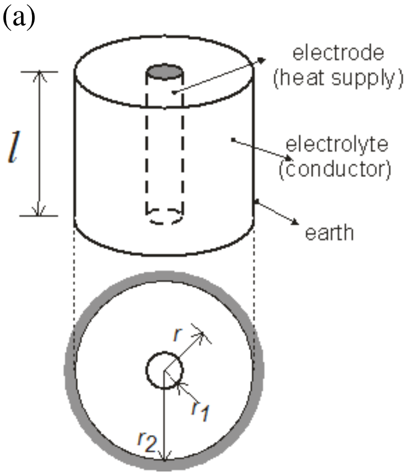
[**6 marks**]

(ii) What is the heating power available when the cylinders are connected to a 240-V ac supply?

[**4 marks**]

---**Solutions**---

(a)



r	=	general radius
r_1	=	radius, inner metal cylinder
r_2	=	radius, outer metal cylinder
l	=	depth of electrolyte
V	=	voltage between cylinder
I	=	total conduction current
σ	=	conductivity
ρ	=	resistivity
E	=	electric field intensity
J	=	current density

$$E = \frac{J}{\sigma} = \frac{I}{2\pi l\sigma}\frac{1}{r} ; \; J = \frac{I}{A} = \frac{I}{2\pi rl}$$

$$V = \int E dr = \frac{I}{2\pi l\sigma}\int_{r_1}^{r_2}\frac{dr}{r} = \frac{I}{2\pi l\sigma}\ln\frac{r_2}{r_1}$$

With $\rho = \dfrac{1}{\sigma}$, $\quad R = \dfrac{V}{I} = \rho \dfrac{\ln \frac{r_2}{r_1}}{2\pi l}$

[10 marks]

(b)
d_1 = 20 cm, d_2 = 35 cm, l =60 cm, ρ = 8000 Ω.cm
(i)

$$R = \rho \frac{\ln \frac{r_2}{r_1}}{2\pi l} = 80 \times \frac{\ln \frac{35}{20}}{2\pi \times 0.6} = \underline{\underline{11.875\ \Omega}}$$

[6 marks]

(ii)

$$P = \frac{V^2}{R} = \frac{240^2}{11.875} = 4850\ \text{W} = \underline{\underline{4.85\ \text{kW}}}$$

[4 marks]

Question 5. [20 marks]

(a) From first principles, derive the expression for the permeance of a uniform magnetic circuit composed of material of permeability μ, length l and having a constant cross-sectional area A .

[6 marks]

(b) A soft-iron ring with relative permeability 1000 has a mean circumference of 800 mm and a cross-sectional area of 500 mm^2 and is wound with coil of 1000 turns. A radial air-gap of 1 mm is cut in the ring.

(i) Calculate the current in the coil required to produce an air-gap flux of 0.5 mWb.

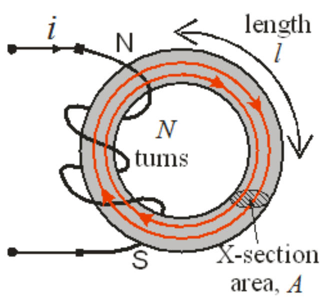
[10 marks]

(ii) What is inductance presented by the coil?

[4 marks]

--**Solutions**--

(a)



l = complete length of magnetic circuit
A = X-section area of magnetic circuit
S = reluctance of magnetic circuit
F = mmf of coil
ϕ = magnetic flux
B = magnetic flux density
H = magnetic field intensity
μ = permeability of magnetic circuit

Permeance: $\Lambda = \dfrac{\phi}{F} = \dfrac{BA}{Hl} = \dfrac{B}{H}\dfrac{A}{l} = \mu\dfrac{A}{l}$

$S = \dfrac{1}{\Lambda} = \dfrac{l}{\mu A}$

[6 marks]

(b)

l_{FE} = 800 mm, l_g = 1 mm, μ_r = 1000, A = 500 mm^2, N = 1000;

(i)

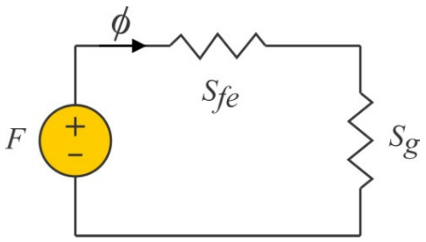
$S = \dfrac{l}{\mu A}$

$$S_{Fe} = \frac{l_{Fe}}{\mu_{Fe}\mu_0 A_{Fe}} = \frac{800\times10^{-3}}{1000\times4\pi\times10^{-7}\times500\times10^{-6}} = 1,273,239 \text{ A/Wb}$$

[2 marks]

$$S_g = \frac{l_g}{\mu_0 A_{Fe}} = \frac{1\times10^{-3}}{4\pi\times10^{-7}\times500\times10^{-6}} = 1,593,149 \text{ A/Wb}$$

[2 marks]



$$S_T = S_{Fe} + S_g = 1,273,239 + 1,591,549 = 2,864,788 \text{ A/Wb}$$

[2 marks]

$$F = \phi S_T = 0.5\times10^{-3}\times2864788 = 1432 \text{ A}$$

[2 marks]

$$NI = 1432 \text{ A} \rightarrow I = \frac{1432}{1000} = 1.423 \text{ A}$$

[2 marks]

(ii)

$$L = \frac{N^2}{S} = \frac{1000^2}{2864788} = \underline{\underline{0.35 \text{ A}}}$$

[4 marks]