The University of Zambia
Department of Mathematics and Statistics
MAT 4119 - Engineering Mathematics III
Tutorial Sheet I

2023

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1. Approximate the following decimal numbers to three digits by using (a) rounding rules (b) chopping (truncation) rules

   (a) $x_1 = 1.34579$ (b) $x_2 = 1.34679$ (c) $x_3 = 1.34479$
   (d) $x_4 = 3.34379$ (e) $x_5 = 2.34579$

2. Compute the absolute error, relative error, and percentage error in approximations of p by $p*$ in each of the following:

   (a) $p = \sqrt{3}, \quad p^* = 1.73$ (b) $p = 10^e, \quad p^* = 530$ (c) $p = e^2, \quad p^* = 7.4.$

3. Find relative maximum error in the function

$$u = \frac{5xy^2}{z^3} \text{ at} x = y = z = 1 \text{ with} \triangle x = \triangle y = \triangle z = 0.001.$$

4. If p is approximated by p^* to 4 significant figures in each of the following, find the largest interval in which p^* must lie.

   (a) $p = 700$ (b) 3.3 (c) 33.3

5. The geometric sequence

$$1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \cdots, \frac{1}{3^n}, \cdots$$

   can be generated recursively by either

   (i) $p_0 = 1, \quad p_1 = \frac{1}{3}, \quad p_n = \frac{5}{6}p_{n-1} - \frac{1}{6}p_{n-2}, \quad n \geq 2, \quad n \in \mathbb{Z}$

   (ii) $q_0 = 1, \quad q_1 = \frac{1}{3}, \quad q_n = \frac{5}{3}q_{n-1} - \frac{4}{9}q_{n-2}, \quad n \geq 2, \quad n \in \mathbb{Z}$

   (a) Compute p_n for $n = 2, 3, \cdots 8$.
   (b) Compute q_n for $n = 2, 3, \cdots 8$.
   (c) Determine whether the procedures above are stable or unstable.

6. Use the fourth Taylor polynomial about $x_0 = 1$ to approximate $\ln(1.1)$ and find the maximum error in the approximation.

7. Use the 4th degree Maclaurine polynomial for $f(x) = \cos x$ to estimate $\cos(0.1)$ and find the maximum error.

8. Use the 3rd degree Taylor polynomial centered at $x = 0$ for $f(x) = e^x$ to estimate e and determine the maximum error.

9. The Maclaurin series for $\arctan s$ converges for $-1 \leq x \leq 1$ and is given by

$$\arctan x = \sum_{i=1}^{\infty} \left((-1)^{i+1} \frac{x^{2i-1}}{2i-1} \right).$$

Since $\tan(\frac{\pi}{4}) = 1$, it follows that the series for π is $\pi = 4 \arctan 1$.

(a) Determine the number of terms the series that need to be summed in order to ensure that

$$|4 \arctan 1 - \pi| < 10^{-3}.$$

(b) How many terms are required to ensure the 10^{-10} accuracy needed for an approximation to π?