The University of Zambia
Department of Mathematics and Statistics
MAT 4119 - Engineering Mathematics III
Tutorial Sheet 3

2023

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1. Solve the following systems of linear equations using Gauss elimination method:

(a) $\begin{cases} x_1 + x_2 - x_3 = 3, \\ 2x_1 - x_2 + 3x_3 = 0, \\ -x_1 - 2x_2 + x_3 = -5 \end{cases}$ (b) $\begin{cases} x_1 + x_2 - x_3 = 3, \\ -x_1 + x_2 + x_3 = 2, \\ x_1 + 3x_2 - x_3 = 8 \end{cases}$ (c) $\begin{cases} -2x_1 + x_2 - x_3 = 1, \\ 3x_1 + x_2 - 3x_3 = -1, \\ x_1 - 2x_2 + x_3 = 5 \end{cases}$

2. Use Gauss elimination with scaled partial pivoting and three-digit rounding arithmetic to solve the following systems of linear equations:

(a) $\begin{cases} 3.3330x_1 + 15920x_2 + 10.333x_3 = 7959, \\ 2.2220x_1 + 16.710x_2 + 9.6120x_3 = 0.965, \\ -1.5611x_1 + 5.1792x_2 - 1.6855x_3 = 2.714 \end{cases}$ (b) $\begin{cases} \pi x_1 + \sqrt{2}x_2 - x_3 + x_4 = 0, \\ ex_1 - 2x_2 + x_3 + 2x_4 = 1, \\ x_1 + x_2 - \sqrt{3}x_3 + x_4 = 2, \\ -x_1 - x_2 + x_3 - \sqrt{5}x_4 = 3 \end{cases}$

(c) $\begin{cases} x_1 - 4x_2 + 4x_3 + 7x_4 = 4, \\ 2x_2 - x_3 = 5, \\ 2x_1 + x_2 + x_3 + 4x_4 = 2 \\ 2x_1 - 3x_2 + 2x_3 - 5x_4 = 9 \end{cases}$ (d) $\begin{cases} x_1 - x_2 + 2x_3 + x_4 = 1, \\ 3x_1 + 2x_2 + x_3 + 4x_4 = 1, \\ 5x_1 - 8x_2 + 6x_3 + 3x_4 = 1, \\ 4x_1 + 2x_2 + 5x_3 + 3x_4 = -1 \end{cases}$

3. Carry out the first three iterations of the Jacobi's Method and Gauss-Siedel Method for the following systems using $\mathbf{X}^{(0)} = (0, 0, 0)^t$

(a) $\begin{cases} 2x_1 - x_2 + x_3 = -1, \\ 3x_1 + 3x_2 + 9x_3 = 0, \\ 3x_1 + 2x_2 + 5x_3 = 4 \end{cases}$ (b) $\begin{cases} 2x_2 + 3x_3 = 0, \\ x_1 - x_2 - x_3 = 0.375, \\ x_1 - x_2 + 2x_3 = 0 \end{cases}$ (c) $\begin{cases} -0.0023x_1 + x_2 - x_3 = 1, \\ 34x_1 + 3x_2 - x_3 = -1, \\ 3x_1 - 2x_2 + x_3 = 0 \end{cases}$

4. The nonlinear system

$$x_1^2 - 10x_1 + x_2^2 = -8, \qquad x_1 x_2^2 + x_1 - 10x_2 + 8 = 0.$$

can be transformed into the fixed-point problem

$$x_1 = g_1(x_1, x_2) = \frac{x_1^2 + x_2^2 + 8}{10}, \quad x_2 = g_2(x_1, x_2) = \frac{x_1 x_2^2 + x_1 + 8}{10}.$$

(a) Show that $\mathbf{G} = (g_1, g_2)^t$ mapping D into $\mathbb{R}^2$ has a unique fixed point in

$$D = \{\mathbf{X} = (x_1, x_2)^t : 0 \le x_i \le 1.5, i = 1, 2\}$$

(b) Apply functional iteration to approximate the solution.

5. The nonlinear system

$$5x_1^2 - x_2^2 = 0, \qquad x_2 - 0.25(\sin x_1 + \cos x_2) = 0$$

has a solution near $(\frac{1}{4}, \frac{1}{4})^t$

(a) Find a function G and a set $D \in \mathbb{R}^2$ such that $G : D \to \mathbb{R}^2$ and G has a unique fixed point in D.

(b) Apply functional iteration to approximate the solution to within 10^{-5} in the l^∞ norm.

6. Use Newton's method with the given $X^{(0)}$ to compute $X^{(2)}$ for each of the following nonlinear systems

(a) $\sin(4\pi x_1 x_2) - 2x_2 - x_1 = 0; \quad \dfrac{4\pi - 1}{4\pi}(e^{2x_1} - e) + 4ex_2^2 - 2ex_1 = 0; \quad X^{(0)} = (0, 0)^t.$

(b) $x_1^3 + x_1^2 x_2 - x_1 x_3 + 6 = 0; \quad e^{x_1} + e^{x_2} - x_3 = 0; \quad x_2^2 - 2x_1 x_3 = 4; \quad X^{(0)} = (-1, -2, 1)^t.$