The University of Zambia
Department of Mathematics and Statistics
MAT 4119 - Engineering Mathematics III
Tutorial Sheet 7

2023

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1. Show that each of the following IVP has a unique solution and find the solution.

   (a) $y' = y\cos t, \quad 0 \le t \le 1, \quad y(0) = 1.$

   (b) $y' = -\frac{2}{t}y + t^2 e^t, \quad 1 \le t \le 2, \quad y(1) = \sqrt{2}e.$

2. Show that the following IVPs are well-posed.

   (a) $y' = 1 - y, \quad 0 \le t \le 2, y(0) = 0,$ (b) $y' = -ty + 4\frac{t}{y}, \quad 0 \le t \le 1, \quad y(0) = 1.$

3. Use Euler's method to approximate the solution for each of the following IVP.

   (a) $y' = \sin t + e^{-t}, \quad 0 \le t \le 1, \quad y(0) = 0, \quad h = 0.5.$

   (b) $y' = -ty + \frac{4t}{y}, \quad 0 \le t \le 1, \quad y(0) = 1, \quad h = 0.25.$

   (c) $y' = 1 + t\sin(ty), \quad 0 \le t \le 2, \quad y(0) = 0, \quad h = 0.1$

4. Use Taylor's methods of orders two and four to approximate the solution to:

   (a) $y' = (\frac{y}{t})^2 + \frac{y}{t}, \quad 1 \le t \le 1.2, \quad y(1) = 1, \quad h = 0.1.$

   (b) $y' = \frac{1}{t}(y^2 + y), \quad 1 \le t \le 3, \quad y(1) = -2, \quad h = 0.5.$

5. Given the IVP
   $$y' = \frac{2}{t}y + t^2 y + t^2 e^t, \quad 1 \le t \le 2, \quad y(1) = 0,$$
   with exact solution $y(t) = t^2(e^t - e)$. Use Taylor's method of orders two and four with $h = 0.1$ to approximate the solution and compare it with the actual values of y.

6. Use the Runge-Kutta methods of order two and four to approximate the solutions to each of the following IVPs, and compare the results to the actual values:

   (a) $y' = \frac{y^2+y}{t}, \quad 1 \le t \le 3, \quad y(1) = -2, \quad h = 0.2$; actual solution is $y(t) = \frac{2t}{1-2t}$

   (b) $y' = -y + t\sqrt{y}, \quad 2 \le t \le 3, \quad y(2) = 2, \quad h = 0.25$; actual solution is
   $y(t) = \left(t - 2 + \sqrt{2}ee^{-\frac{t}{2}}\right)^2$