The University of Zambia Department of Mathematics and Statistics MAT 4119 - Engineering Mathematics III Tutorial Sheet I

March 2024

1. Approximate the following decimal numbers to three digits by using (a) rounding rules (b) chopping (truncation) rules

(a)
$$x_1 = 1.34579$$
 (b) $x_2 = 1.34679$ (c) $x_3 = 1.34479$ (d) $x_4 = 3.34379$ (e) $x_5 = 2.34579$

(b)
$$x_2 = 1.34679$$

(c)
$$x_3 = 1.34479$$

(d)
$$x_4 = 3.34379$$

(e)
$$x_5 = 2.34579$$

2. Compute the absolute error, relative error, and percentage error in approximations of pby p* in each of the following:

(a)
$$p = \sqrt{3}$$
, $p^* = 1.73$ (b) $p = 10^e$, $p^* = 530$ (c) $p = e^2$, $p^* = 7.4$.

(b)
$$p = 10^e$$
, $p^* = 530$

(c)
$$p = e^2$$
, $p^* = 7.4$

3. Find relative maximum error in the function

$$u = \frac{5xy^2}{z^3}$$
 at $x = y = z = 1$ with $\Delta x = \Delta y = \Delta z = 0.001$.

4. If p is approximated by p^* to 4 significant figures in each of the following, find the largest interval in which p^* must lie.

(a)
$$p = 700$$

5. Use the fifth Taylor polynomial about $x_0 = 1$ to approximate $\ln(1.1)$ and find the maximum error in the approximation.

6. Use the 5th degree Maclaurine polynomial for $f(x) = \cos x$ to estimate $\cos(0.1)$ and find the maximum error.

7. Use the 4th degree Taylor polynomial centered at x=0 for $f(x)=e^x$ to estimate e and determine the maximum error.

8. The Maclaurin series for $\arctan s$ converges for $-1 \le x \le 1$ and is given by

$$\arctan x = \sum_{i=1}^{\infty} \left((-1)^{i+1} \frac{x^{2i-1}}{2i-1} \right).$$

Since $\tan(\frac{\pi}{4}) = 1$, it follows that the series for π is $\pi = 4 \arctan 1$. Determine the number of terms of the series that need to be summed in order to ensure that the error in approximating π is less than 10^{-4} .