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1. For each of the following functions f, show that there is a root to the given equation $f(x) = 0$ on the given interval. Hence, approximate the root within an accuracy of

   (i) 3 significant figures
   (ii) 3 decimal places.

   (a) $f(x) = (x - 0.3)(x - 0.5)$ on $[0.31, 1.00]$ (b) $f(x) = x^3 - 4x - 9$ on $[2, 3]$
   (c) $f(x) = 2x - \log_{10} x - 7$ on $[3, 4]$ (d) $f(x) = x - 2^{-x}$ on $[0, 1]$.
   (e) $f(x) = 4x^2 - 1 - e^{\frac{x^2}{2}}$ on $[1, 3]$

2. (a) By sketching
   $$y = \cos x \text{ and } y = x^3 - 1$$
   on the same coordinate system, show that $f(x) = \cos x - x^3 + 1 = 0$ in some interval.
   (b) Using part (a), find the interval where the root lies.
   (c) Use bisection method to find an approximate value of the solution to $f(x) = 0$ in the interval found in part (b).

3. (a) By sketching the graphs of
   $$y = 4\sin x \text{ and } y = e^x$$
   on the same coordinate system, find the number of solutions to the equation
   $$f(x) = 4e^{-x}\sin x - 1 = 0$$
   in the interval $[0, \pi]$.
   (b) Show that one zero of $f(x) = 0$ lies on the interval $[0, 0.5]$.
   (c) Hence, use bisection method to perform five iterations to approximate the root of $f(x) = 0$ in the interval $[0, 0.5]$.

4. Find a bound for the number of iterations needed to achieve an approximation to an accuracy of

   (i) 10^{-4} (4 decimal places)
   (ii) 4 significant figures

   to the solution of each of the following equations in the given interval while using the bisection method:

   (a) $x^3 + x - 4 = 0$, in $[1, 4]$ (b) $x^3 - x - 1 = 0$ in $[1, 2]$
   (c) $2x^6 - 5x^4 + 2 = 0$ in $[0, 1]$ (d) $x\sin x - 1 = 0$ in $[0, 2]$.