

# PROBLEM SET

## 1-5 PERIOD, FUNDAMENTAL PERIOD

The *fundamental period* is the smallest positive period. Find it for

- $\cos x, \sin x, \cos 2x, \sin 2x, \cos \pi x, \sin \pi x, \cos 2\pi x, \sin 2\pi x$
- $\cos nx, \sin nx, \cos \frac{2\pi x}{k}, \sin \frac{2\pi x}{k}, \cos \frac{2\pi nx}{k}, \sin \frac{2\pi nx}{k}$
- If  $f(x)$  and  $g(x)$  have period  $p$ , show that  $h(x) = af(x) + bg(x)$  ( $a, b$ , constant) has the period  $p$ . Thus all functions of period  $p$  form a **vector space**.
- Change of scale.** If  $f(x)$  has period  $p$ , show that  $f(ax)$ ,  $a \neq 0$ , and  $f(x/b)$ ,  $b \neq 0$ , are periodic functions of  $x$  of periods  $p/a$  and  $bp$ , respectively. Give examples.
- Show that  $f = \text{const}$  is periodic with any period but has no fundamental period.

## 6-10 GRAPHS OF $2\pi$ -PERIODIC FUNCTIONS

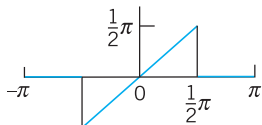
Sketch or graph  $f(x)$  which for  $-\pi < x < \pi$  is given as follows.

- $f(x) = |x|$
- $f(x) = |\sin x|, f(x) = \sin |x|$
- $f(x) = e^{-|x|}, f(x) = |e^{-x}|$
- $f(x) = \begin{cases} x & \text{if } -\pi < x < 0 \\ \pi - x & \text{if } 0 < x < \pi \end{cases}$
- $f(x) = \begin{cases} -\cos^2 x & \text{if } -\pi < x < 0 \\ \cos^2 x & \text{if } 0 < x < \pi \end{cases}$
- Calculus review.** Review integration techniques for integrals as they are likely to arise from the Euler formulas, for instance, definite integrals of  $x \cos nx$ ,  $x^2 \sin nx$ ,  $e^{-2x} \cos nx$ , etc.

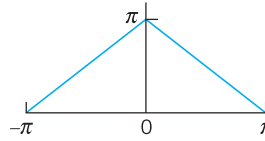
## 12-21 FOURIER SERIES

Find the Fourier series of the given function  $f(x)$ , which is assumed to have the period  $2\pi$ . Show the details of your work. Sketch or graph the partial sums up to that including  $\cos 5x$  and  $\sin 5x$ .

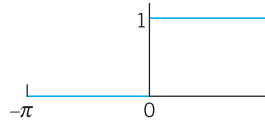
- $f(x)$  in Prob. 6
- $f(x)$  in Prob. 9
- $f(x) = x^2 \quad (-\pi < x < \pi)$
- $f(x) = x^2 \quad (0 < x < 2\pi)$
- 



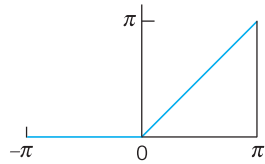
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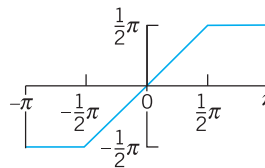
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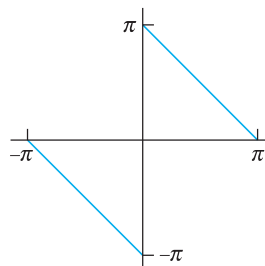
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20.



21.



## PROBLEM SET

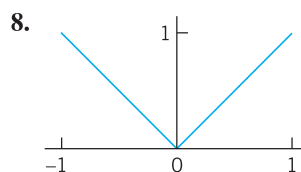
### 1–7 EVEN AND ODD FUNCTIONS

Are the following functions even or odd or neither even nor odd?

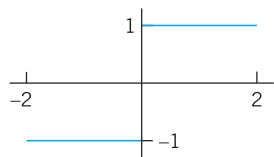
1.  $e^x$ ,  $e^{-|x|}$ ,  $x^3 \cos nx$ ,  $x^2 \tan \pi x$ ,  $\sinh x - \cosh x$
2.  $\sin^2 x$ ,  $\sin(x^2)$ ,  $\ln x$ ,  $x/(x^2 + 1)$ ,  $x \cot x$
3. Sums and products of even functions
4. Sums and products of odd functions
5. Absolute values of odd functions
6. Product of an odd times an even function
7. Find all functions that are both even and odd.

### 8–17 FOURIER SERIES FOR PERIOD $p = 2L$

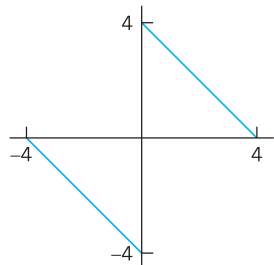
Is the given function even or odd or neither even nor odd? Find its Fourier series. Show details of your work.



9.



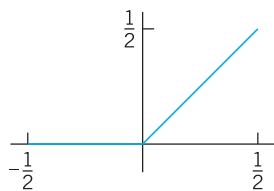
10.



11.  $f(x) = x^2 \quad (-1 < x < 1), \quad p = 2$

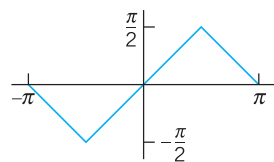
12.  $f(x) = 1 - x^2/4 \quad (-2 < x < 2), \quad p = 4$

13.



14.  $f(x) = \cos \pi x \quad (-\frac{1}{2} < x < \frac{1}{2}), \quad p = 1$

15.



16.  $f(x) = x|x| \quad (-1 < x < 1), \quad p = 2$

17.

