The University of Zambia
Department of Mathematics and Statistics
MAT 4119 - Engineering Mathematics III
Tutorial Sheet 3

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1. For each of the following, the equation $f(x) = 0$ has been written in the form $x = g(x)$. Determine which ones converge to a fixed-point in the specified interval:

   (a) $g(x) = \dfrac{5}{x^2} + 2$ on $[2.5, 3]$.

   (b) $g(x) = \pi + 0.5 \sin\left(\dfrac{x}{2}\right)$ on $[0, 2\pi]$.

   (c) $g(x) = 2^{-x}$ on $\left[\dfrac{1}{3}, 1\right]$.

   (d) $g(x) = \dfrac{2 + x^2 - e^x}{3}$ on $[0, 1]$.

   (e) $g(x) = (20 - x)^{\frac{1}{3}}$ on $[2, 3]$.

2. Most functions can be rearranged in several ways to give $x = g(x)$ with which to begin the fixed-point method. For $f(x) = e^x - 2x^2 = 0$, one $g(x)$ is $x = \pm\sqrt{\dfrac{e^x}{2}}$.

   (a) Show that this converges to the root near 1.5 if the positive value is used and to the root near -0.5 if the negative value is used.

   (b) There is a third root near 2.6. Show that we do not converge to this root even though values near the root such as $x_0 = 2.5$ or $p_0 = 2.7$ are used to begin the iteration.

   (c) Find another rearrangement that does converge correctly to the third root.

3. Use the function $g(x) = (20 - x)^{\frac{1}{3}}$ to solve the equation $x^3 + x = 20$, using the fixed-point iteration method in the interval $[2, 3]$ with $x_0 = 2.4$ to a 5% error.

4. Find the real root of $2 - 4x + \cos x = 0$, by Newton Raphson method up to four decimal places, assuming $x_0 = 0.5$

5. Use the Newton-Raphson method and the Secant method to find solutions accurate to within 10^{-4} for each of the following problems, starting with the given x_0.

   1. $x^3 + 3x^2 - 1 = 0, \quad x_0 = -3.$
   2. $4e^{-x} \sin x - 1 = 0, \quad x_0 = 0.336.$
   3. $\ln(x - 1) + \cos(x - 1) = 0, \quad x_0 = 1.3.$
   4. $2 \sin x - 2^{\frac{x}{4}} - 1 = 0, \quad x_0 = -5.$

6. Newton-Raphson method is to be applied for approximating a root of the nonlinear equation $x^4 - x - 10 = 0$.

   (a) How many solutions of the nonlinear equation are there in $[1, \infty)$? Are they simple?

   (b) Find an interval $[1, b]$ that contains the smallest positive solution of the nonlinear equation.

   (c) Compute five iterations of Newton-Raphson method, for each of the initial guesses $x_0 = 1, \quad x_0 = 2, \quad x_0 = 100$. What are your observations?

7. Find the multiplicity of each given zero of $f(x) = 0$. Hence, use the Modified Newton-Raphson's method to approximate the same zero starting with the given x_0.

   (a) $f(x) = (x+1)^3,\ x = -1,\ x_0 = 0.$ (b) $f(x) = (x-1)(e^{x-1}-1),\ x = 1,\ x_0 = 0.$

   (c) $f(x) = \left(x - \dfrac{\pi}{2}\right)^2 (\cot x - 1),\ x = \dfrac{\pi}{4},\ x_0 = \dfrac{\pi}{6}.$