The University of Zambia
Department of Mathematics and Statistics
MAT 4119 - Engineering Mathematics III
Tutorial Sheet 5

2024

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1. For the given functions $f(x)$, let $x_0 = 0, \quad x_1 = 0.6$, and $x_2 = 0.9$. Construct Lagrange interpolation polynomials of degree at most two to approximate $f(0.45)$, and find the absolute error.

   (a) $f(x) = \cos x$ (b) $f(x) = \sqrt{x+1}$ (c) $f(x) = \ln(x+1)$ (d) $f(x) = \tan x$

2. Let $f(x) = \sqrt{x - x^2}$ be a function and $P_2(x)$ be the Lagrange interpolation polynomial on $x_0 = 0, \quad x_1$ and $x_2 = 1$. Find the largest value of the number $x_1 \in (0, 1)$ for which $f(0.5) - P_2(0.5) = -0.25$.

3. Use the Newton forward-difference formula to construct interpolating polynomials of degree one, two, and three for the following data. Approximate the specified value using each of the polynomials.

   (a) $f(0.43)$ if $f(0) = 1, f(0.25) = 1.64872, f(0.5) = 2.71828, f(0.75) = 4.48169$
   (b) $f(0.18)$ if $f(0.1) = -0.29004986, f(0.2) = -0.56079734, f(0.3) = -0.81401972$ and $f(0.4) = -1.0526302$

4. Use the Newton backward-difference formula to construct interpolating polynomials of degree one, two, and three for the following data. Approximate the specified value using each of the polynomials.

5. Using $\sin(0.1) = 0.9983$ and $\sin(0.2) = 0.19867$, find an approximate value of $\sin(0.18)$ by Lagrange interpolation. Obtain a bound on the truncation error.

6. Evaluate $\sqrt{155}$ by using Lagrange interpolation formula from the following data:

x	150	152	154
y=$\sqrt{x}$	12.247	12.239	12.410

7. The following table gives the normal weight of a baby during the first six months of life. Use Newton's divided difference formula to estimate the weight of the baby at the age of 4 months.

Age in months	0	2	3	5	6
Weight in Kg	3.0	3.7	4.5	5.2	6.3

8. Approximate $f(0.05)$ and $f(0.65)$ using the following data and the Newton forward- or backward- difference formula:

x	0.0	0.2	0.4	0.6	0.8
f(x)	1.00000	1.22140	1.49182	1.82212	2.22554

9. Using Newton forward- or backward- interpolation find the annual premium at the ages of 27 and 33 from the following data:

Age in years	24	28	32	36	40
Annual premium	28.06	30.19	32.75	34.94	40

10. Given the data $(0, 0), (1, 1), (2, 4)$, fit a quadratic spline to the function f defined by this set of points and use it to approximate $f(1.5)$.

11. Given the data $f(0) = 0, f(1) = 1, f(2) = 2$,
    (a) construct the natural cubic spline S that interpolates the data.
    (b) determine the clamped cubic spline S that interpolates the data and satisfies $S'(0) = S'(2) = 1$.

12. A natural cubic spline S on $[0, 2]$ is defined by

$$\begin{cases} 1 + 2x - x^3, & \text{if } x \in [0, 2] \\ 2 + b(x-1) + c(x-1)^2 + d(x-1)^3, & \text{if} x \in [1, 2]. \end{cases}$$

    Find the values of b, c, and d.