The University of Zambia

School of Natural Sciences

Department of Mathematics & Statistics

MAT 4119 - Engineering Mathematics III

Tutorial Sheet 3 23/08/2024

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1. Use Newton's method to find a solution to the following nonlinear systems in the given domain. Iterate until $\left\|\boldsymbol{X}^{(k)}-\boldsymbol{X}^{(k-1)}\right\|_\infty < 10^{-6}$

   (a) $3x_1^2 - x_2^2 = 0,\qquad 3x_1x_2^2 - x_1^3 - 1 = 0,\qquad$ Use $\boldsymbol{X}^{(0)} = (1,1)^t$

   (b) $\ln\left(x_1^2 + x_2^2\right) = \sin(x_1x_2) + \ln 2\pi,\qquad e^{x_1-x_2} + \cos(x_1x_2) = 0.\qquad$ Use $\boldsymbol{X}^{(0)} = (2,2)^t$

   (c) $x_1^3 + x_1^2x_2 - x_1x_3 + 6 = 0,$
   $e^{x_1} + e^{x_2} - x_3 = 0,$
   $x_2^2 - 2x_1x_3 = 4.$
   . Use $\boldsymbol{X}^{(0)} = (-1,-2,1)^t.$

   (d) $9x_2 + \sqrt{x_1^2 + \sin x_3 + 1.06} = -0.9,$
   $6x_1 - 2\cos\left(x_2x_3\right) - 1 = 0,$
   $60x_3 + 3e^{-x_1x_2} = 3 - 10\pi$
   Use $\boldsymbol{X}^{(0)} = (0,0,0)^t$

2. Solve the following systems of linear equations using Gauss elimination method:

   (a) $x_1 + x_2 - x_3 = 3$
   $2x_1 - x_2 + 3x_3 = 0$
   $-x_1 - 2x_2 + x_3 = -5$

   (b) $x_1 + x_2 - x_3 = 3$
   $-x_1 + x_2 + x_3 = 2$
   $x_1 + 3x_2 - x_3 = 8.$

3. Use Gauss elimination with scaled partial pivoting and three-digit rounding arithmetic to solve the following systems of linear equations and compare with the actual solution:

(a) $3.3330x_1 + 15920x_2 + 10.333x_3 = 7959$
$2.2220x_1 + 16.710x_2 + 9.6120x_3 = 0.965$
$-1.5611x_1 + 5.1792x_2 - 1.6855x_3 = 2.714$
Actual solution: $(1, 0.5, -1)^t$

(b) $\pi x_1 + \sqrt{2}x_2 - x_3 + x_4 = 0$
$e^1 x_1 - x_2 + x_3 + 2x_4 = 1$
$x_1 + x_2 - \sqrt{3}x_3 + x_4 = 2$
$-x_1 - x_2 + x_3 - \sqrt{5}x_4 = 3$
Actual solution: $(1.35, -4.68, -4.03, -1.66)^t$.

4. Carry out the first three iterations of the Jacobi's Method and Gauss-Siedel Method for the following systems using $X^{(0)} = (0, 0, 0)^t$:

(a) $2x_1 - x_2 + x_3 = -1$
$3x_1 + 3x_2 + 9x_3 = 0$
$3x_1 + 3x_2 + 5x_3 = 4$

(b) $2x_2 + 4x_3 = 0$
$x_1 - x_2 - x_3 = 0.375$
$x_1 - x_2 + 2x_3 = 0$

5. The nonlinear system

$$x^2 + y = 11, \quad y^2 + x = 7$$

has four solutions.

(a) Approximate the solutions graphically.

(b) Use the approximations from part (a) as initial approximations for an appropriate function iteration, and determine the solutions to within 10^{-5} in the l_∞ norm.

6. The nonlinear system

$$x_1^2 - 10x_1 + x_2^2 = -8, \quad x_1 x_2^2 + x_1 - 10x_2 + 8 = 0$$

can be transformed into the fixed-point problem

$$x_1 = g_1(x_1, x_2) = \frac{x_1^2 + x_2^2 + 8}{10}, \quad x_2 = g_2(x_1, x_2) = \frac{x_1 x_2^2 + x_1 + 8}{10}.$$

(a) Show that $G = (g_1, g_2)^t$ mapping D into $\mathbb{R}^2$ has a unique fixed point in

$$D = \{X = (x_1, x_2)^t : 0 \leq x_i \leq 1.5, i = 1, 2\}.$$

(b) Apply functional iteration to approximate the solution.

7. The nonlinear system

$$5x_1^2 - x_2^2 = 0, \quad x_2 - 0.25(\sin x_1 + \cos x_2) = 0$$

has a solution near $(\frac{1}{4}, \frac{1}{4})^t$.

(a) Find a function G and a set D in $\mathbb{R}^2$ such that $G : D \to \mathbb{R}^2$ and G has a unique fixed point in D.

(b) Apply functional iteration to approximate the solution to within 10^{-5} in the l_∞ norm.

8. Use Newton's method with the given $X^{(0)}$ to compute $X^{(2)}$ for each of the following nonlinear systems:

(a) $\sin(4\pi x_1 x_2) - 2x_2 - x_1 = 0$

$$\left(\frac{4\pi - 1}{4\pi}\right)\left(e^{2x_1} - e^1\right) + 4e^1 x_2^2 - 2e^1 x_1 = 0$$

$X^{(0)} = (0, 0)^t$

(b) $x_1^3 + x_1^2 x_2 - x_1 x_3 + 6 = 0$
$e^{x_1} + e^{x_2} - x_3 = 0$
$x_2^2 - 2x_1 x_3 = 4$

$X^{(0)} = (-1, -2, 1)^t.$