

**The University of Zambia**  
**Department of Mathematics and Statistics**  
**MAT 4119 - Engineering Mathematics III**  
**Tutorial Sheet 7**

2024

1. Use the Trapezoidal and Simpson's rules to approximate the following integrals. Compare the approximations to the actual value and find a bound for the error in each case, if possible.

(a)  $\int_1^2 \ln x \, dx$       (b)  $\int_0^{\frac{\pi}{3}} \cos^2 x \, dx$       (c)  $\int_0^{\frac{\pi}{4}} e^{3x} \sin 2x \, dx$       (d)  $\int_1^{1.5} x^2 \ln x \, dx$

2. Use the table below to find an approximation to  $\int_{1.1}^{1.5} e^x dx$ , using:

- (a) the Trapezoidal rule with  $x_0 = 1.1$  and  $x_1 = 1.5$ ;  
(b) Simpson's rule with  $x_0 = 1.1$ ,  $x_1 = 1.3$ , and  $x_2 = 1.5$ .

$x$	1.1	1.3	1.5
$e^x$	3.0042	3.6693	4.4817

3. Approximate the following integrals using Composite Trapezoidal rule:

(a)  $\int_0^2 \frac{2}{x^2+4} dx$ ;  $n = 6$       (b)  $\int_0^{\frac{3\pi}{8}} \tan x \, dx$ ;  $n = 8$       (c)  $\int_e^{e+1} \frac{1}{x \ln x} dx$ ;  $n = 8$ .

- (d) Determine the value of  $n$  and of  $h$  required to approximate the integrals in (a) to (c) to within  $10^{-5}$

4. Approximate the following integrals using Composite Simpson's rule:

(a)  $\int_3^5 \frac{2}{\sqrt{x^2-4}} dx$ ;  $n = 8$       (b)  $\int_0^{\pi} x^2 \cos x \, dx$ ;  $n = 6$       (c)  $\int_{0.75}^{1.75} (\sin^2 x - 2x \sin x + 1) dx$ ;  $n = 8$ .

- (d) Determine the value of  $n$  and of  $h$  required to approximate the integrals in (a) to (c) to within  $10^{-5}$

5. Suppose that  $f(0) = 1$ ,  $f(0.5) = 2.5$ ,  $f(1) = 2$  and  $f(0.25) = f(0.75) = \alpha$ . Find  $\alpha$  if the Composite Trapezoidal rule with  $n = 4$  gives the value 1.75 for  $\int_0^1 f(x) \, dx$ .