The University of Zambia

School of Natural Sciences

Department of Mathematics & Statistics

2020/2021 Academic Year

End of Year Final Examinations

# MAT 4119 - Engineering Mathematics III

Time allowed : **Three (3) hours** **Full marks : 100**

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**Instructions:**

- Indicate your **computer number** on all answer booklets.
- There are seven (7) questions in this examination. Attempt **any five (5)** questions. All questions carry equal marks.
- **Full credit** will only be given when **all necessary work** is shown.

*This question paper consists of 6 pages.*

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1. (a). The Taylor polynomial of order one for the function

$$f(x) = \frac{1}{3+3^x}$$

about $x_0 = 1$ is

$f(x_0)$ $f'(x_0)(x-x_0)$

$$P_1(x) = \frac{1}{6} - \frac{\ln 3}{12}(x-1).$$

Use $P_1(x)$ to approximate $f(\log_3 2)$. [4]

(b) (i) Find the actual error involved in using $P_1(x)$ to approximate $f(\log_3 2)$. [2]

(ii) Find an upper bound of the error in part (a). [7]

(c) Let $f(x) = x^4 + x - 2$. Determine the number of iterations necessary to solve $f(x) = 0$ by bisection method with accuracy within $10^{-3}$ in the interval $[0.5, 1.5]$. [7]

**Turn Over/...**

2. (a) (i) By sketching the graphs of

$$y = \arcsin(x-1) \text{ and } y = e^{-x}$$

on the same coordinate system, show that there is a root of the equation

$$\arcsin(x-1) - e^{-x} = 0$$ [3]

(ii) Starting with $P_0 = 1$, use the Newton-Raphson method to solve the equation in part (i) to 1 significant figure. [5]

(b) The function

$$f(x) = \tan^{-1}(x-1) + x - x^2$$

has a zero at $x = 1$.

(i) Determine the multiplicity of the zero $x = 1$. [3]

(ii) Use the modified Newton-Raphson method to perform one iteration to solve the equation $f(x) = 0$ starting with $P_0 = 0.5$. [4]

(c) The following is a quadratic spline interpolating $(-1,0)$, $(0,1)$ and $(1,3)$:

$$S(x) = \begin{cases} ax^2 + x + b, & x \in [-1,0] \\ cx^2 + x + 1, & x \in [0,1]. \end{cases}$$

Find the values of the real constants $a$, $b$ and $c$. [5]

$\cos^2 t + \sin^2 x =$

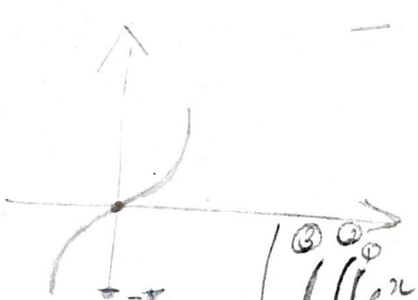
3. (a) Solve the following system of linear equations using Gauss elimination method:

$$x_1 + x_2 - x_3 = 3$$
$$2x_1 - x_2 + 3x_3 = 0$$
$$-x_1 - 2x_2 + x_3 = -5.$$

[8]

$\sec^2$

$-\left(\sin^{-1}(x-1) - e^{-x}\right.$



$-\frac{\pi}{2}$

$x^2 - 2x + 1$

$$\frac{\left(\left(\left(e^x \sin^{-1}(x-1)\right) - 1\right) \times \sqrt{2x - x^2}\right)}{\left(e^x + \sqrt{(\quad)}\right)}$$

Turn Over/...

(b) The system of non-linear equations

$$e^{x_1} + x_2 = 1 \text{ and } x_1^2 + x_2^2 = 4$$

can be written as a fixed-point problem on

$$D = \{X = (x_1, x_2)^t : -2 \le x_1 \le 2,\ -2 \le x_2 < 1\}.$$

One pair of equations is

$$x_1 = \ln(1 - x_2), \qquad x_2 = -\sqrt{4 - x_1^2}$$

and another pair is

$$x_1 = \ln(1 - x_2), \qquad x_2 = \frac{4 - x_1^2}{x_2}.$$

Given that the system has a solution near $(1, -1.7)^t$, determine which pair will converge to a solution and perform one iteration. [8]

(c) Suppose we wish to evaluate

$$\int_{0.25}^{1} x \sin\left(\frac{1}{x^2}\right) dx$$

using Trapezoidal rule. Find an upper bound of the error given that $\max\limits_{\xi \in [0.25, 1.00]} |f''(\xi(x))| \approx 1662.4.$ [4]

<u>NOTE</u>: Error function: $E(f) = -\frac{h^3}{12} f''(\xi(x))$

4. (a) The mesh analysis formulation of equations for a particular circuit of five resistors, driven by a 6 Volt battery gives the following equations:

$$10\,000 I_1 - 6\,000 I_3 = -6$$

$$12\,000 I_2 - 3\,000 I_3 = 6$$

$$-6\,000 I_1 - 3\,000 I_2 + 21\,000 I_3 = 0,$$

where $I_i$, $i = 1, 2, 3$ is current. Starting with $I^{(0)} = (0, 0, 0)^t$, use Gauss-Seidel iterative method with five-decimal places rounding to approximate $I_i$ until

$$\frac{||I^{(k)} - I^{(k-1)}||_\infty}{||I^{(k)}||_\infty} < 10^{-1}$$

[8]



Turn Over/...

**(b)** Laboratory tests on a flocculant suspension in a settling column give the following values of the percentage of solids removed ($S$) as a function of the sampling time ($t$):

300 1380 1980 2940

| $t$(min) | 5 | 23 | 33 | 49 |
|---|---|---|---|---|
| $S$(%) | 9 | 57 | 64 | 68 |

Find the percent removal for detention time of 660 seconds using a Lagrange interpolating polynomial. [7]

**(c)** A number $P^*$ approximates the number $P$ to 3 significant figures and is such that the absolute error is 0.001. Find the range of values of $P$. [5]

5. **(a)** A structural engineer is designing a temporary structure that will be in place for one year. To account for wind load in the design, the engineer uses the 34−year past record of annual maximum wind speed with the mean value ($\bar{Y}$) of 85 $km/hr$ and standard deviation ($S_Y$) of 20. The engineer decides to use a wind speed of 116 $km/hr$ for the design wind loading. The 34−year record suggests that the annual maximum wind speed is normally distributed, so the $z$−value is 1.55. The engineer wishes to estimate the probability that the design loading will exceed 116 $km/hr$ and the normal probabilities he has available for $z$ values from 1.5 to 2.0 are shown in the table below:

| $z$ | 1.5 | 1.6 | 1.7 | 1.8 |
|---|---|---|---|---|
| $f(z)$ | 0.0668 | 0.0548 | 0.0446 | 0.0359 |

Use the appropriate Newton difference formula to estimate the probability for $z = 1.55$. [8]

**(b)** The table below shows some values of $f(x) = x\sin\left(\frac{1}{x^2}\right)$:

| $x$ | 0.25 | 0.5 | 0.75 | 1.00 |
|---|---|---|---|---|
| $f(x)$ | -0.0728 | -0.378 | 0.734 | 0.841 |

Approximate the value of $f''(0.75)$. [5]

(c) The system reliability function $R(t)$, is given by $R(t) = 1 - F(t)$, where

$$F(t) = \int f(t)\,dt$$

is the probability that the component will fail in the interval from 0 to $t$, and $f(t)$ is the probability density of the time to failure of the system. Estimates of $f(t)$ are given below:

| $t$(days) | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| $f(t)$ | 0.33 | 0.24 | 0.17 | 0.12 | 0.09 | 0.06 | 0.04 | 0.03 | 0.02 | 0.01 |

Find the reliability function at $t = 6$ days. [7]

6. (a) Show that the Initial-Value Problem (IVP)

$$\frac{dy}{dt} = t^2 y + 1, \quad 0 \le t \le 1, \; y(0) = 1$$

is well-posed on $D = \{(t, y) : 0 \le t \le 1, \; -\infty < y < \infty\}$. [4]

(b) Use the Runge-Kutta method of order four to solve the IVP in part (a) at $y(0.2)$ using 5 subintervals of $[0, 1]$ [8]

(c) Use Euler's method to solve the IVP in part (a) at $y(0.8)$ using 5 subintervals of $[0, 1]$ [8]

7. (a) Evaluate each of the following, giving your answer in the form $a + ib$, $a, b \in \mathbb{R}$:

(i) $Log(-i)^i$ (ii) $\sin\left(\frac{\pi}{3} + i\sqrt{3}\right)$ (iii) $\tanh(-i)$ [2, 2, 2]

(b) (i) Evaluate the following limit:

$$\lim_{z \to -\pi i} e^{\frac{z^2 + \pi^2}{z + \pi i}}$$

[3]

(ii) Determine the set where the function

$$f(x + iy) = (x^3 - 3xy^2 - x + 1) + i(3x^2 y - y^3 - y)$$

is analytic and find the derivative. [4]

Turn Over/...

(c) Evaluate each of the following:

(i) $\int_C (\bar{z}+1)\,dz,$

where $C$ is a smooth arc $y = x^3$ from $z = 0$ to $z = \sqrt[3]{\pi}+i\pi$ [4]

(ii) $\oint_C \frac{e^{z^2}}{z^3}\,dz,$

around a positively oriented contour $C$, given by $\{z \in \mathbb{C} : |z| = 5\}$ [3]

**END OF EXAMINATION!**

MAT 4119

$f(x) = \frac{1}{3+3^x}$ $x_0 = 1$ $P_1(x) = \frac{1}{6} - \frac{\ln 3}{12}(x-1)$

- $P_1(x)\ f\left(\frac{\log 2}{\log 3}\right) = \frac{1}{6} - \frac{\ln 3}{12}\left(\frac{\log 2}{\log 3} - 1\right) = 0.200455425$

b) ii) $f(\log_3 2) = \frac{1}{(3+3^{\log_3 2})} = \frac{1}{3+2} = \frac{1}{5}$

$\left|\frac{1}{5} - 0.200455425\right| = 0.000455426$

iii) $f(x) = \frac{1}{3+3^x} = (3+3^x)^{-1}$

$f'(x) = -1 \cdot 3^x \ln 3\ (3+3^x)^{-2}$

$f''(x) = -\ln 3\left(3^x \ln 3\ (3+3^x)^{-2} + 3^x(3^x \ln 3)(3+3^x)^{-3}(-2)\right)$

$= -\ln 3\left(3^x \ln 3\ (3+3^x)^{-2} + 3^{2x}\ln 3\ (3+3^x)^{-3}(-2)\right)$

$= (\ln 3)^2\left(-3^x(3+3^x)^{-2} + 3^{2x}(3+3^x)^{-3}\right)$

$= (\ln 3)^2\left(\frac{-3^x(3+3^x) + 3^{2x}}{(3+3^x)^3}\right)$

$= (\ln 3)^2 \frac{\left(-3^{x+1} - \cancel{3^{2x}} + \cancel{3^{2x}}\right)}{(3+3^x)^3}$

$= -\frac{3(\ln 3)^2 \cdot 3^x}{(3+3^x)^3}$

$R_1(x) = \left|\frac{-3(\ln 3)^2 \cdot 3^{\xi(x)}}{2(3+3^{\xi(x)})^3}(\log_3 2 - 1)\right|$ $\log_3 2 \leq \xi \leq 1$

$\xi(x) = \log_3 2$ $\left|\frac{-3(\ln 3)^2 \cdot 2}{2(3+2)^3}(\log_3 2 - 1)\right| = \cancel{0.02526694}$ $0.010690774$

$\xi(x) = 1$ $\left|\frac{-3(\ln 3)^2 \cdot 3}{2(3+3)^3}(\log_3 2 - 1)\right| = 0.009280186$

$R_3(x) \leq 0.010690774$

Chlm

c) $f(x) = x^4 + x - 2$

$\frac{b-a}{2^n} < 10^{-3}$

$\frac{1.5 - 0.5}{2^n} < 10^{-3}$

$\frac{1}{2^n} < 10^{-3}$

$\frac{1}{10^{-3}} < 2^n$

$1000 < 2^n$

$\lg 1000 < n \lg 2$

$n > 9.9657842...$

$n \approx 10$-iterations

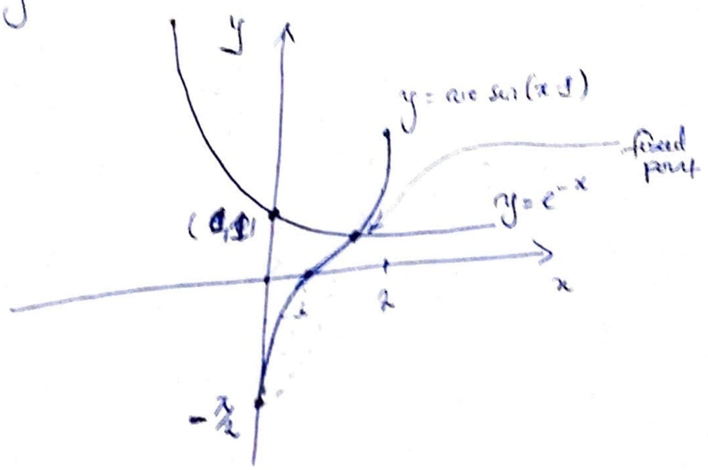
$f(0.5) = 0.5^4 + 0.5 - 2$
$= -1.4375$

$f(1.5) = 1.5^4 + 1.5 - 2$
$= 1.5625$

$f(1.5) f(0.5) < 0$

root between

2. a) i $y = \arcsin(x-1)$ and $y = e^{-x}$




ii) $P_0 = 1$

$f(x) = \sin^{-1}(x-1) - e^{-x}$

$f'(x) = \frac{1}{\sqrt{1-(x-1)^2}} + e^{-x}$

$P_1 = P_0 - \frac{f(P_0)}{f'(P_0)}$

$P_1 = 1 - \frac{\sin^{-1}(0) - e^{-1}}{\frac{1}{\;} + e^{-1}} = 1.268941421$

$\frac{|P_{pres} - P_{prev}|}{|P_{pres}|} < 10^{-5} \times 10^{-1}$

$\epsilon = 0.5$

$\frac{|1.268941421 - 1|}{1.268941421} < 0.5$

$0.21 < 0.5$

$P \approx \underline{\underline{1}}$

$f(x) = \tan^{-1}(x-1) + x - x^2.$

1

$f'(x) = \frac{1}{1+(x-1)^2} + 1 - 2x = \frac{1}{1+x^2-2x+1} + 1 - 2x.$

$f'(1) = 1 + 1 - 2 = 0$

$f''(x) = (-1)(2x-2)(x^2-2x+2)^{-2} - 2$

$f''(1) = -1(2-2)(1-2+2)^{-2} - 2$

$f''(1) = -2$

multiplicity of 2.

ii) ~~$P_n = P_{n-1} - f(P_{n-1}) f(P_{n-1})$~~

$$P_n = P_{n-1} - \frac{f(P_{n-1}) f'(P_{n-1})}{[f'(P_{n-1})]^2 - f(P_{n-1}) f''(P_{n-1})}$$

$f(0.5) = \tan^{-1}(0.5-1) + 0.5 - 0.5^2 = -0.213647609$

$f'(0.5) = \frac{1}{1+(0.5)^2} + 1 - 2(0.5) = 0.8.$

$f''(0.5) = (-1)(1-2)(0.5^2 - 1 + 2)^{-2} - 2.$

$= \frac{1}{1.5625} - 2 = 1.36$

~~$P_1 = P_0 \; 0.5 - (-0.213647609)(0.8)$~~

$$P_1 = 0.5 - \frac{(-0.213647609)(0.8)}{0.8^2 - (-0.213647609)(1.36)}$$

$= 0.5 + \frac{0.170918087}{0.930560748}$

$= \underline{0.683672143}$

$(-1, 0)$, $(0, 1)$

c)

$$S(x) = \begin{cases} ax^2 + x + b & x \in [-1, 0] \\ cx^2 + x + 1 & x \in [0, 1] \end{cases}$$

$S(1) = 3 = c + 1 + 1$

$c = 1$

$S_0(-1) = a - 1 + b = 0$

$S_0(0) = b = 1$

$S_1(0) = 1$

$S_1'(x) = 2cx + 1$

$a - 1 + 1 = 0$

$a = 0$

$S_0'(x) = 2ax + 1$

$S_0'(0) = 1$

$a = 0$

$b = 1$

$c = 1$

$$S(x) = \begin{cases} x + 1 & x \in [-1, 0] \\ x^2 + x + 1 & x \in [0, 1] \end{cases}$$

3. a)

$x_1 + x_2 - x_3 = 3$

$2x_1 - x_3 + 3x_2 = 0$

$-x_1 - 2x_2 + x_3 = -5$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 3 \\ 2 & -1 & 3 & 0 \\ -1 & -2 & 1 & -5 \end{array}\right]$$

$$\xrightarrow[r_3 \to r_3 + r_1]{r_2 \to r_2 - 2r_1} \left[\begin{array}{ccc|c} 1 & 1 & -1 & 3 \\ 0 & -3 & 5 & -6 \\ 0 & -1 & 0 & -2 \end{array}\right] \xrightarrow{r_3 \to 3r_3 - r_2} \left[\begin{array}{ccc|c} 1 & 1 & -1 & 3 \\ 0 & -3 & 5 & -6 \\ 0 & 0 & -5 & 0 \end{array}\right]$$

$-5x_3 = 0$

$x_3 = 0$

$-3x_2 + 5x_3 = -6$

$x_2 = 2$

$x_1 + x_2 - x_3 = 3$

$x_1 + 2 = 3$

$x_1 = 1$

$X = (1, 2, 0)$

$x_1 = \ln(1-x_2)$ $\quad x_2 = -\sqrt{4-x_1^2}$ $\quad D = \{x = (x_1, x_2)^t : -2 \le x_1 \le 2, -2 \le x_2 < 1\}$

$x_1 = g_1(x_2) = \ln(1-x_2)$

$g_1(-2) = \ln(1+2) = \ln 3 = 1.098612289 \in [-2, 2]$

$g_1(0.5) = \ln(0.5) = -0.69314718 \in [-2, 2]$

$\left|\cancel{\frac{\partial g_1}{\partial x_1}}\right| = 0$ $\quad x_2 = -\sqrt{4-x_1^2} = g_2(x_1)$

$g_2(-2) = -\sqrt{4-2^2} = 0 \in [-2, 1)$

$\frac{\partial g_2}{\partial x}$ $\quad g_2(2) = -\sqrt{4-2^2} = 0 \in [-2, 1)$

$g_2(1) = -\sqrt{4-1} = -1.732 \in [-2, 1)$ converges.

$g_2(x_1, x_2) = \frac{4-x_1^2}{x_2}$

$g_2(-2, x_2) = \frac{4-4}{x_2} = 0$

$g_2(-1, 0.5) = \frac{4-1}{0.5} = \frac{3}{0.5} = 6 \notin [-2, 1)$ does not converge on this interval

$(1, -1.7)^t$.

$x_1 = \ln(1-x_2)$ $\quad x_2 = -\sqrt{4-x_1^2}$

$x_1 = \ln(1+1.7)$ $\quad x_2 = -\sqrt{4-1}$

$= 0.993251773$ $\quad x_2 = -1.732$

$(0.993251773, -1.73050808)$

$10000\,I_1 - 6000\,I_3 = -6$ $\qquad \dfrac{\| I^{k} - I^{(k-1)} \|_\infty}{\| I^{(k)} \|_\infty} < 0.1$

$12000\,I_2 - 3000\,I_3 = 6$

$-6000\,I_1 - 3000\,I_2 + 21000\,I_3 = 0$

$a_{11} = 10\cdot000 \qquad a_{22} = 12000 \qquad a_{33} = 21000$

$$I_1 = \frac{(6000\,I_3 - 6)}{10000} = \frac{3}{5} I_3 - 0.0006$$

$$I_2 = \frac{(3000\,I_3 + 6)}{12000} = \frac{I_3}{4} + 0.0005$$

$$I_3 = \frac{6000\,I_1 + 3000\,I_2}{21000} = \frac{2\,I_1 + I_2}{7}$$

$$I_1^{(1)} = \tfrac{3}{5}(0) - 0.0006 = -0.0006.$$

$$I_2^{(1)} = \frac{0}{4} + 0.0005 = 0.0005 \qquad I = (-0.0006, 0.0005, -0.0001)^t$$

$$I_3^{(1)} = \frac{2(-0.0006) + 0.0005}{7} = -0.0001$$

$$\max \frac{\| (-0.0006 - 0),\ (0.0005 - 0),\ (-0.0001 - 0) \|_\infty}{\| -0.0006, 0.0005, -0.0001 \|_\infty} < 10^{-1}$$

$$1 > 0.1$$

$$I_1^{(2)} = \frac{3}{5}(-0.0001) - 0.0006 = -0.00066.$$

$$I_2^{(2)} = \frac{-0.0001}{4} + 0.0005 = 0.000475$$

$$I_3^{(2)} = \frac{2(-0.00066) + 0.000475}{7} = -0.000120714$$

$$\frac{\| (-0.00066 - (-0.0006)),\ (0.000475 - 0.0005),\ (-0.0001 - (-0.000120714)) \|_\infty}{\| -0.00066, 0.000475, -0.000120714 \|_\infty} < 0.1$$

$$\frac{0.00006}{0.00066} < 0.1 \qquad 0.09090909 < 0.1$$

$$I^{(2)} = (-0.00066, 0.000475, -0.000120714)^t$$

... test 2

$P_3(x) = f$

... os
... 2605
... 11 minutes

$$P_3(x) = \frac{9(11-23)(11-33)(11-49)}{(5-23)(5-33)(5-49)} + \frac{57(11-5)(11-33)(11-49)}{(23-5)(23-33)(23-49)} + \frac{64(11-5)(11-23)(11-49)}{(33-5)(33-23)(33-49)}$$

$$+ \frac{68(11-5)(11-23)(11-33)}{(49-5)(49-23)(49-33)}$$

$$= \frac{9(-10032)}{-22179} + \frac{57(5016)}{4680} + \frac{64(2736)}{-4480} + \frac{68(1584)}{18304}$$

$$= 4.070877857 + 61.09230769 - 39.08571429 + 5.884615385$$

$$P_3(11) = 31.96208664$$

$$= 32\%.$$

$$\frac{|P - P^*|}{|P|} \leq 5 \times 10^{-3} \qquad |P - P^*| = 0.001$$

$$0.2 + 0.001 = 0.201$$

$$0.2 - 0.001 = 0.199$$

$$\frac{0.001}{|P|} \leq 5 \times 10^{-3}$$

$$0.199 \leq P \leq 0.201$$

$$\frac{0.001}{5 \times 10^{-3}} \leq |P|$$

$$0.2 \leq |P|$$

5. a) $x = 1.55$ test 2

$$P_n(x) = P_n(x_0 + sh) = f(x_0) + \sum_{k=1}^{n} s(s-1)\ldots(s-k+1)h^k f[x_0, x_1, \ldots x_k]$$

$x_0 = 1.5$ $h = 0.1$ --- find $x_0$ and $h$.

$1.55 = 1.5 + s(0.1)$ --- determine $s$

$s = 0.5$ . . . (1)

| $x$ | $f(x)$ | 1st | 2nd | 3rd |
|---|---|---|---|---|
| 1.5 | $f(x_0)$ 0.0668 | | | |
| | | $f[x_0,x_1]$ −0.12 | | |
| 1.6 | $f(x_1)$ 0.0548 | | $f[x_0,x_1,x_2]$ 0.09 | |
| | | $f[x_1,x_2]$ −0.102 | | $f[x_0,x_1,x_2,x_3]$ −0.05 FL |
| 1.7 | $f(x_2)$ 0.0446 | | 0.075 $f[x_1,x_2,x_3]$ | |
| | | $f[x_2,x_3]$ −0.087 | | |
| 1.8 | $f(x_3)$ 0.0359 | | | |

$$P_3(1.5) = P_3(1.5 + 0.5(0.1)) = 0.0668 + (0.5)(0.1)(-0.12) + (0.5)(0.5-1)(0.1)^2(0.09) + (0.5)(0.5-1)(0.5-2)(-0.05)(0.1)^3$$

$$= 0.0668 + (-0.006) + (-0.000225) + 0.00001875$$

$$= \underline{0.0606}$$

b) $f(x) = x\sin(1/x^2)$ $\qquad f''(x_0) = \frac{1}{h^2}\left[f(x_0 - h) - 2f(x_0) + f(x_0 + h)\right]$

$h = 0.25$

$$f''(0.75) = \frac{1}{0.25^2}\left[-0.378 - 2(0.734) + 0.841\right]$$

$$f''(0.75) = \underline{-16.08}$$

c. Trapezoidal Rule

$$\int_a^b f(x)\,dx = h/2\left[f(a) + 2\sum_{j=1}^{n-1} f(x_j) + f(b)\right]$$

$$\int_0^6 f(t)\,dt = \frac{1}{2}\left[0.33 + 2(0.24 + 0.17 + 0.12 + 0.09 + 0.06) + 0.04\right]$$

$$F(t) = \int_0^6 f(t)\,dt = 0.865$$

$$R(t) = 1 - 0.865 = 0.135$$

Simpsons Rule

$$\int_a^b f(x)\,dx = h/3\left[f(a) + 2\sum_{j=1}^{n/2-1} f(x_{2j}) + 4\sum_{j=1}^{n/2} f(x_{2j-1}) + f(b)\right]$$

$$\int_0^6 f(t)\,dt = \frac{1}{3}\left[0.33 + 2(0.17 + 0.09) + 4(0.24 + 0.12 + 0.06) + 0.04\right]$$

$$= \frac{1}{3}\left[0.33 + 0.52 + 1.68 + 0.04\right]$$

$$= 0.8567$$

$$R(t) = 1 - 0.8567 = 0.143$$

$\frac{\partial f}{\partial}$

a) $\frac{dy}{dt} = t^2 y + 1, \; 0 \le t \le 1, \; y(0) = 1$

$$|f(t, y_1) - f(t, y_2)| < L\,|y_1 - y_2|$$

$$|f(t, y_1) - f(t, y_2)| = |(t^2 y_1 + 1) - (t^2 y_2 + 1)| = |t^2 y_1 - t^2 y_2| = |t^2|\,|y_1 - y_2|$$

$$|t^2_{y_1} - t^2_{y_2}| < |t^2|\,|y_1 - y_2| < 1\,|y_2 - y_2|$$

$L = 1$

~~As not a constant~~

or $\left|\frac{\partial f}{\partial y}\right| = |t^2| < 1$

L is not constant so the function is not well posed

b. $w_0 = \alpha$

$k_1 = hf(t_i, w_i)$

$k_2 = hf(t_i + h/2, w_i + \frac{1}{2}k_1)$

$k_3 = hf(t_i + h/2, w_i + \frac{1}{2}k_2)$

$k_4 = hf(t_{i+1}, w_i + k_3)$

$w_{i+1} = w_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

$y(0.2)$ $[0,1]$

$N = 5$ $y(0) = 1$

$\frac{dy}{dt} = t^2y + 1$

$t = 0$

$t = 0.1$

$h = \frac{1}{5} = 0.2$

$h/2 = \frac{0.2}{2} = 0.1$

$w_0 = 1$

$k_1 = 0.2\left(0^2(1) + 1\right) = 0.2$

$k_2 = 0.2\left(0.1^2(1 + 0.1) + 1\right) = 0.2022$

$k_3 = 0.2\left(0.1(1 + 0.1011) + 1\right) = 0.222022$

$k_4 = 0.2\left(0.2^2(1 + 0.222022) + 1\right) = 0.209776176$

$w_1 \approx y(0.2)$

$w_1 = 1 + \frac{1}{6}(0.2 + 2(0.2022) + 2(0.222022) + 0.209776176)$

$w_1 = 1.2097$

$f(t, y) = t^2y + 1$

c. $y(0.8)$ $h = 0.2$ $t_0 = 0$ $t_1 = 0.2$ $t_2 = 0.4$ $t_3 = 0.6$ $t_4 = 0.8$ $t_5 = 1$

$w_0 = 1$ $w_{i+1} = w_i + hf(t_i, w_i)$

$y(0.2) \approx w_1 = 1 + 0.2\left(0(1) + 1\right) = 1.2$

$y(0.4) \approx w_2 = 1.2 + 0.2\left(0.2^2(1.2) + 1\right) = 1.4096$

$y(0.6) \approx w_3 = 1.4096 + 0.2\left(0.4^2(1.4096) + 1\right) = 1.6547072$

$y(0.6) \approx w_3 = 1.6547072 + 0.2\left(0.6^2(1.6547072) + 1\right) = 1.973846118$

$y(0.8) \approx w_4 = 1.6547072 + 0.2\left(0.6^2(1.6547072) + 1\right)$

7.